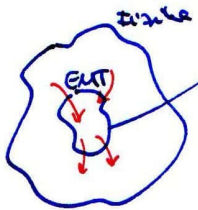


Zombory László: Elektromágneses terek

Bücs: A matematikai villamosmérnöki alkelemiszorral

Fitichev összefoglaló



elektromos és mágneses jelenségek + mérnöki megközelítés

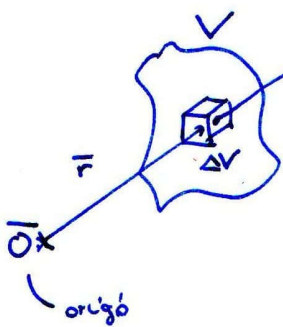
- matematikai modell
- fizikai mennyiségek és egyenletek

\downarrow (mértékszám) • (mértékegység)

Az elektromágneses tér közvetlen forrásai (töltés, áram)

- töltés: elemi egysége: $e = -1,6 \cdot 10^{-19} \text{ C}$
 huroktöltés: $\longrightarrow \uparrow$
 jele: Q
 egysége: $[Q] = \text{C} = \text{As}$

a) Térfogati töltéssűrűség:

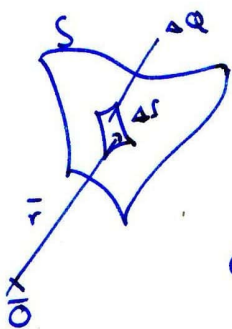


$$\lim_{\Delta V \rightarrow 0} \frac{\Delta Q}{\Delta V} \triangleq \rho(\vec{r})$$

$$[\rho] = \frac{\text{C}}{\text{m}^3}$$

$$Q_V = \int_V \rho(\vec{r}) dV$$

b) Felületi töltéssűrűség:

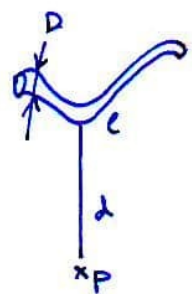


$$\lim_{\Delta S \rightarrow 0} \frac{\Delta Q}{\Delta S} \triangleq \sigma(\vec{r})$$

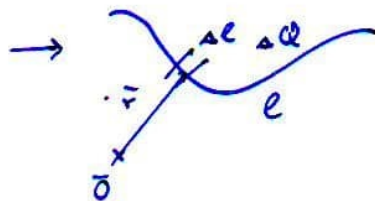
$$[\sigma] = \frac{\text{C}}{\text{m}^2}$$

$$Q_S = \int_S \sigma(\vec{r}) \cdot dS$$

c) Vona (matti) töltéssűrűség



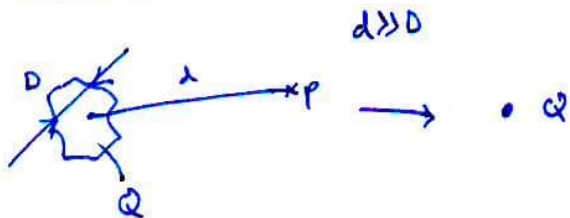
$c \gg D$
 $d \gg D$



$$\lim_{\Delta l \rightarrow 0} \frac{\Delta Q}{\Delta l} \triangleq q(F)$$

$$[q] = \frac{C}{m}$$

d) Ponttöltés

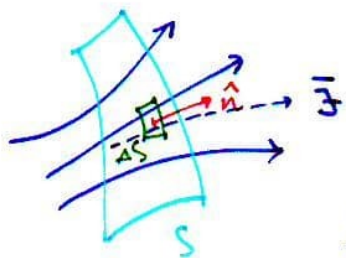


$d \gg D$

Áram

• töltéshordozók rendezett mozgása - $[I] = A$

a) Terfogatit áram sűrűség



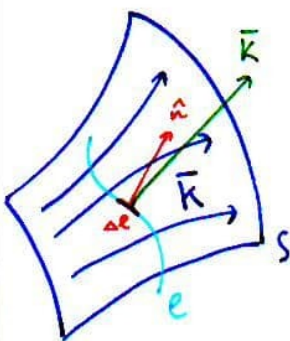
normális komponens:

$$J_n = \vec{J} \cdot \hat{n}$$

$$\lim_{\Delta S \rightarrow 0, \Delta t \rightarrow 0} \frac{\Delta Q}{\Delta S \cdot \Delta t} \triangleq J_n$$

össztörvény: $I_S = \int_S J_n \cdot dS = \int_S \vec{J} \cdot d\vec{S}$ ← felületelem vektora
 $d\vec{S} = dS \cdot \hat{n}$

b) Felületi áram sűrűség



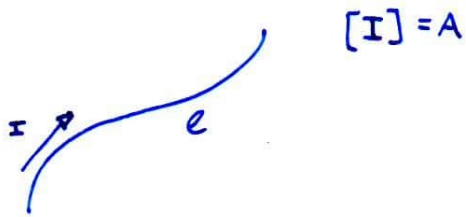
$$I_e = \int_e \vec{K} \cdot (\hat{n} \, d\ell) = \int_e K_n \, d\ell$$

$$K_n = \vec{K} \cdot \hat{n}$$

$$[K] = \frac{A}{m}$$

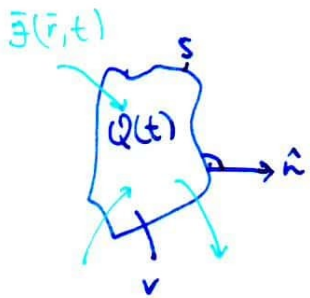
$$\lim_{\Delta e \rightarrow 0, \Delta t \rightarrow 0} \frac{\Delta Q}{\Delta e \cdot \Delta t} = K_n$$

c) Vonaláram



Töltésmegmaradás elve:

↳ folytonossági egyenlet



$$\frac{dQ(t)}{dt} = -I(t)$$

$$\frac{d}{dt} \int_V \rho(\vec{r}, t) dV = - \oint_S \vec{E}(\vec{r}, t) \cdot d\vec{S}$$

Folytonossági egyenlet
integrális alakja

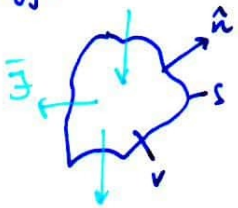
$$\rightarrow \frac{d}{dt} \int_V \rho(\vec{r}, t) dV = - \int_V \operatorname{div} \vec{E}(\vec{r}, t) dV$$

Gauss-tétel

$$\int_V \left(\frac{\partial}{\partial t} \rho(\vec{r}, t) + \operatorname{div} \vec{E}(\vec{r}, t) \right) dV = 0$$

$\equiv 0$ Folytonossági egyenlet (differenciális alak)

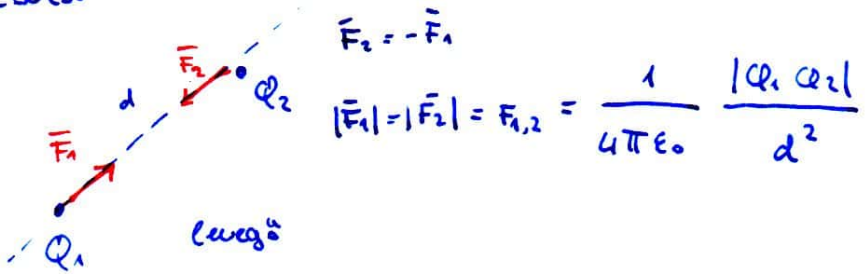
Megj.:



$$\operatorname{div} \vec{E} \hat{=} \lim_{V \rightarrow 0} \frac{\oint \vec{E} \cdot d\vec{S}}{V}$$

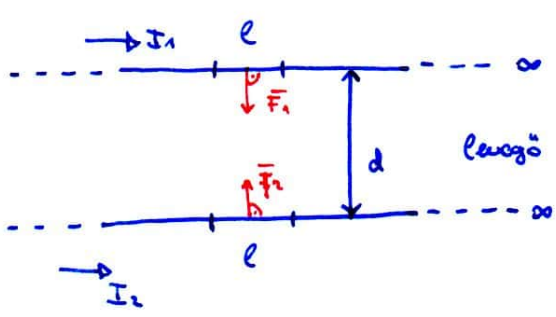
Erőhatások

a) Coulomb-tr.



ϵ_0 : vákuum permittivitása $\approx 8,854 \cdot 10^{-12} \frac{As}{Vm}$

b) Ampere-tr.



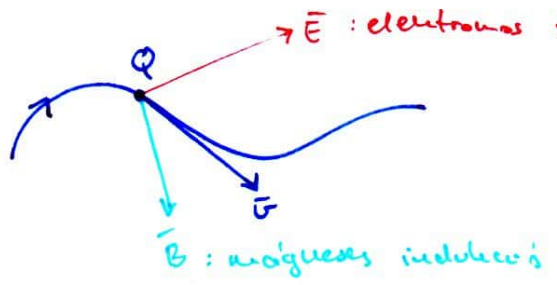
$$|\vec{F}_1| = |\vec{F}_2| = F_{12} = \frac{\mu_0 |I_1 \cdot I_2| l}{2\pi d}$$

μ_0 : vákuum permeabilitása

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$$

$I_1 \cdot I_2 > 0 \Rightarrow$ vonzó erő
 $I_1 \cdot I_2 < 0 \Rightarrow$ taszító erő

c) Lorentz-tr.



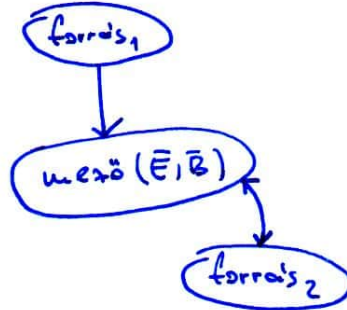
$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$

\vec{E}, \vec{B} : "álló" mezők

\vec{v} : elektron sebessége

re Erőhatások

- a) Coulomb-erő tv. } talvokkatas
- b) Ampere-tv. } talvokkatas
- c) Lorentz-erő → közzekatas



• E definíciója:

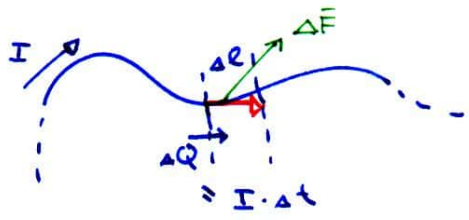
$$\vec{v} \equiv \vec{0} : \boxed{\vec{E} \triangleq \frac{\vec{F}}{Q}}$$

$$[E] = \dots = \frac{V}{m}$$

• B definíciója:

$$\vec{v} \equiv \vec{0} : \vec{F} = q\vec{v} \times \vec{B} \rightarrow \vec{B} = \dots$$

alternatív definíció:



$$\Delta Q \cdot \vec{v} = I \cdot \Delta t \cdot \vec{v}$$

Δl → vonalelem

$$\Delta \vec{F} = \Delta Q \cdot (\vec{v} \times \vec{B}) = I \cdot (\Delta \vec{l} \times \vec{B})$$

A7 elektromos és mágneses mező levezetése

a) Elektrosztatika Gauss-tétele

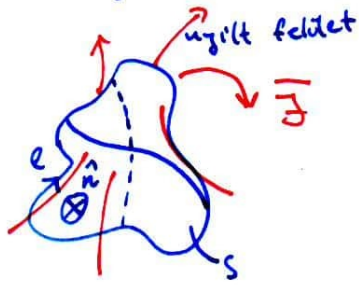


$$\int_V \rho(\vec{r}) dV = \oint_S \vec{D} \cdot d\vec{S}$$

↑
elektromos eltolás vektora

(levegő: $\vec{D} = \epsilon_0 \cdot \vec{E}$)

b) Gerjesztési törvény:



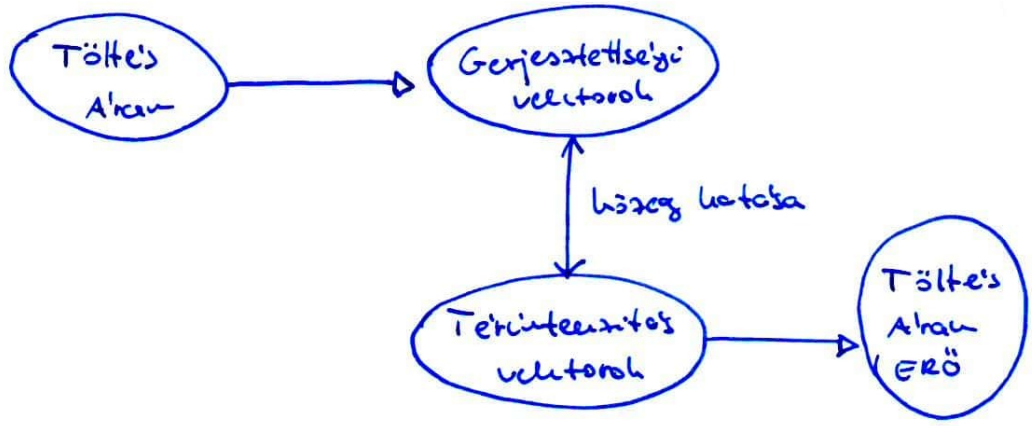
$$\int_S \vec{E} \cdot d\vec{S} = \oint_C \vec{H} \cdot d\vec{l}$$

↑
mágneses térerősség vektora

(Levegő + v, ami nem ferromágneses közeg: $\vec{B} = \mu_0 \cdot \vec{H}$)

\vec{E}, \vec{B} : térerősségi vektorok

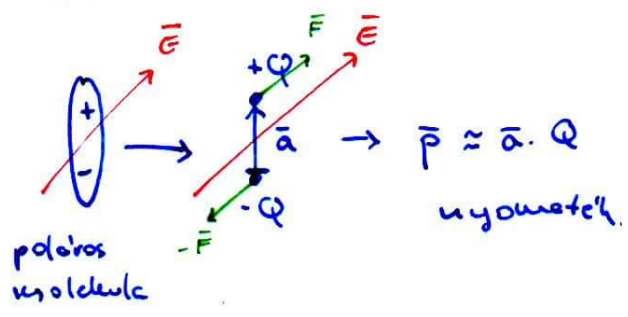
\vec{D}, \vec{H} : gerjesztési vektorok

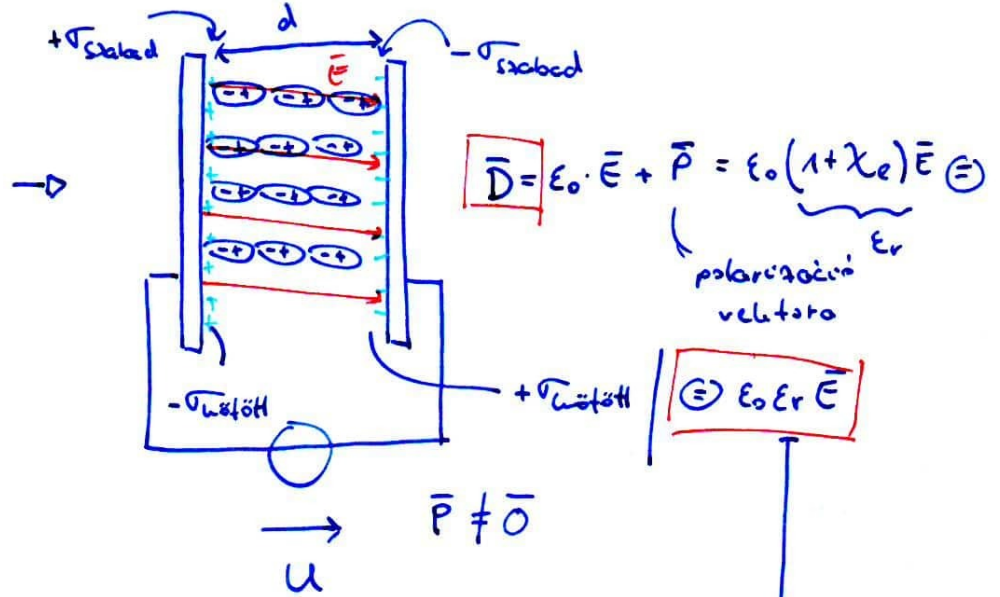
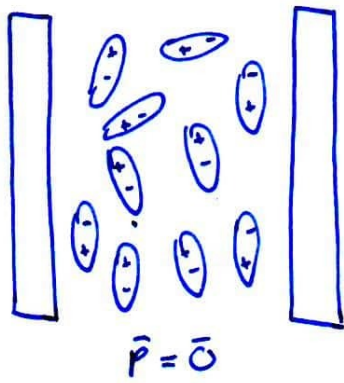


A közeg hatása

- Vákuumban, levegőben: $\vec{D} = \epsilon_0 \cdot \vec{E}$, $\epsilon_0 = 8,854 \cdot 10^{-12} \frac{As}{Vm}$
 $\vec{B} = \mu_0 \cdot \vec{H}$, $\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$

• Anyagban: Polarizáció lehet fel





$$\bar{D} = \epsilon_0 \cdot \bar{E} + \bar{P} = \epsilon_0 (1 + \chi_e) \bar{E} \quad \text{polarizáció vektora}$$

ϵ_r

$$\epsilon_0 \epsilon_r \bar{E}$$

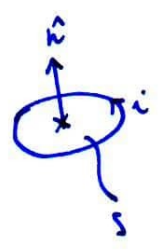
$$\bar{P} = \frac{\sum \bar{p}}{V}$$

$$\bar{P} = \epsilon_0 \cdot \chi_e \cdot \bar{E} \quad (1 + \chi_e = \epsilon_r : \text{relatív permittivitás})$$

elektromos susceptibilitás

• Mágneses polarizáció

elfordulásra képes részecskék



$$\bar{m} = \hat{n} \cdot S \cdot I$$

mágnesesség vektor

$$\bar{M} = \frac{\sum \bar{m}}{V}$$

$$\bar{B} = \mu_0 \cdot \bar{H} + \mu_0 \cdot \bar{M} \quad (\bar{H} = \frac{1}{\mu_0} \bar{B} - \bar{M})$$

$$\bar{M} = \chi_m \cdot \bar{H}$$

$$\Rightarrow \bar{B} = \mu_0 (1 + \chi_m) \bar{H} = \mu_0 \mu_r \bar{H}$$

mágneses susceptibilitás

relatív permeabilitás

lineáris
válasz

$\chi_m \gtrsim 0$: paramágnes

$\chi_m \lesssim 0$: diamágnes

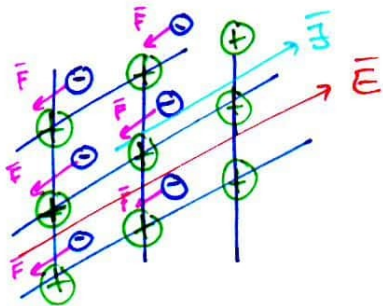
$\chi_m \gg 0$: ferromágnes

Egységei:

$$[D] = [P] = \frac{C}{m^2}$$

$$[H] = [M] = \frac{A}{m}$$

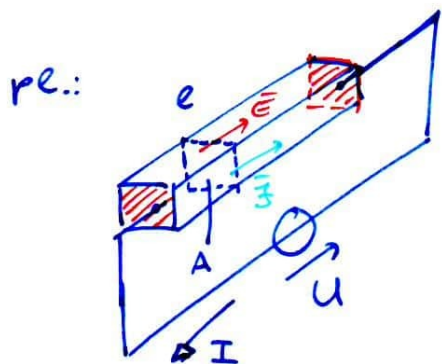
• Elektromos vezeték



$\vec{J} = \sigma \vec{E}$: differenciális Ohm-tör.

fajlagos vezetőképesség

$$[\sigma] = \frac{[J]}{[E]} = \frac{A/m^2}{V/m} = \frac{S}{m}$$



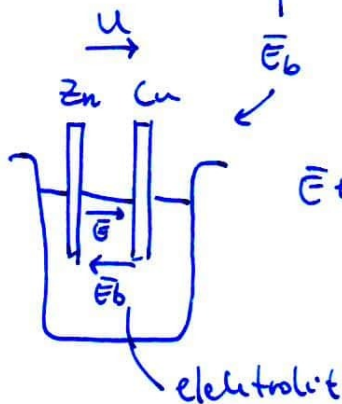
$$E = \frac{U}{l}$$

$$J = \sigma E \rightarrow I = J \cdot A$$

$$\left. \begin{array}{l} E = \frac{U}{l} \\ J = \sigma E \rightarrow I = J \cdot A \end{array} \right\} \frac{U}{I} = \frac{E \cdot l}{\sigma E A} = \frac{l}{\sigma A}$$

Beitárolt mennyiség:

$$\vec{J} = \sigma \cdot (\vec{E} + \vec{E}_b) + \vec{J}_b$$



\vec{J}_b : beiktatott áram-sűrűség / áram (Van de Graaf generator)

$$\vec{E} + \vec{E}_b = \vec{0} \Rightarrow \vec{J} = \vec{0}$$

A Maxwell-egyenletek teljes rendszere

Differenciális alak

Integrális alak

I. $\text{rot } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ $\xrightarrow{\text{Stokes}}$

$$\oint_e \vec{H} d\vec{l} = \int_s (\vec{J} + \frac{\partial \vec{D}}{\partial t}) d\vec{S}$$

II. $\text{rot } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$ $\xrightarrow{\text{Stokes}}$

$$\oint_e \vec{E} d\vec{l} = - \int_s \frac{\partial \vec{B}}{\partial t} d\vec{S}$$

III. $\text{div } \vec{B} = 0$ $\xrightarrow{\text{Gauss}}$

$$\oint_s \vec{B} d\vec{S} = 0 \left(\equiv \int_v \text{div } \vec{B} dV \right)$$

IV. $\text{div } \vec{D} = \rho$ $\xrightarrow{\text{Gauss}}$

$$\oint_s \vec{D} d\vec{S} = \int_v \rho dV$$

V. Konstansok és egyenletek

Lineáris, izotrop

(nem lineáris, anizotrop)

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$\vec{D} = \mathcal{D} \{ \vec{E} \}$$

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

$$\vec{B} = \mathcal{B} \{ \vec{H} \}$$

$$\vec{J} = \sigma (\vec{E} + \vec{E}_b) + \vec{J}_b$$

$$\vec{J} = \mathcal{J} \{ \vec{E} \}$$

VI.

$$w = \frac{1}{2} \vec{E} \cdot \vec{D} + \frac{1}{2} \vec{H} \cdot \vec{B} : \text{Energiasűrűség}$$

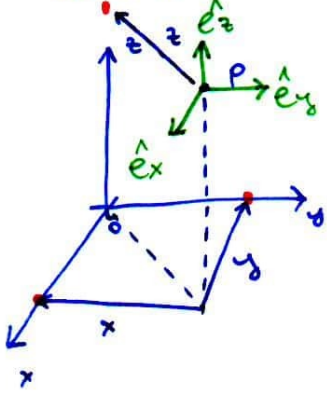
VII.

$$\vec{F} = q (\vec{E} + \vec{v} \times \vec{B}) : \text{Erőhatás}$$

Koordináta-rendszer

3D-s pont \leftrightarrow 3 „érték”

• Descartes:

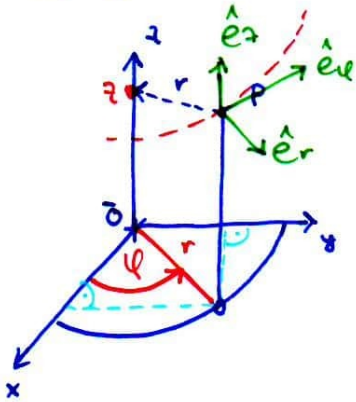


$$|\hat{e}_x| = |\hat{e}_y| = |\hat{e}_z| \equiv 1$$

$$\hat{e}_x \perp \hat{e}_y \perp \hat{e}_z$$

$$\left[\hat{e}_x \times \hat{e}_y = \hat{e}_z \right] : \text{jobbsodrosú koord. rendszer}$$

• Hengerkoordináta-rendszer



$$\hat{e}_z \perp \hat{e}_r \perp \hat{e}_\phi$$

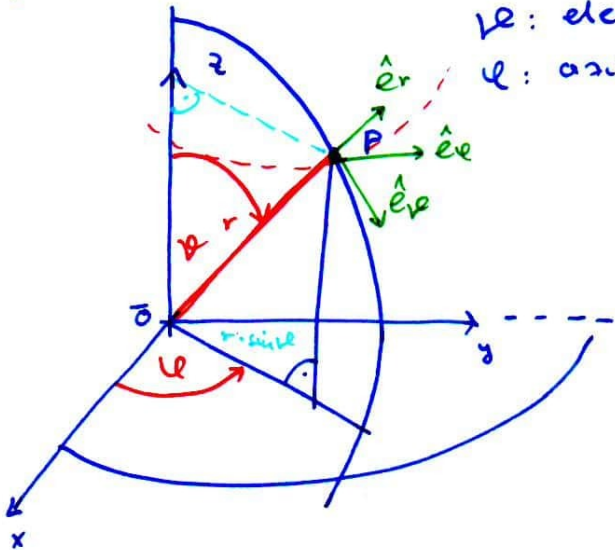
$$\hat{e}_z \times \hat{e}_r = \hat{e}_\phi : \text{jobbsodrosú koord. r.}$$

$$z = z$$

$$x = r \cdot \cos \phi$$

$$y = r \cdot \sin \phi$$

• Gömbkoordináta-rendszer



θ : elevációs szög

ϕ : azimutális szög

$$\hat{e}_r \perp \hat{e}_\theta \perp \hat{e}_\phi$$

$$\hat{e}_r \times \hat{e}_\theta = \hat{e}_\phi$$

$$x = (r \cdot \sin \theta) \cos \phi$$

$$y = (r \cdot \sin \theta) \sin \phi$$

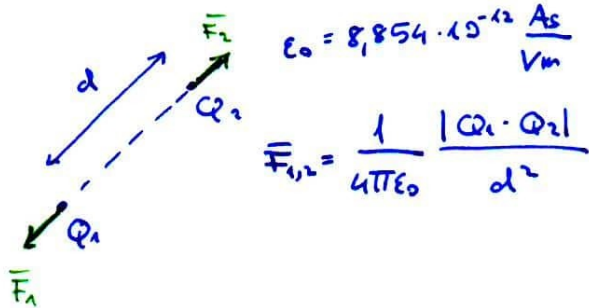
$$z = r \cdot \cos \theta$$

Elektrosztatika példatár: 2.1-2.7

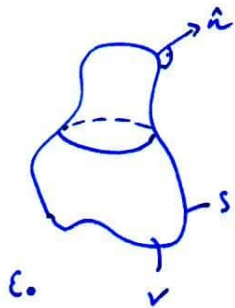
statisztika: nyugvó töltések: $\frac{\partial}{\partial t} \equiv 0$

↓
elektromos tereket képeznek $\vec{E} \triangleq \frac{\vec{F}}{Q} \leftarrow$ próbatöltés $[E] = \frac{V}{m}$

Coulomb-tör:

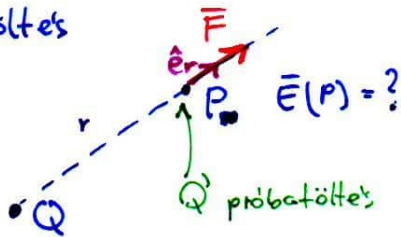


Gauss-tör:



$$\oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \sum_V Q \quad (\text{homogén közegben})$$

1 Ponttöltés



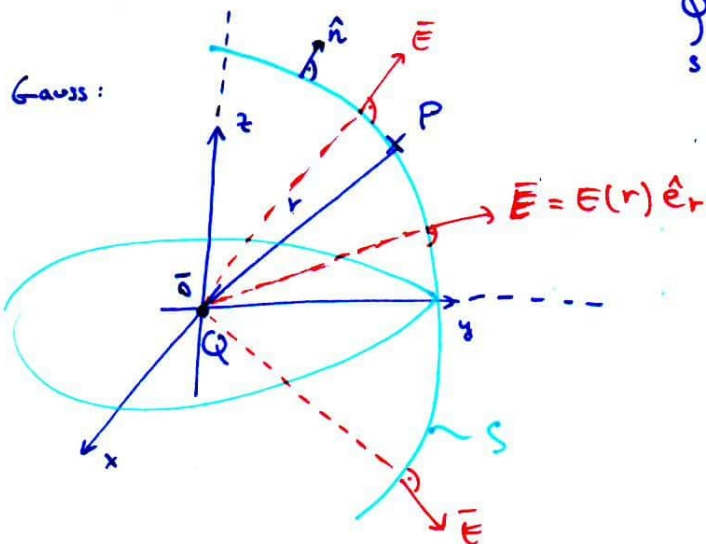
a) $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{|Q \cdot Q'|}{r^2} \cdot \hat{e}_r \quad (\text{Coulomb-tör})$

$\vec{E} = \frac{\vec{F}}{Q'} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r^2} \cdot \hat{e}_r$

b)

Levegő ϵ_0

Gauss:



$$\oint_S \vec{E} \cdot d\vec{S} = \oint_S (E(r) \hat{e}_r) \cdot (dS \cdot \hat{n}) =$$

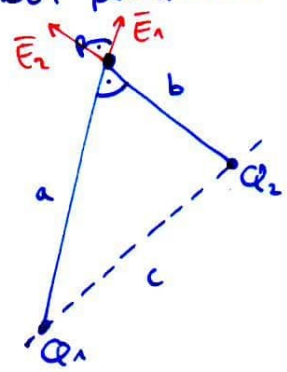
$$= \oint_S E(r) \underbrace{\hat{e}_r \cdot \hat{n}}_1 \cdot dS = E(r) \underbrace{\oint_S dS}_S =$$

gömbfelület

$$= E(r) \cdot 4r^2\pi =$$

$$= \frac{1}{\epsilon_0} Q \quad \Rightarrow \quad \underline{\underline{E(r) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r^2}}}$$

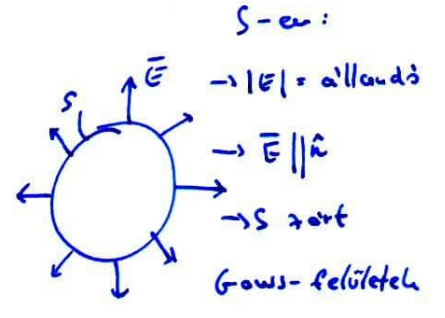
2 Két ponttöltés



$a = 4\text{m}$
 $b = 3\text{m}$
 $c = 5\text{m}$
 $Q_1 = 3\mu\text{C}$
 $Q_2 = 8\mu\text{C}$

$$|\vec{E}_1| = \frac{Q_1}{4\pi\epsilon_0} \cdot \frac{1}{a^2} = \dots$$

$$|\vec{E}_2| = \frac{Q_2}{4\pi\epsilon_0} \cdot \frac{1}{b^2} = \dots$$



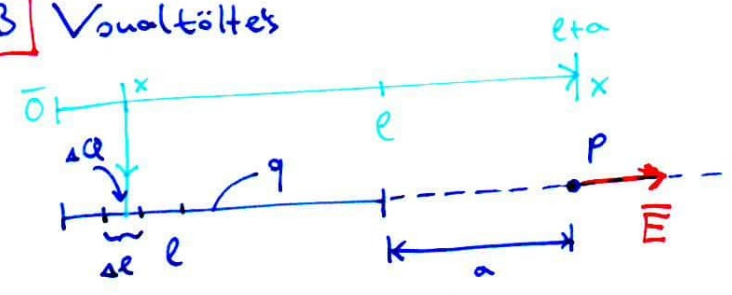
$$|\vec{E}|^2 = |\vec{E}_1|^2 + |\vec{E}_2|^2 \rightarrow |\vec{E}| = \underline{\underline{8,16 \frac{\text{kV}}{\text{m}}}}$$

ϵ_0 (levegő)

$$[E] = \frac{[Q]}{[\epsilon_0] \cdot [a]^2} = \frac{C = As}{\frac{As}{Vm} \cdot m^2} = \frac{V}{m}$$

superpozíció elve \Leftrightarrow lineáris hálózat

3 Vonaltöltés



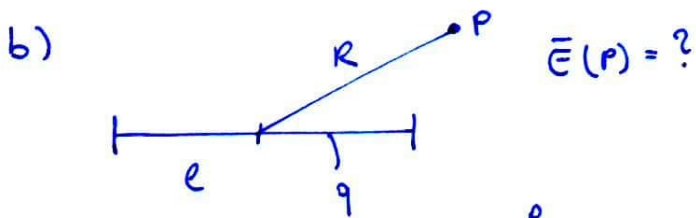
a) $\vec{E}(P) = ?$

$$\Delta E = \frac{\Delta Q}{4\pi\epsilon_0} \cdot \frac{1}{[(l+a)-x]^2} \xrightarrow{\rightarrow \infty} dE = \frac{dQ}{4\pi\epsilon_0} \cdot \frac{1}{(l+a-x)^2}$$

$$\underline{\underline{E}} = \int dE = \int_{x=0}^{x=l} \frac{dQ}{4\pi\epsilon_0} \cdot \frac{1}{(l+a-x)^2} = \frac{q}{4\pi\epsilon_0} \int_{x=0}^{x=l} \frac{dx}{(l+a-x)^2} =$$

$(dx \cdot q) = dQ$

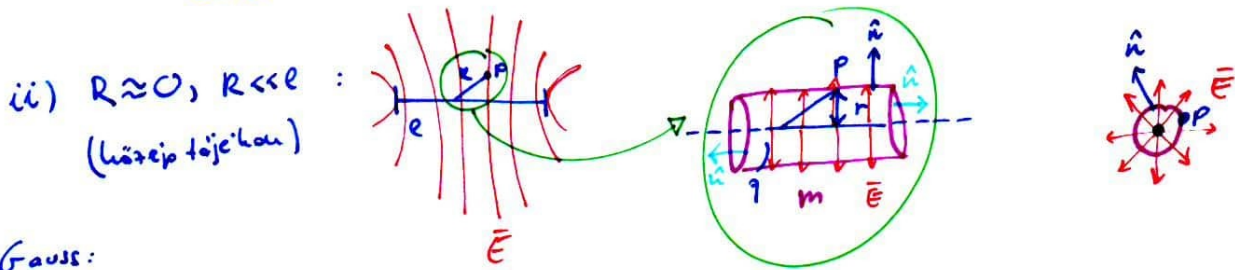
$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{l+a-x} \right]_0^l = \underline{\underline{\frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{l+a} \right)}}$$



Közéltételek:

i) $R \gg l$: $Q = l \cdot q$ $E \approx \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{R^2}$

ℳ Ell.: $\lim_{a \rightarrow \infty} \frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{l+a} \right) = \lim_{a \rightarrow \infty} \frac{q}{4\pi\epsilon_0} \frac{l}{a(l+a)} \approx \frac{q \cdot l}{4\pi\epsilon_0} \frac{1}{a^2 + al} \approx \frac{q \cdot l}{4\pi\epsilon_0} \cdot \frac{1}{a^2}$



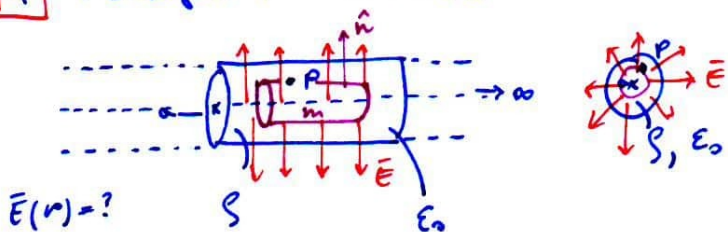
Gauss:

$$\oint \vec{E} d\vec{s} = \int_{S_{\text{polár}}} \vec{E} d\vec{s} + \int_{S_{\text{oldp}}} \vec{E} d\vec{s} + \int_{S_{\text{fedlap}}} \vec{E} d\vec{s} = \frac{1}{\epsilon_0} \cdot m \cdot q$$

$\underbrace{\int_{S_{\text{polár}}} \vec{E} d\vec{s} + \int_{S_{\text{oldp}}} \vec{E} d\vec{s} + \int_{S_{\text{fedlap}}} \vec{E} d\vec{s}}_{\phi}$
 \downarrow
 $E(r) \cdot m \cdot 2r\pi$

$\Rightarrow E(r) = \frac{q}{2\pi\epsilon_0} \cdot \frac{1}{r}$

4 Törésgáti töltéseloszlás

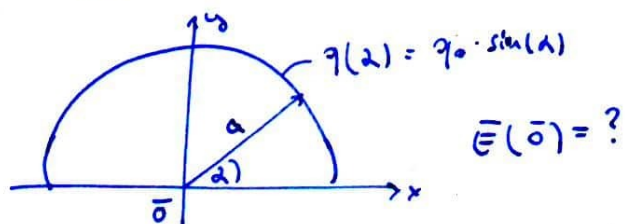


$$E(r) \cdot 2r\pi \cdot m = \frac{1}{\epsilon_0} \sum_V Q = \frac{\rho \cdot V}{\epsilon_0}$$

$$= \rho \cdot r^2\pi \cdot m \cdot \frac{1}{\epsilon_0}$$

$\Rightarrow E(r) = \frac{\rho \cdot r}{2\epsilon_0}$

HF:



A Maxwell-egyenletek

$$\text{rot } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{div } \vec{B} = 0$$

$$\text{div } \vec{D} = \rho$$

$$\vec{D} = \epsilon_0 \vec{E}, \vec{B} = \mu \vec{H}, \vec{J} = \sigma(\vec{E} + \vec{E}_B) + \vec{J}_0$$

beiktatott elektromos ter

$$w = \frac{1}{2} \vec{E} \cdot \vec{D} + \frac{1}{2} \vec{H} \cdot \vec{B} = \frac{1}{2} \epsilon |\vec{E}|^2 + \frac{1}{2} \mu |\vec{H}|^2$$



1. aszimmetria:

mágneses áram } ≠
mágneses töltés }

2. differenciális alak => lokális szemlélet

3. integrális alak => globális szemlélet

↳ allokáltság (differenciál egyenlet nem jó a diff. alak)

4. ellentmondásmentes, teljes

5. evolúciós egyenletek: jelenbeli állapotból lehet következtetni a jövőbelire

↳ kauzalitás

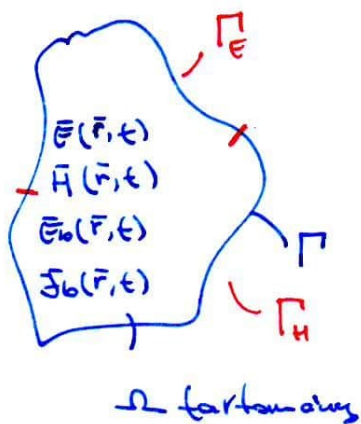
6. határok: statisztikus sokaság legyen (kvantum hatást nem érnek le az egyenletek)

7. posztulátumok (nem vesethető le a fizika más törvényeiből)

A₂ egyenletek egyértelmű megoldhatósága

$$\vec{J} = \sigma (\vec{E} + \vec{E}_b) + \vec{J}_b$$

forrás



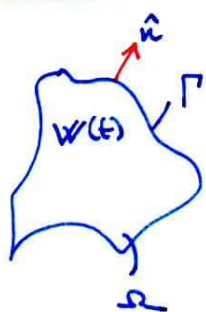
t_0 : kezdeti pillanat
 $t_1 > t_0$

Adott:

- $\vec{E}(\vec{r}, t_0), \vec{H}(\vec{r}, t_0), \vec{r} \in \Omega$: KEZDETI FELTÉTELEK
 - források: $\vec{E}_b(\vec{r}, t), \vec{J}_b(\vec{r}, t), \vec{r} \in \Omega, t \in [t_0, t_1]$: GERJESÍTÉSEK
 - $\boxed{\Gamma_E}$: $\vec{E}_t|_{\Gamma_E}(t)$
 - $\boxed{\Gamma_H}$: $\vec{H}_t|_{\Gamma_H}(t)$
- tangenciális komponensek időfüggése : PEREMFELTÉTELEK

$\Rightarrow \vec{E}(\vec{r}, t_1), \vec{H}(\vec{r}, t_1)$ a teljes tartományon egyértelműen meghatározható.

A₂ energiaáramlás



$w(t)$: elektromágneses energia

$$W(t) = \int w \, d\Omega$$

$$[w] = \frac{J^2}{\omega^3}, [W] = J$$

1. $\text{rot } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad / \cdot \vec{E} \Rightarrow \text{I} - \text{II}$

2. $\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad / \cdot \vec{H}$

$$\vec{E} \cdot \text{rot } \vec{H} - \vec{H} \cdot \text{rot } \vec{E} = \vec{J} \cdot \vec{E} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}$$

a zavarosság: $-\text{div}(\vec{E} \times \vec{H}) =$ \rightarrow

$$\mathbf{E} \cdot \text{rot} \mathbf{H} - \mathbf{H} \cdot \text{rot} \mathbf{E} = \mathbf{j} \cdot \mathbf{E} + \epsilon \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \quad (\bar{\mathbf{E}} = \frac{1}{r}(\mathbf{j} - \mathbf{j}_b) - \bar{\mathbf{E}}_b)$$

$$(\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H})$$

~~$$\text{div}(\bar{\mathbf{E}} \times \bar{\mathbf{H}}) = \frac{\mathbf{j} \cdot \bar{\mathbf{E}}}{r} - \frac{\mathbf{j} \cdot \bar{\mathbf{j}}_b}{r} - \mathbf{j} \cdot \bar{\mathbf{E}}_b$$~~

$$-\text{div}(\bar{\mathbf{E}} \times \bar{\mathbf{H}}) = \frac{\mathbf{j} \cdot \bar{\mathbf{j}}}{r} - \frac{\mathbf{j} \cdot \bar{\mathbf{j}}_b}{r} - \mathbf{j} \cdot \bar{\mathbf{E}}_b + \epsilon \bar{\mathbf{E}} \cdot \frac{\partial \bar{\mathbf{E}}}{\partial t} + \mu \bar{\mathbf{H}} \cdot \frac{\partial \bar{\mathbf{H}}}{\partial t}$$

$$\left(\frac{\partial w}{\partial t} = \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon |\bar{\mathbf{E}}|^2 + \frac{1}{2} \mu |\bar{\mathbf{H}}|^2 \right) = \epsilon |\bar{\mathbf{E}}| \cdot \frac{\partial |\bar{\mathbf{E}}|}{\partial t} + \mu |\bar{\mathbf{H}}| \cdot \frac{\partial |\bar{\mathbf{H}}|}{\partial t} \right) = \frac{\partial w}{\partial t}$$

$$\Rightarrow -\frac{\partial w}{\partial t} = \text{div}(\bar{\mathbf{E}} \times \bar{\mathbf{H}}) + \frac{|\bar{\mathbf{j}}|^2}{r} - \left(\frac{\mathbf{j} \cdot \bar{\mathbf{j}}_b}{r} + \mathbf{j} \cdot \bar{\mathbf{E}}_b \right) \int_{\Omega} d\Omega$$

$$\Rightarrow -\frac{d}{dt} \int_{\Omega} w d\Omega = \int_{\Omega} \text{div}(\bar{\mathbf{E}} \times \bar{\mathbf{H}}) d\Omega + \int_{\Omega} \frac{|\bar{\mathbf{j}}|^2}{r} d\Omega - \int_{\Omega} \left(\frac{\mathbf{j} \cdot \bar{\mathbf{j}}_b}{r} + \mathbf{j} \cdot \bar{\mathbf{E}}_b \right) d\Omega$$

E.M. energi voltazása

$\oint_{\Gamma} (\bar{\mathbf{E}} \times \bar{\mathbf{H}}) d\bar{\Gamma}$

Γ -n átszerezés
E.M. teljesítmény

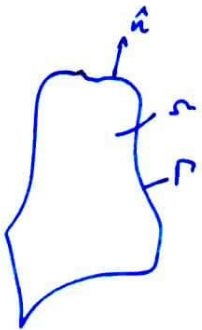
Gauss

> 0

vektor hosszait
integráljuk (> 0)
Joule-hő

hűlés hatásos teljesítménye
(+ és - is lehet)

$$-\frac{d}{dt} \int_{\Omega} w d\Omega = \int_{\Omega} \text{div}(\bar{\mathbf{E}} \times \bar{\mathbf{H}}) d\Omega + \int_{\Omega} \frac{|\bar{\mathbf{j}}|^2}{r} d\Omega - \int_{\Omega} \left(\frac{\mathbf{j} \cdot \bar{\mathbf{j}}_b}{r} + \mathbf{j} \cdot \bar{\mathbf{E}}_b \right) d\Omega$$



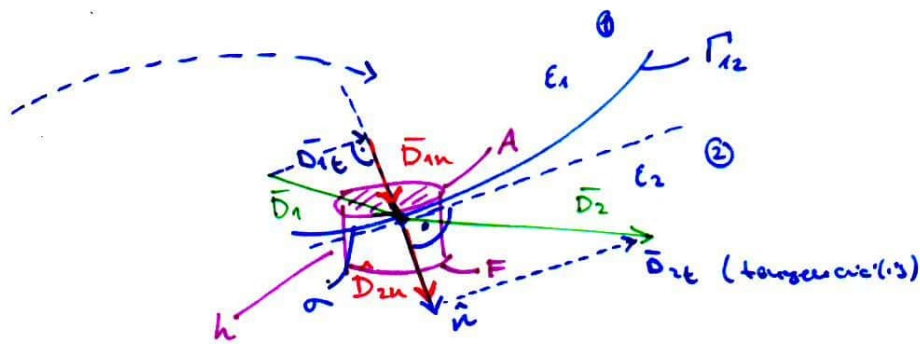
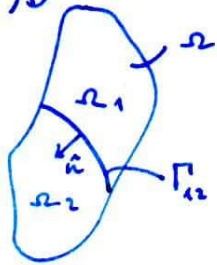
$$\oint_{\Gamma} (\bar{\mathbf{E}} \times \bar{\mathbf{H}}) d\bar{\Gamma} : \Gamma\text{-n átszerezés teljesítmény}$$

$\bar{\mathbf{S}}$: Poynting-vektor : teljesítmény sűrűség vektor

$$[\bar{\mathbf{S}}] = \frac{W}{m^2}$$

Folytonossági feltételek hővezetésben

1.) \vec{D} :



$$\oint_F \vec{D} d\vec{F} = \int_{\Gamma_{12}} \sigma d\Gamma$$

$$\left. \begin{array}{l} h \rightarrow 0 \\ A \rightarrow 0 \end{array} \right\} \int_{\text{palaist}} \vec{D} d\vec{F} + A \cdot D_{2n} - A D_{1n} = A \cdot \sigma$$

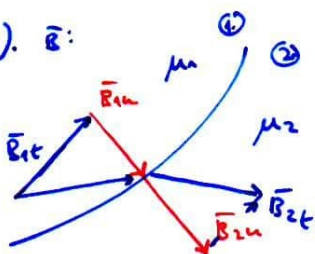
$\underbrace{\hspace{10em}}_{\phi, \text{ ha } h \rightarrow 0}$

$$\Rightarrow \boxed{D_{2n} - D_{1n} = \sigma}$$

$$\vec{D}_2 \cdot \hat{n} - \vec{D}_1 \cdot \hat{n} = \sigma$$

$$\boxed{(\vec{D}_2 - \vec{D}_1) \cdot \hat{n} = \sigma} \quad : \text{vektoros alak}$$

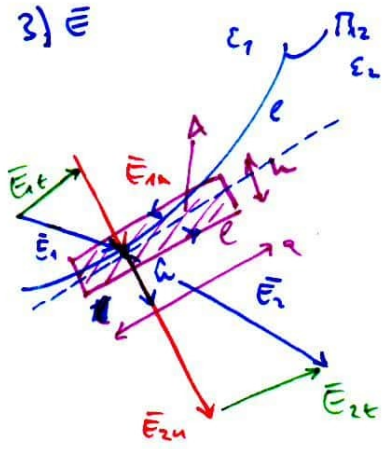
2.) \vec{B} :



$$B_{2n} - B_{1n} = 0$$

$$\boxed{(\vec{B}_2 - \vec{B}_1) \cdot \hat{n} = 0}$$

3) \vec{E}



$$\vec{E}_{2t} - \vec{E}_{1t} = 0$$

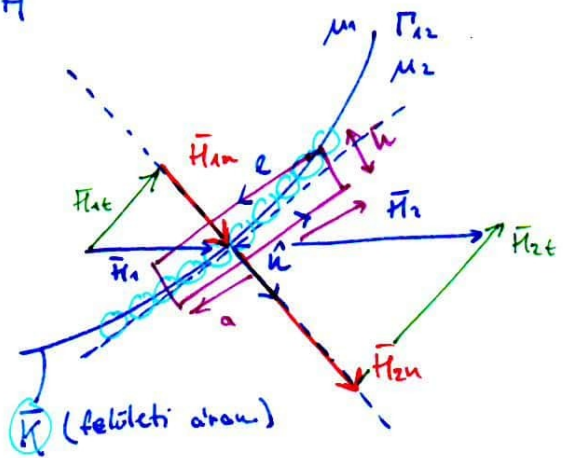
$$\oint_C \vec{E} d\vec{l} = \int_A \left(-\frac{\partial B}{\partial t}\right) d\vec{A}$$

$$\left. \begin{matrix} h \rightarrow 0 \\ a \rightarrow 0 \end{matrix} \right\} a E_{2t} - a E_{1t} = 0$$

$E_{2t} - E_{1t} = 0$: \vec{E} is always irrotational

$$\underbrace{\vec{E}_2 \times \hat{n}}_{\vec{E}_{2t}} - \underbrace{\vec{E}_1 \times \hat{n}}_{\vec{E}_{1t}} = 0 \Rightarrow \boxed{(\vec{E}_2 - \vec{E}_1) \times \hat{n} = 0}$$

4.) \vec{H}



$$\oint_C H d\vec{l} = \int_A \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) d\vec{A}$$

$$\left. \begin{matrix} h \rightarrow 0 \\ a \rightarrow 0 \end{matrix} \right\}$$

$$\begin{aligned} a H_{2t} - a H_{1t} &= a K \\ H_{2t} - H_{1t} &= K \end{aligned}$$

$$\boxed{(\vec{H}_2 - \vec{H}_1) \times \hat{n} = \vec{K}}$$

Az elektrodinamika felosztása

- Stacionárius terek $\begin{cases} \rightarrow \text{elektrosztatika} \\ \rightarrow \text{magnetosztatika} \end{cases}$: töltéseloszlás elmozdulása nélkül

idő-függetlenség: $\frac{\partial}{\partial t} = 0$ és $\vec{J} \equiv \vec{0}$

- Stacionárius terek $\begin{cases} \rightarrow \text{stacionárius áram (osk.) tere} \\ \rightarrow \text{stacionárius áramú mágneses tere} \end{cases}$ } egyirányú áramok

$$\frac{\partial}{\partial t} = 0, \quad \vec{J} \neq \vec{0}$$

~~Stacionárius terek~~

- Kvázi-stacionárius terek : $\frac{\partial}{\partial t} \neq 0$

- Hullékter

Tereh E.
15.09.24.

Az elektrodinamika összefoglaló

Cél: Maxwell-eg. m.o.-ólal megkötésre (kötéltételek)

$$\left. \begin{aligned} \text{rot } \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} & \vec{D} &= \epsilon_0 \vec{E} + \vec{P} \\ \text{rot } \vec{E} &= - \frac{\partial \vec{B}}{\partial t} & \vec{B} &= \mu_0 (\vec{H} + \vec{M}) \\ \text{div } \vec{B} &= 0 & \vec{J} &= \nabla \cdot (\vec{E} + \vec{E}_b) + \vec{J}_b \\ \text{div } \vec{D} &= \rho \end{aligned} \right\} \text{nem változhatnak}$$

I. Stacionárius terek ($\frac{\partial}{\partial t} = 0, \vec{J} \equiv \vec{0}$)

• Elektrosztatika egyenletei:

$$\left. \begin{aligned} \text{rot } \vec{E} &= \vec{0} \\ \text{div } \vec{D} &= \rho \\ \vec{D} &= \epsilon_0 \vec{E} + \vec{P} \end{aligned} \right\} \text{Stacionárius terek}$$

forrás

• Magnetosztatika:

$$\left. \begin{aligned} \text{rot } \vec{H} &= \vec{0} \\ \text{div } \vec{B} &= 0 \\ \vec{B} &= \mu_0 (\vec{H} + \vec{M}) \end{aligned} \right\}$$

forrás

⊗ Nagyfesz. technika, kapacitások

⊕ Államok mágneses, villamos gépek

II. Stacionárius terek ($\frac{\partial}{\partial t} = 0, \vec{J} \neq \vec{0}$)

• Stacionárius áramok ↑ veszd! • Stacionárius mágneses terek

$$\left. \begin{aligned} \text{rot } \vec{E} &= \vec{0} \\ \text{div } \vec{J} &= \vec{0} \\ \vec{J} &= \nabla \cdot (\vec{E} + \vec{E}_b) + \vec{J}_b \end{aligned} \right\} \text{forrás}$$

Egyenlőségfeltétel

$$\left. \begin{aligned} \text{rot } \vec{H} &= \vec{J} \\ \text{div } \vec{B} &= 0 \\ \vec{B} &= \mu_0 (\vec{H} + \vec{M}) \end{aligned} \right\} \text{forrás}$$

⊗ rot H = J => div rot H = div J = 0 | ⊕ Ellenőrzés, mágneses terek ⊕ Villamos gépek

III. Kvázi-stacionaritás terelő ($\frac{\partial}{\partial t} \neq 0, \vec{J} \neq \vec{0}$)

$$\begin{aligned} \text{rot } \vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\ \text{rot } \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \text{div } \vec{D} &= \bar{\rho} \\ \text{div } \vec{J} &= \bar{\rho}' \end{aligned}$$

\downarrow "lassú"
 elhanyagolható
 $|\vec{J}| \gg \left| \frac{\partial \vec{D}}{\partial t} \right|$

$$\begin{aligned} \vec{D} &= \mu_0 (\vec{H} + \vec{M}) \\ \vec{J} &= \sigma (\vec{E} + \vec{E}_b) + \vec{J}_b \end{aligned}$$

$$\vec{J} = \sigma \vec{E}, \quad \vec{D} = \epsilon \vec{E}$$

$$|\sigma \vec{E}| \gg \left| \epsilon \frac{\partial \vec{E}}{\partial t} \right|$$

$$\text{"} \sigma \gg \epsilon \frac{\partial}{\partial t} \text{"}$$

jó vezetőben (forrás) E.M. tér

AKKAMKISZORÍTÁS =

IV. Elektromágneses hullámok ($\frac{\partial}{\partial t} \neq 0, \vec{J} \neq \vec{0}$)

• Egyenletek:

Maxwell-egy. teljes rendszer

$\frac{\partial \vec{D}}{\partial t}$ szerepe nagyon jelentős

pl.: szigetelőben ($\vec{J} = \vec{0}$)

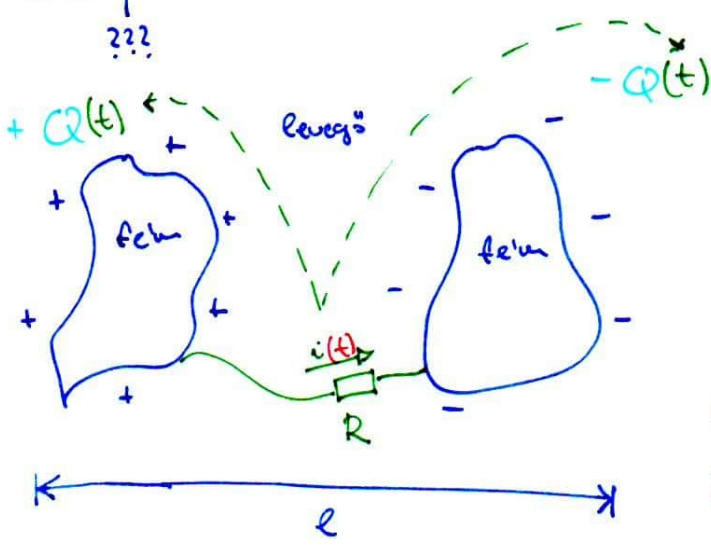
$$\begin{aligned} \text{rot } \vec{H} &= -\frac{\partial \vec{D}}{\partial t} \\ \text{rot } \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{D} &= \epsilon \vec{E} \end{aligned}$$

$\mu \vec{H}$

: egymást hajtó hullámok

csatolás, hullámvezetők

$$\frac{\partial}{\partial t} = 0 \quad \text{fizikai tartalom?}$$



$Q(t)$: $Q(t)$ mennyiség

$$c = 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$$

$$\tau = \frac{l}{c} : \text{levegőtartózkodás idő}$$

"levegőtartózkodás mérték"

$$|Q(t+\tau) - Q(t)| \ll |Q(t)| \Rightarrow \text{"\epsilon" hossz\text{e}l\text{e}ben Q(t) \approx \text{konstans}}$$

$$\Rightarrow \text{Elektrosztatikai esetek} : \frac{\partial}{\partial t} = 0$$

Elektrosztatika

Nyugodó töltések $\rightarrow \frac{\partial}{\partial t} = 0, \vec{J} \equiv \vec{0}$

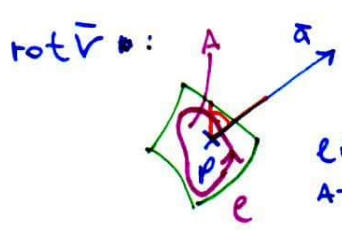
- $\text{rot } \vec{E} = \vec{0}$
- $\text{div } \vec{D} = \rho$
- $\vec{D} = \epsilon \vec{E}$, levegő: $\epsilon = \epsilon_0$

$$\oint_C \vec{E} \cdot d\vec{l} = 0 \quad ; \quad \text{örvénymentes}$$

$$\oint_V \vec{D} \cdot d\vec{F} = \int_V \rho \, dV \quad ; \quad \text{forrásos}$$

Megj.: (ROT)

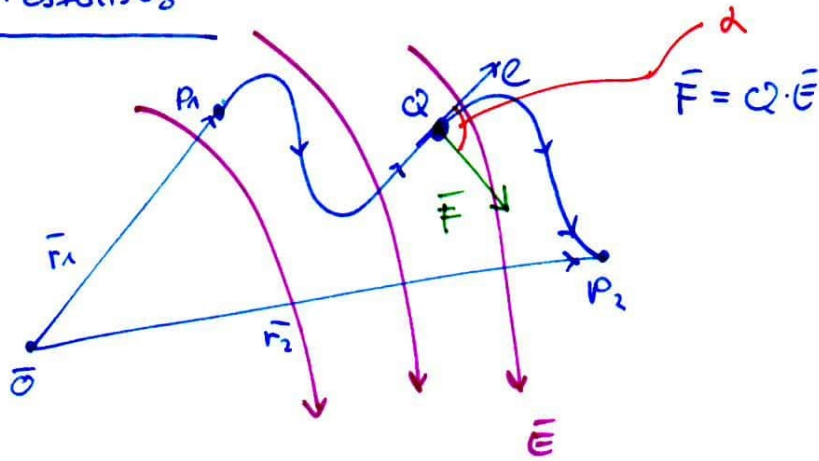
$$\vec{V}(x, y, z) = \vec{V}_x(x, y, z) \hat{e}_x + \vec{V}_y(x, y, z) \hat{e}_y + \vec{V}_z(x, y, z) \hat{e}_z$$



$$\lim_{A \rightarrow 0} \frac{\oint \vec{V} \cdot d\vec{l}}{A} = (\text{rot } \vec{V}) \cdot \frac{\vec{a}}{|\vec{a}|}$$

\vec{a} irányú, egység hossz

Feszültség



A mező által végzett munka:

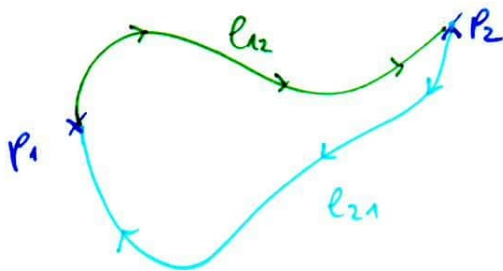
$$W_{12} = \int_{e_1=P_1}^{P_2} \overline{F} \cdot d\overline{e} \cdot \cos \alpha = \int_e Q \overline{E} d\overline{e} = Q \int_e \overline{E} d\overline{e} = Q \int_e \underbrace{(-\text{grad} \varphi)}_{-\frac{\partial \varphi}{\partial e} |d\overline{e}|} d\overline{e} =$$

$$= Q \cdot \int_e -d\varphi = Q \underbrace{(\varphi_1 - \varphi_2)}_{\triangleq U_{12} \text{ feszültség}}$$

volt definíciója: egyenlőségi: kölcsönösen végzett munka:

$$V = \frac{W}{Q}$$

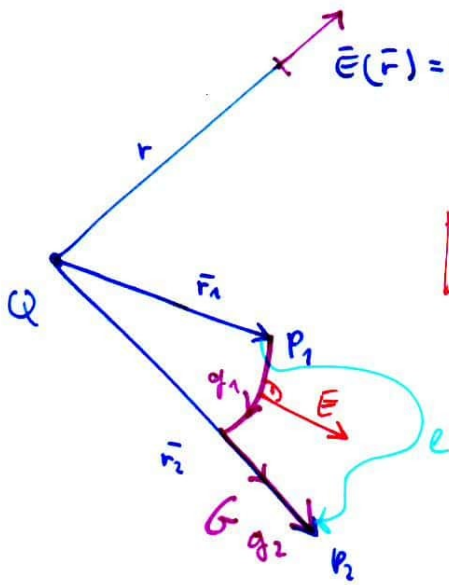
Feszültség útól való függetlensége:



$$\oint_e \overline{E} d\overline{e} = \oint_{e_1} \overline{E} d\overline{e} + \oint_{e_2} \overline{E} d\overline{e} = \oint$$

$$U_{12} \equiv -U_{21}$$

Ponttöltés potenciáltere



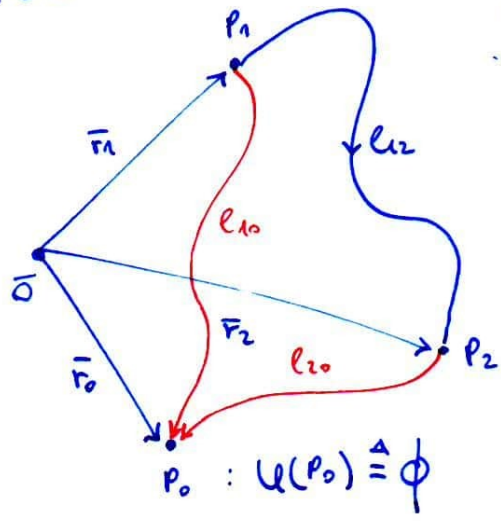
$$U_{12} = ? \quad \int_{r_1}^{r_2} \vec{E} d\vec{e} = \int_{r_1}^{r_2} \vec{E} d\vec{e} = \int_{r_1}^{r_2} \vec{E} d\vec{e} + \int_{r_2}^{r_1} \vec{E} d\vec{e} =$$
$$= \phi$$

$$= \int_{r_2}^{r_1} \underbrace{\hat{e}_r \cdot \frac{Q}{4\pi\epsilon} \cdot \frac{1}{r^2}}_{\vec{E}} \cdot \underbrace{\hat{e}_r dr}_{d\vec{e}} \quad (\hat{e}_r \cdot \hat{e}_r \equiv 1)$$

$$\Rightarrow \int_{r_2}^{r_1} \frac{Q}{4\pi\epsilon} \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon} \left[-\frac{1}{r} \right]_{r_1}^{r_2} = \frac{Q}{4\pi\epsilon} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Referenciapont választása

$\varphi(\vec{r})$



$$U_{12} = \int_{l_{12}} \vec{E} d\vec{e} = \varphi(P_1) - \varphi(P_2)$$

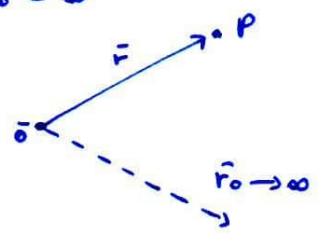
↑
-grad φ

$$\left. \begin{aligned} \varphi(P_1) &= \int_{l_{10}} \vec{E} d\vec{e} + \underbrace{\varphi(P_0)}_{\equiv \phi} \\ \varphi(P_2) &= \int_{l_{20}} \vec{E} d\vec{e} + \underbrace{\varphi(P_0)}_{\equiv \phi} \end{aligned} \right\} \text{Potenciálhoz a referenciapontot választjuk.}$$

PE.: ponttöltés esetén

$$U_{12} = \frac{Q}{4\pi\epsilon} \left(\frac{1}{r_1} - \frac{1}{r_2} \right); \quad P_0 \rightarrow \infty, \quad \vec{r}_0 \hat{=} \infty$$

$$\boxed{\varphi(P) = \frac{Q}{4\pi\epsilon} \left(\frac{1}{r} - \frac{1}{\infty} \right) = \frac{Q}{4\pi\epsilon} \cdot \frac{1}{r}}$$



Laplace - Poisson egyenlet

$$\text{rot } \vec{E} = \vec{0} \Rightarrow \vec{E} = -\text{grad } \varphi$$

$$\begin{aligned} \text{div } \vec{D} &= \rho \\ \vec{D} &= \epsilon \vec{E} \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{div } \epsilon \vec{E} &= \rho \\ \text{div } \epsilon (-\text{grad } \varphi) &= \rho \end{aligned}$$

$$\boxed{-\text{div } \epsilon \text{ grad } \varphi = \rho} \quad : \text{L-P egyenlet}$$

Ha ϵ konstans (homogén közeg) $\Rightarrow \underbrace{\text{div grad}}_{\Delta} \varphi = -\frac{\rho}{\epsilon}$

„div grad” = „ Δ ” Laplace operator

$$\Delta \varphi = -\frac{\rho}{\epsilon}$$

Megj: Δ hiszámításra Descartes - koordin. r. - ban

$$\psi(x, y, z) \rightarrow \Delta \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

Töltésmentes térben: $\rho \equiv 0$

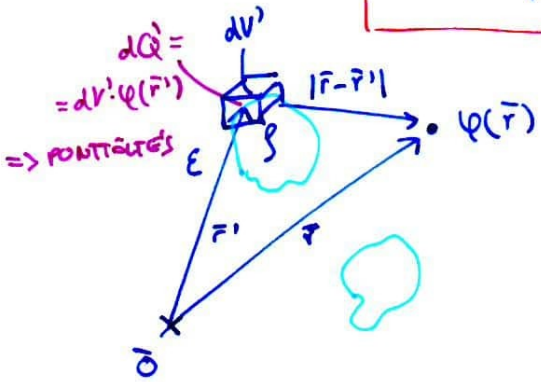
-díringrad $\psi = 0$
 $\Delta \psi = 0$ } Laplace egyenletek: másodrendű parciális differenciálegyenlet (PDE)

A Laplace-Poisson szabadtéri megoldása

$\hookrightarrow \epsilon = \text{konstans} \in$ teljes végtelen tér

$$\Delta \psi = -\frac{\rho}{\epsilon} \rightarrow \psi(\vec{r}) = \int_{V(\infty)} \frac{1}{4\pi\epsilon|\vec{r}-\vec{r}'|} \cdot \underbrace{\rho(\vec{r}')}_{dQ'} dV' : \text{kovariáns integrál}$$

← gerjesztés



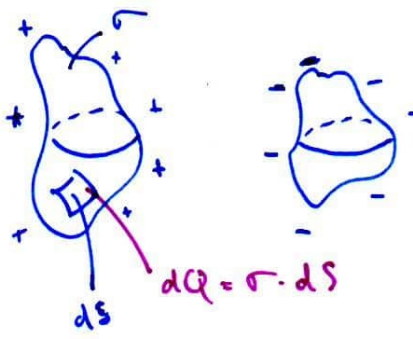
Megjegyzés: Lineáris törvény igaz a szuperpozíciós elv.

$$\Rightarrow d\psi = \frac{dQ}{4\pi\epsilon|\vec{r}-\vec{r}'|} \rightarrow \psi = \int_{V(\infty)} d\psi$$

jelen: $y(t) = \int_{t(0)}^{t(\infty)} h(t-\tau)u(\tau) d\tau$

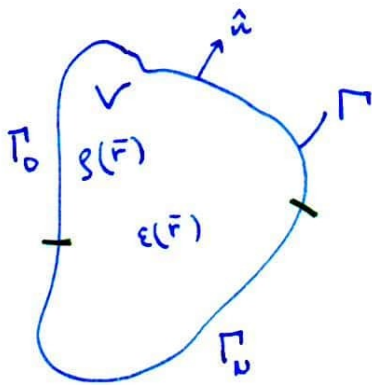
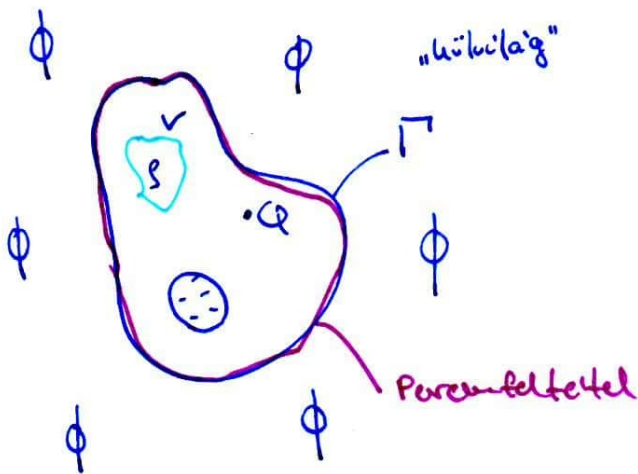
* : GREEN-függvény (impulzus válasz)

Gyákorlatban: $\rho = 0$, de $\sigma \neq 0$



$$\psi(\vec{r}) = \int_S \frac{1}{4\pi\epsilon|\vec{r}-\vec{r}'|} \cdot \underbrace{\sigma(\vec{r}') \cdot dS'}_{dQ'}$$

Az elektrosztatikus peremérték-feladatok



$$\nabla: -\text{div} \underbrace{\epsilon(\vec{r})}_{\text{adott}} \text{grad} \underbrace{\varphi(\vec{r})}_{?} = \underbrace{\rho(\vec{r})}_{\text{adott}} : \text{parciális diff. e.}$$

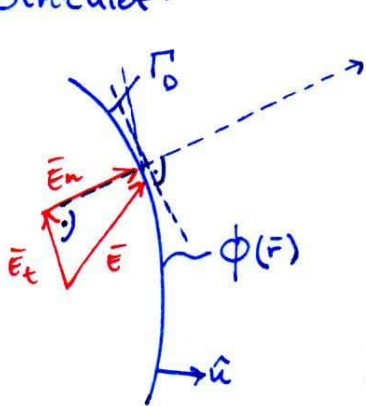
$$\Gamma_D: \varphi|_{\Gamma_D} = \underbrace{\phi(\vec{r})}_{\text{adott}} : \text{Dirichlet-peremfeltétel}$$

$$\Gamma_N: \epsilon(\vec{r}) \cdot \frac{\partial \varphi}{\partial n} \Big|_{\Gamma_N} = \underbrace{\sigma(\vec{r})}_{\text{adott}} : \text{Neumann-peremfelt.}$$

Allítás: PDE megoldása egyértelmű D. és N. peremfeltételek mellett.

A peremfeltételek fizikai jelentése

Dirichlet:



$$\vec{E} = -\text{grad} \varphi$$

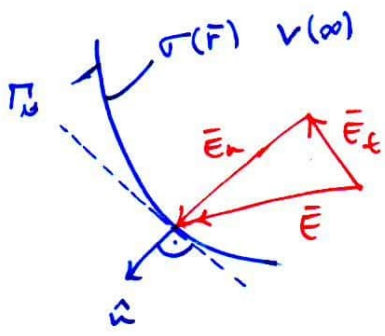
$$\vec{E}_t|_{\Gamma_D} = -\text{grad}_t \phi(\vec{r})$$

! [fémlektrodák felszínei: $\phi = \text{konstans}$
 $\Rightarrow \vec{E}_t = \vec{0}$

Megj.: $\text{grad}_t u(\vec{r})$

$$\text{grad}_t u(\vec{r}) = \text{grad} u(\vec{r}) - \hat{n}(\hat{n} \cdot \text{grad} u(\vec{r}))$$

Neumann:



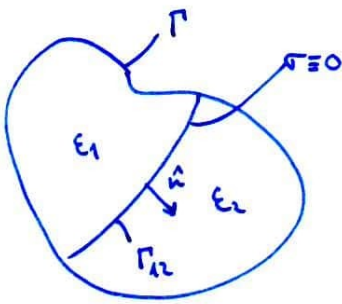
$$\vec{E} = -\text{grad} \psi$$

$$\vec{E}_n = -\hat{n} \cdot \text{grad} \psi = -\frac{\partial \psi}{\partial n} \quad / \cdot \epsilon(\vec{r})$$

$$\left[\frac{\partial \psi}{\partial n} \right]_{\Gamma_0} = -\epsilon(\vec{r}) \left. \frac{\partial \psi}{\partial n} \right|_{\Gamma_0} = \sigma(\vec{r}) \quad ; \text{ felületi töltéssűrűség}$$

\hat{n} kifele mutat

Belső peremek:



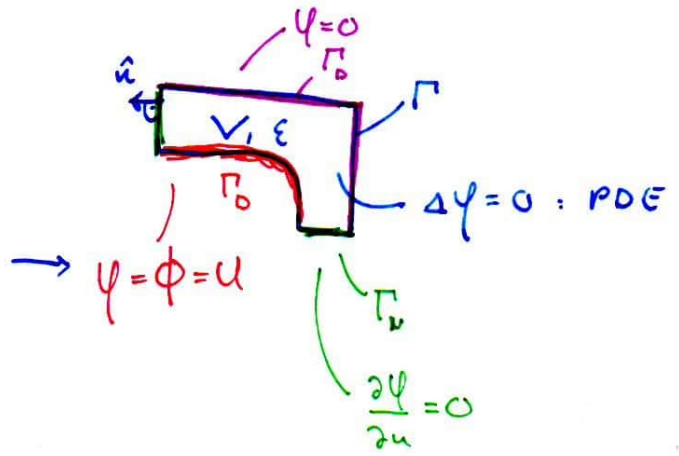
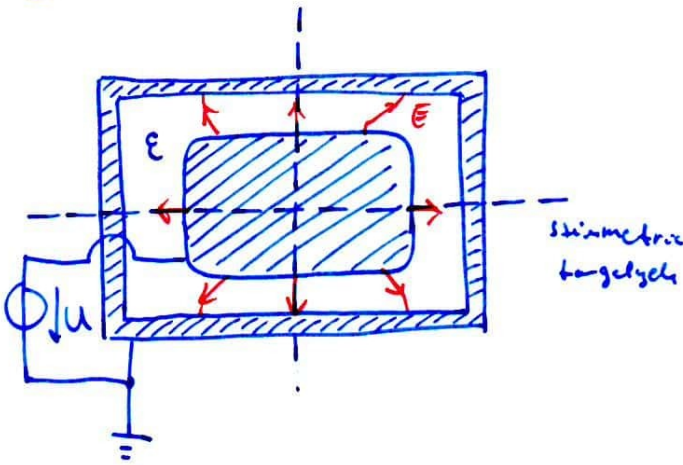
$$\Gamma_{12}: E_t^{(1)} \Big|_{\Gamma_{12}} = E_t^{(2)} \Big|_{\Gamma_{12}} \rightarrow \psi^{(1)} \Big|_{\Gamma_{12}} = \psi^{(2)} \Big|_{\Gamma_{12}}$$

perem mentén meggyeznek a potenciálok

$$D_n^{(1)} \Big|_{\Gamma_{12}} = D_n^{(2)} \Big|_{\Gamma_{12}} \rightarrow \epsilon_1 \frac{\partial \psi^{(1)}}{\partial n} \Big|_{\Gamma_{12}} = \epsilon_2 \frac{\partial \psi^{(2)}}{\partial n} \Big|_{\Gamma_{12}}$$

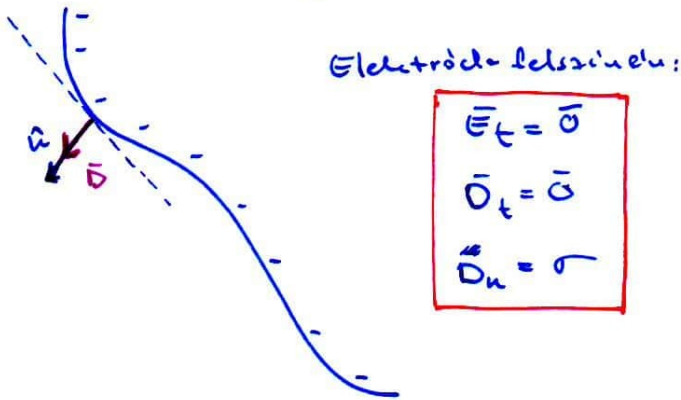
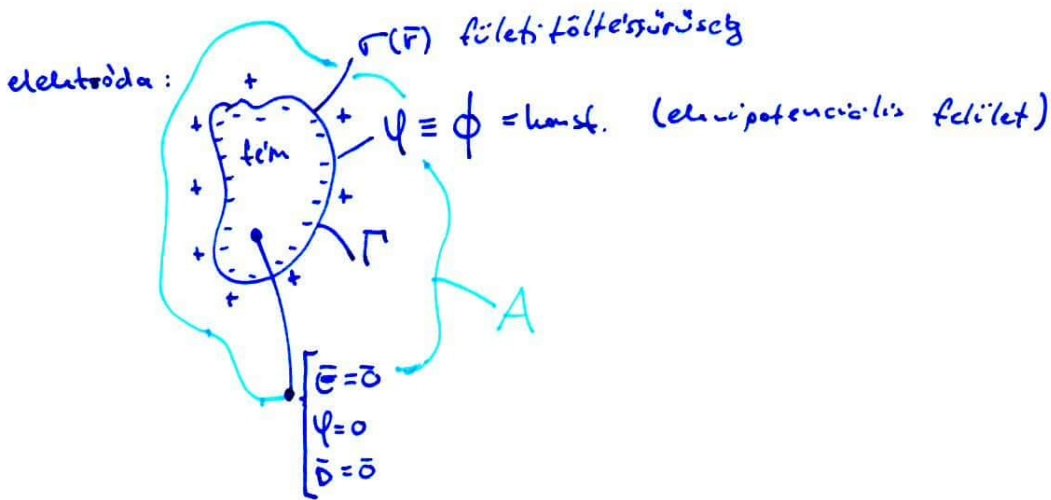
$$\epsilon_1 \frac{\partial \psi^{(1)}}{\partial n} \Big|_{\Gamma_{12}} = \epsilon_2 \frac{\partial \psi^{(2)}}{\partial n} \Big|_{\Gamma_{12}}$$

(P2) Kábelben kialakuló tér



⇒ egyértelműen megoldható

Elektrodarendszerek

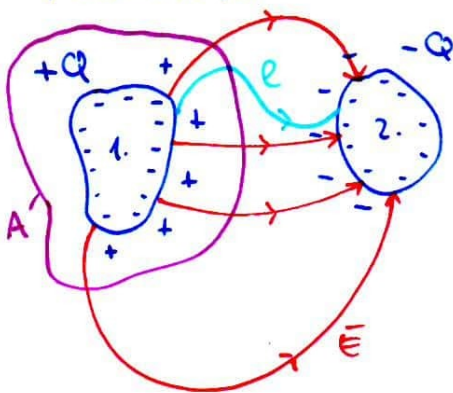


Össztöltés:

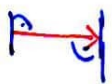
Gauss

$$Q = \oint_{\Gamma} \sigma d\Gamma = \oint_A \vec{D} d\vec{A}$$

a) Elektrodapár:



$\sum Q = 0$ (zárt rendszer)



lineáris közegben:



KAPACITÁ'S

$$U_{12} = \int_e \vec{E} d\vec{l}$$

$$Q = \oint_A \vec{D} d\vec{A}$$

$$C \stackrel{\Delta}{=} \frac{Q}{U_{12}} = \text{konstans}$$

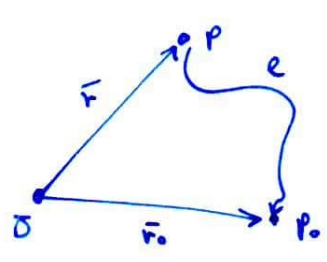
$C = 1F$ (farad)

Elektrosztatika

$$\text{rot } \vec{E} = \vec{0} \iff \oint_P \vec{E} d\vec{e} = 0$$



$$\vec{E} = -\text{grad } \psi$$

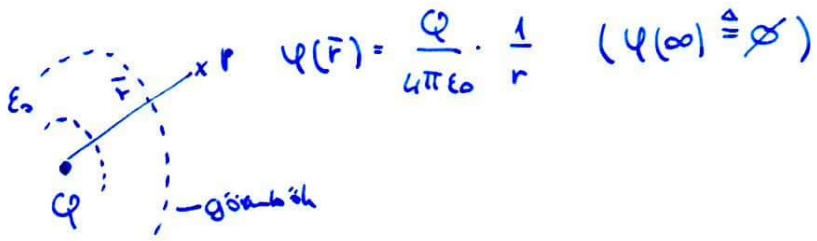


$$\psi(P) = \int_{P=P_0}^{P=P} \vec{E} d\vec{e} + \underbrace{\psi(P_0)}_{\equiv \phi}$$

kapacitás:

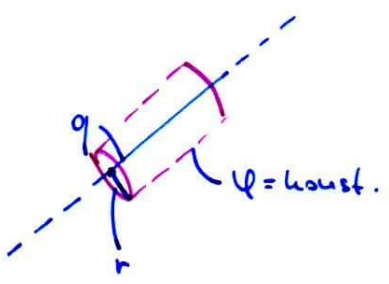
$$C = \frac{Q}{U}$$

pl.: 1) ponttöltés



$$\psi(r) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r} \quad (\psi(\infty) \triangleq \phi)$$

2) végtelen hosszú egyenes vezető



$$\psi(r) = \frac{q}{2\pi\epsilon_0} \cdot \ln \frac{r_0}{r}$$

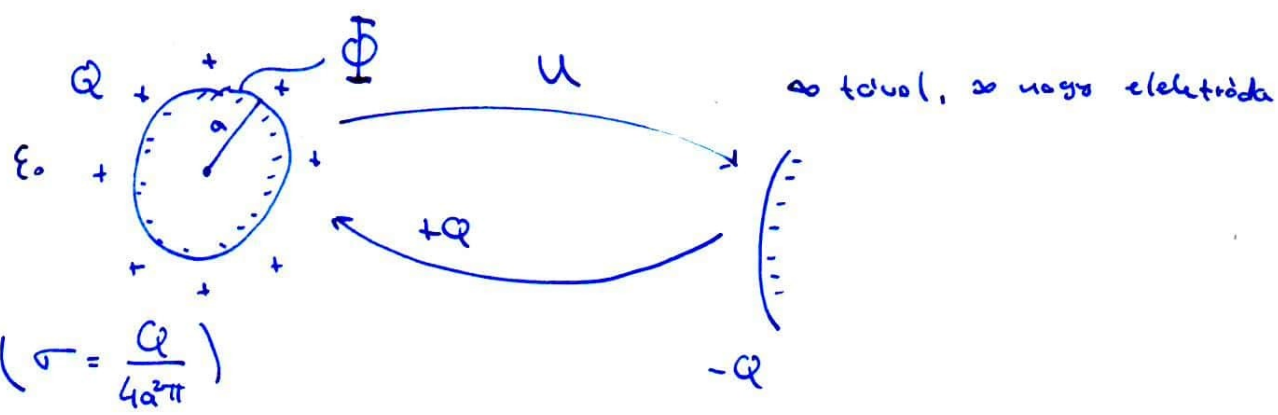
$$\psi(r_0) \triangleq \phi$$

$$r_0 := k_1 r_0 \quad (k_1 > 0)$$

$$\Rightarrow \psi'(r) = \frac{q}{2\pi\epsilon_0} \ln \frac{k_1 r_0}{r} = \underbrace{\frac{q}{2\pi\epsilon_0} \ln \frac{r_0}{r}}_{\psi(r)} + \underbrace{\frac{q}{2\pi\epsilon_0} \ln k_1}_{\text{konst.}}$$

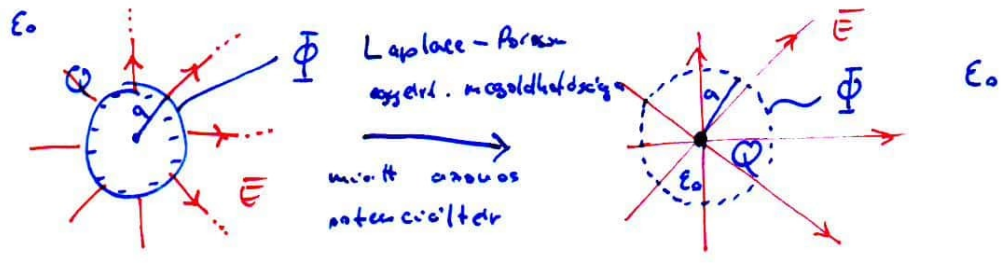
$\Rightarrow r_0$ megválasztása lényegtelen

1 Mágisban álló féngömb kapacitása



$C = \frac{Q}{U} = ? \Rightarrow U = ?$

Helyettesítő töltesek módszerre:



$\varphi(\infty) = \Phi$

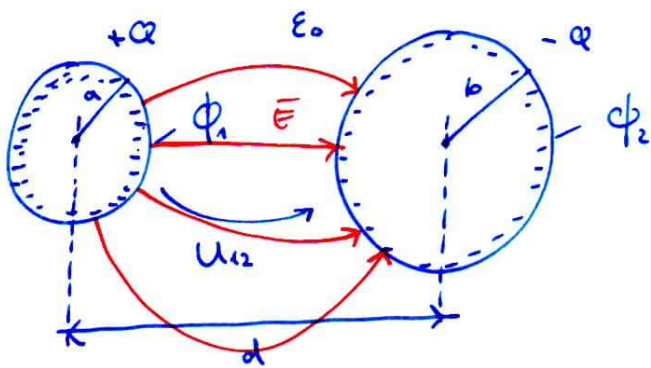
$\varphi(\infty) = 0$

$\varphi(r) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r} \rightarrow \Phi = \varphi(r=a) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{a}$: gömb felszín potenciálja

$U = \Phi - 0 = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{a}$

$\Rightarrow C = 4\pi\epsilon_0 \cdot a$: ∞-re van eltolított kapacitása

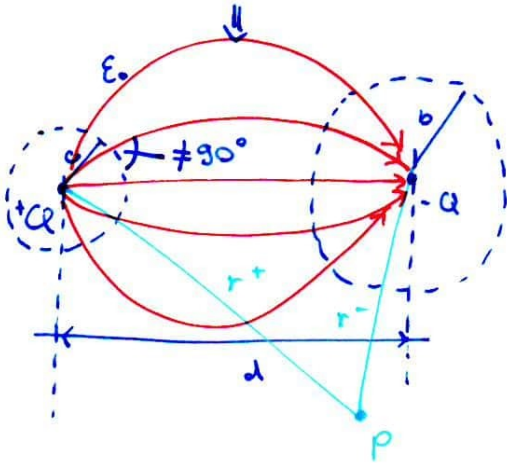
2 két gömb közötti kapacitás



$$C_{12} = \frac{Q}{U_{12}}$$

Q: a közt költések

→ : nem létezően leírni a b



→ : NEM nem létezően leírni a b

⇒ feltételezés: $a, b \ll d$

$$\varphi(P) = \frac{+Q}{4\pi\epsilon_0} \cdot \frac{1}{r^+} + \frac{-Q}{4\pi\epsilon_0} \cdot \frac{1}{r^-}$$

$$\left. \begin{array}{l} \phi : r^+ \cong a \\ r^- = [d-a, d+a] \end{array} \right\} \phi_1 \cong \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{a} + \frac{-Q}{4\pi\epsilon_0} \cdot \frac{1}{d}$$

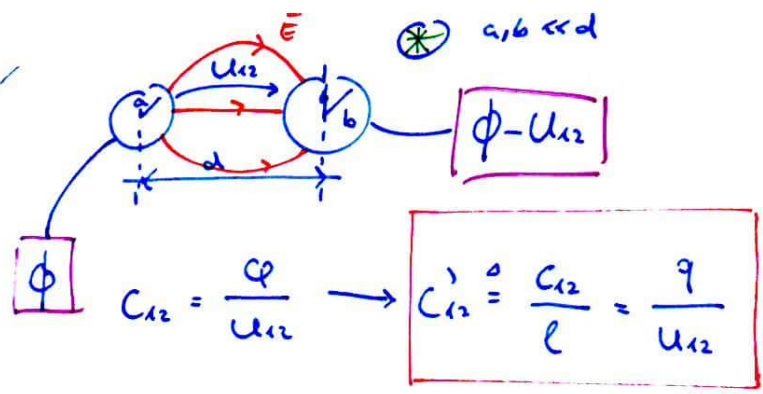
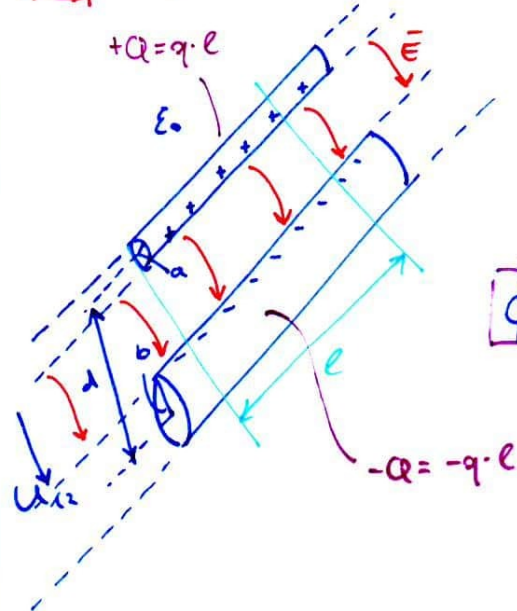
$$\phi_2 \cong \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{d} + \frac{-Q}{4\pi\epsilon_0} \cdot \frac{1}{b}$$

$$U = \phi_1 - \phi_2 = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d} \right)$$

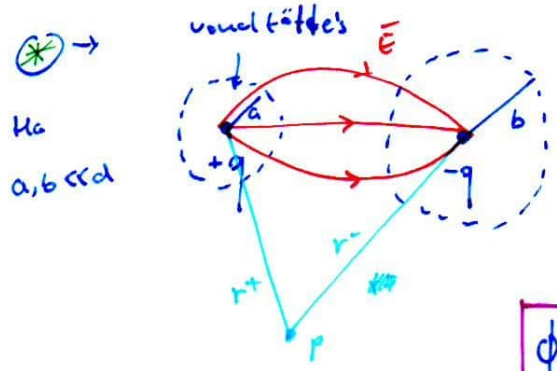
$$C_{12} = \frac{Q}{U_{12}} = \frac{4\pi\epsilon_0}{\frac{1}{a} + \frac{1}{b} - \frac{2}{d}}$$

Ha $a \rightarrow \infty$
 $b \rightarrow \infty \Rightarrow$ **1** feladat $\Rightarrow C_{12} \rightarrow 4\pi\epsilon_0 a$

3. Vezeték hosszú vezetékpar hosszegységére való kapacitása



$$C_{12} = \frac{Q}{U_{12}} \rightarrow C'_{12} = \frac{C_{12}}{l} = \frac{q}{U_{12}}$$



$$\varphi(P) = \frac{+q}{2\pi\epsilon_0} \cdot l \cdot \ln \frac{r_0^+}{r} + \frac{-q}{2\pi\epsilon_0} \cdot l \cdot \ln \frac{r_0^-}{r}$$

$$\phi = \frac{q}{2\pi\epsilon_0} \cdot \left(\ln \frac{r_0^+}{a} - \ln \frac{r_0^-}{d} \right) : \text{I.}$$

$$\phi - U_{12} = \frac{q}{2\pi\epsilon_0} \cdot \left(\ln \frac{r_0^+}{d} - \ln \frac{r_0^-}{b} \right) : \text{II.}$$

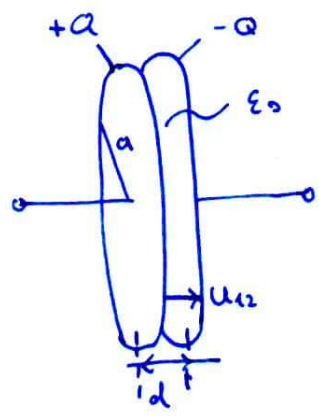
$$\text{I.} - \text{II.} : U_{12} = \frac{q}{2\pi\epsilon_0} \cdot \left(\ln \frac{r_0^+}{a} - \ln \frac{r_0^-}{d} - \ln \frac{r_0^+}{d} + \ln \frac{r_0^-}{b} \right) \text{ (3)}$$

$$\ln \frac{r_0^+ \cdot d \cdot d \cdot r_0^-}{a \cdot r_0^- \cdot r_0^+ \cdot b} = \ln \frac{d^2}{ab}$$

$$\text{(3)} \frac{q}{2\pi\epsilon_0} \cdot \ln \frac{d^2}{ab}$$

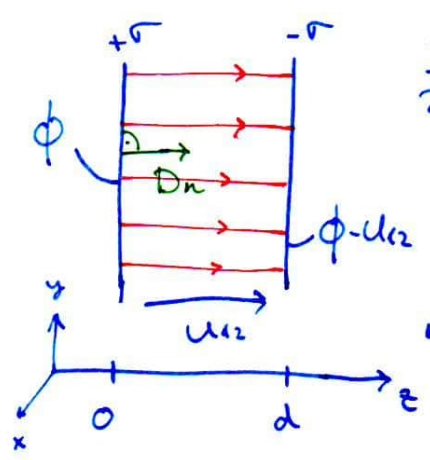
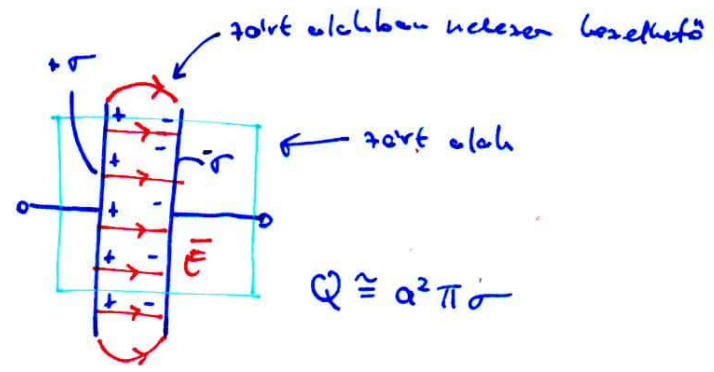
$$\Rightarrow C'_{12} = \frac{2\pi\epsilon_0}{\ln \frac{d^2}{ab}}$$

4 Silikonkondensator



$$C_{12} = \frac{Q}{U_{12}} = ?$$

$$l \ll a$$



$$\frac{\partial}{\partial x} \equiv \frac{\partial}{\partial y} \equiv 0$$

$$\Delta \psi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{d^2 \psi}{dz^2} = 0$$

$$[0, d]: \frac{d^2 \psi}{dz^2} = 0 \quad \text{: diff. e.}$$

$$z=0: \psi = \phi$$

$$z=d: \psi = \phi - U_{12}$$

1-dimensional perpendicul. Felder

$$\rightarrow \int dz: \frac{d\psi}{dz} + A = 0$$

$$\iint dz: \psi(z) + Az + B = 0$$

$$z=0: \phi + A \cdot 0 + B = 0$$

$$z=d: \phi - U_{12} + A \cdot d + B = 0$$

$$\text{I. - II.} : U_{12} - A \cdot d = 0 \Rightarrow A = \frac{U_{12}}{d}$$

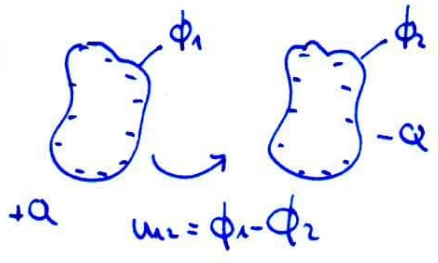
$$\sigma = D_n = E_n \cdot \epsilon_0 = \epsilon_0 \left(-\frac{d\psi}{dz} \right)$$

$$-\frac{d\psi}{dz} = A \Rightarrow \sigma = +\epsilon_0 \cdot \frac{U_{12}}{d}$$

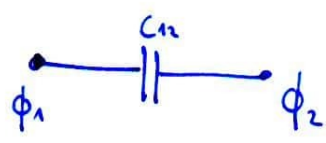
↑
grad-wert

$$\Rightarrow Q = a^2 \pi \sigma = \underbrace{\epsilon_0 \frac{a^2 \pi}{d}}_{C_{12}} \cdot U_{12}$$

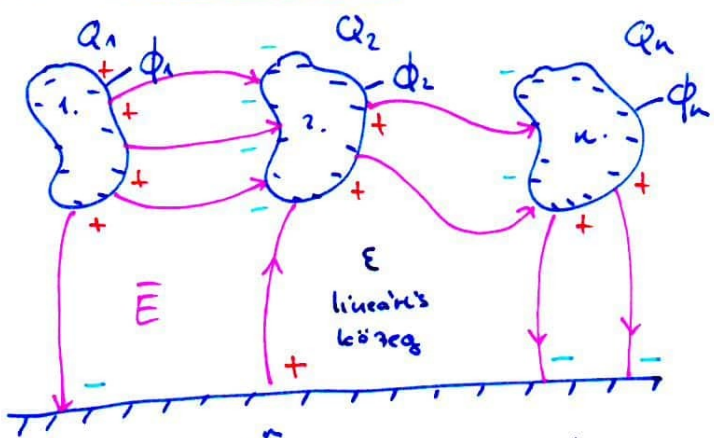
Emel:



$$C_{12} = \frac{Q}{u}$$



Többelektródos rendszerek



$$\phi_j = \sum_{i=1}^n p_{ij} Q_i$$

p_{ij} : potenciál/együttható
 $j = 1, \dots, n$

$$Q_0 = - \sum_{i=1}^n Q_i$$

$$\phi_0 \triangleq 0$$

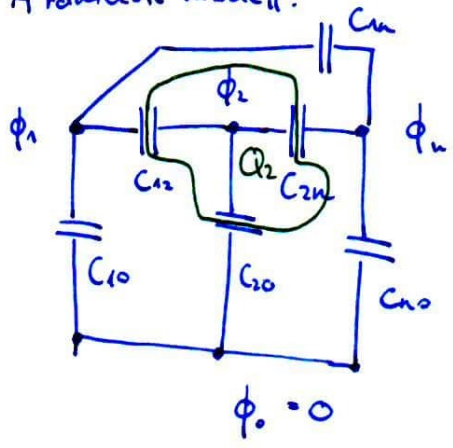
$$\underline{\phi} = \underline{p} \cdot \underline{Q}$$

$$\underline{Q} = \underline{c} \cdot \underline{\phi}$$

$\underline{p} = [p_{ij}]_{ij}$, $p_{ij} \equiv p_{ji}$ reciprocitás miatt

$\underline{c} = \underline{p}^{-1}$: kapacitív együttható mátrix

A'raunhőri modell:



$$Q_j = C_{j0} (\phi_j - 0) + \sum_{\substack{i=1 \\ i \neq j}}^n C_{ji} (\phi_j - \phi_i)$$

$$\underline{Q} = \underline{c} \cdot \underline{\phi}$$

$$\begin{bmatrix} Q_1 \\ \vdots \\ Q_n \end{bmatrix} = \begin{bmatrix} C_{11} & \dots & C_{1n} \\ \vdots & & \vdots \\ C_{n1} & \dots & C_{nn} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_n \end{bmatrix}$$



re ✖

$$Q_j = C_{j0}(\phi_j - 0) + \sum_{\substack{i=1 \\ i \neq j}}^n C_{ji}(\phi_j - \phi_i), \quad j = 1, \dots, n$$

$$C_{ij} = \left(C_{j0} + \sum_{\substack{i=1 \\ i \neq j}}^n C_{ji} \right)$$

$$C_{ij} \equiv C_{ji} = -C_{ji}$$

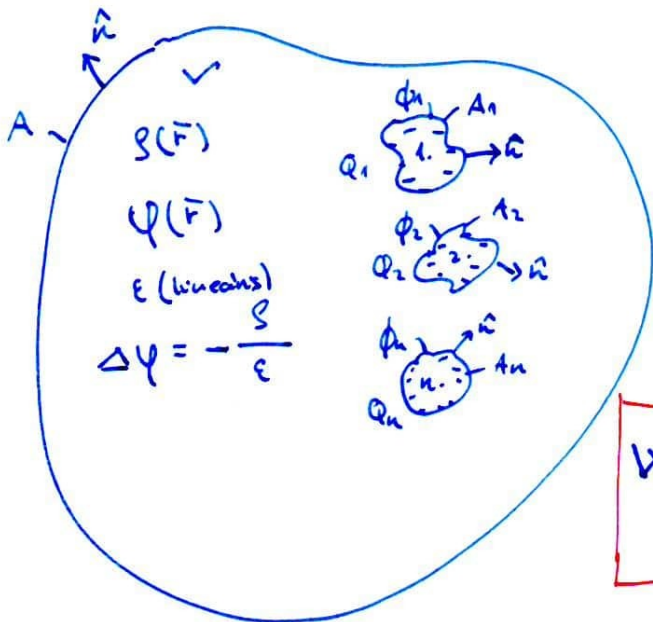
Resztkapacitások:

• földkapacitások: $C_{j0} \quad j = 1, \dots, n$

• fölkapacitások: $C_{ij} \quad i \neq j, i \neq 0, j \neq 0$

Gyakorlatban: kondenzátor

Az elektrosztatikus mező energiája



$$W_E = ? \quad V\text{-ben}$$

$$1.) w = \frac{1}{2} \vec{E} \cdot \vec{D} + \frac{1}{2} \vec{D} \cdot \vec{E} = \frac{1}{2} \epsilon |\vec{E}|^2$$

$$\vec{D} = \epsilon \cdot \vec{E}$$

$$W_E = \int_V w dV$$

$$2.) w = \frac{1}{2} \epsilon |\text{grad } \varphi|^2$$

$$\text{azonosság: } \text{div}(\varphi \text{grad } \varphi) = \underbrace{\text{grad } \varphi \cdot \text{grad } \varphi}_{|\text{grad } \varphi|^2} + \underbrace{\varphi \cdot \text{div grad } \varphi}_{\Delta \varphi}$$

$$W_E = \int_V \frac{1}{2} \epsilon [\text{div}(\varphi \text{grad } \varphi) - \varphi \Delta \varphi] dV =$$

$$\Delta \varphi = -\frac{\rho}{\epsilon}$$

Gauss

$$= \oint_A \frac{1}{2} \epsilon (\varphi \text{grad } \varphi) \cdot d\vec{A} - \sum_{i=1}^n \oint_{A_i} \frac{1}{2} \epsilon (\varphi \text{grad } \varphi) \cdot d\vec{A} + \int_V \frac{1}{2} \epsilon \rho dV$$

↗

$$W_E = \oint_A \frac{1}{2} \epsilon (\nabla \phi \cdot \nabla \phi) d\bar{A} - \sum_{i=1}^n \oint_{A_i} \frac{1}{2} \epsilon (\nabla \phi \cdot \nabla \phi) d\bar{A} + \int_V \frac{1}{2} \rho \phi dV \quad \text{①}$$

$-\vec{D} \rightarrow -D_n dA = -\sigma dA$

$$\text{②} \quad \underbrace{\oint_A \frac{1}{2} \epsilon (\nabla \phi \cdot \nabla \phi) d\bar{A}}_{\phi(\infty)=0, A \rightarrow \infty} + \underbrace{\sum_{i=1}^n \oint_{A_i} \frac{1}{2} \rho \phi dA}_{\sum_{i=1}^n \frac{1}{2} \phi_i \cdot Q_i} + \int_V \frac{1}{2} \rho \phi dV \quad \text{③}$$

$$\phi(\infty) = 0 \quad A \rightarrow \infty$$

$$\sum_{i=1}^n \frac{1}{2} \phi_i \cdot Q_i$$

$$\text{④} \quad \boxed{\sum_{i=1}^n \frac{1}{2} \phi_i \cdot Q_i + \int_V \frac{1}{2} \rho \phi dV}$$

gyakorlatban: $\rho = 0$

Alkalmazások:

1. Kondenzátor:

$$\left. \begin{aligned} Q_1 &= -Q_2 \\ U_{12} &= \phi_1 - \phi_2 \end{aligned} \right\} W_E = \frac{1}{2} Q \phi_1 + \frac{1}{2} (-Q \phi_2) = \frac{1}{2} Q \cdot U_{12} = \frac{1}{2} \frac{Q^2}{C_{12}} = \frac{1}{2} C_{12} U_{12}^2$$

2. Többelektródos rendszer:

$$W_E = \sum_{i=1}^n \frac{1}{2} \left(\sum_{j=1}^n c_{ij} \cdot Q_{ij} \right) Q_i = \frac{1}{2} \underline{Q}^T \cdot \underline{C} \cdot \underline{Q} \geq 0$$

↑
pozitív definit

$$W_E = \sum_{i,j} \frac{1}{2} c_{ij} (\phi_i - \phi_j)^2$$

↑
összes kapacitás

A helyettesítő töltések módszere

Cél: $\Delta\varphi = 0$ megoldása, ha elektródák potenciálja vagy töltés adott

Eszköz: elektróda + felületi töltés \Rightarrow szabad térben egyenlő töltés

peremfeltétel \longrightarrow számos peremfeltétel

$\Delta\varphi = 0$ egyenletű
megoldása

Egyenlő töltés elnevezésű potenciálja

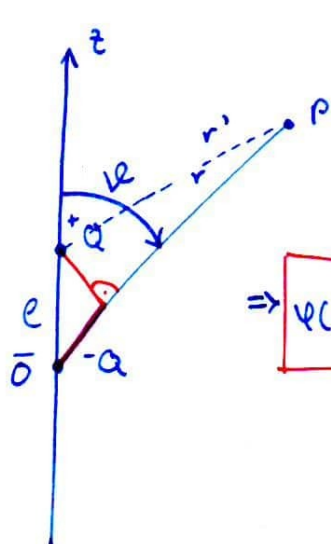
1.) Ponttöltés: $\varphi(r) = \frac{Q}{4\pi\epsilon} \cdot \frac{1}{r}$ r : gömb sugar

\hookrightarrow elvipotenciális koncentrikus gömbfelületek

2.) Végtelen hosszú egyenes vonaltöltés: $\varphi(r) = \frac{q}{2\pi\epsilon} \ln \frac{r_0}{r}$

r : hengerrugó, r_0 : referencia pont
elvipotenciális koncentrikus hengerpálya

3.) Elektrosztatikus dipólus:



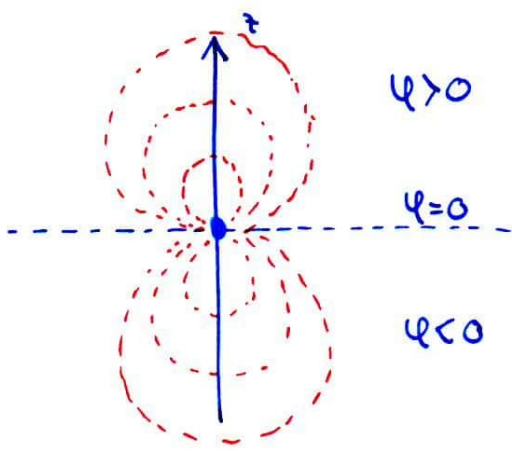
$$\varphi(P) = \frac{Q}{4\pi\epsilon} \left(\frac{1}{r'} - \frac{1}{r} \right)$$

$$\text{Ha } l \ll r : r' = r - l \cdot \cos\theta$$

$$\Rightarrow \varphi(P) = \frac{Q}{4\pi\epsilon} \frac{l \cdot \cos\theta}{r(r - l \cos\theta)} = \frac{p}{4\pi\epsilon} \cdot \frac{\cos\theta}{r^2}$$

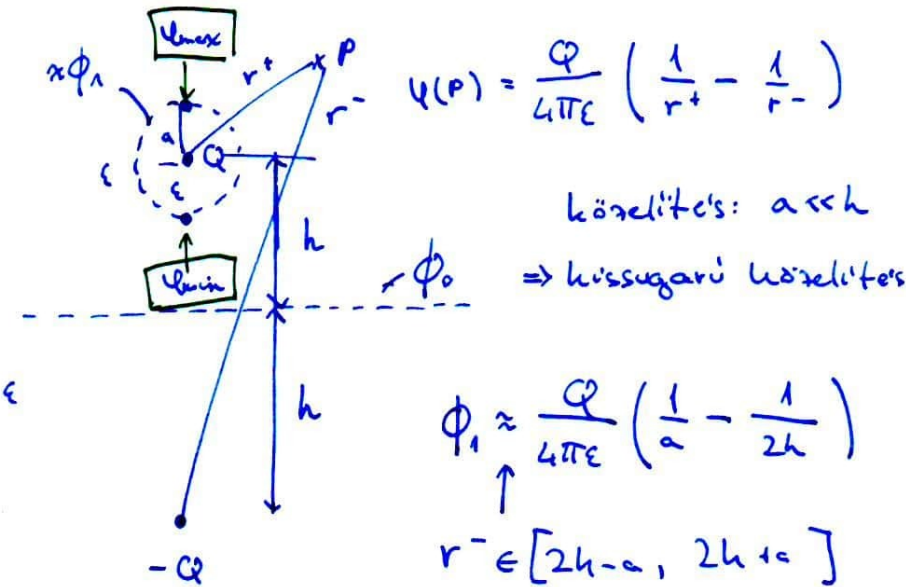
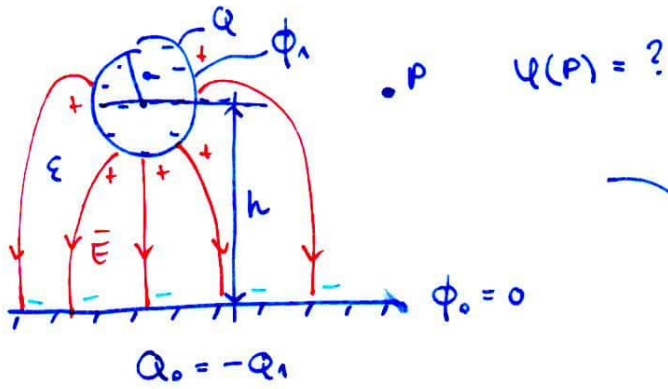
$$\left. \begin{array}{l} l \rightarrow 0 \\ Q \rightarrow \infty \end{array} \right\} Q \cdot l = \text{const.} = p$$

$$\vec{p} = Q \cdot l \cdot \hat{e}_z : \text{dipólus nyomaték vektor}$$



• dipólus

1. példa: gömb + ∞ felület



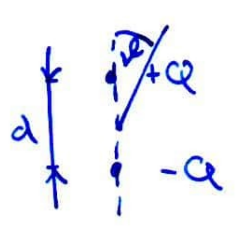
hiba: $\frac{\psi_{max} - \psi_{min}}{\phi_1} \approx \frac{a^2}{2h^2}$

általában:

$$C_{10} = \frac{Q}{\phi_1 - \phi_0} = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{2h}}$$

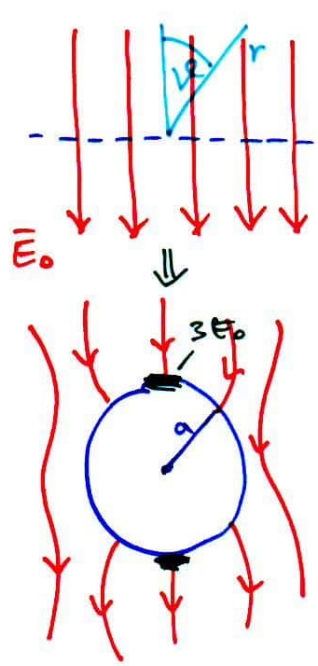
Tereh E.
15.10.14.

Ismétlés:



$P = Q \cdot d$
 $\varphi(r, \theta) = \frac{P}{4\pi\epsilon} \cdot \frac{\cos\theta}{r^2}$

pelda: Homogén térbe helyezett felhőgömb



$\vec{E} = -\hat{e}_z \cdot E_0$

$\varphi(z=0) = 0$ $\varphi(z) = E_0 z$

- gömb torzítja a teret
- $\vec{E} = \vec{E}_0 + \vec{E}_g$
- $\varphi = \varphi_0 + \varphi_g$

$\varphi(r=a, \theta) = \text{const.} = 0$ (polárkoordináta)

$\varphi_0(r, \theta) = E_0 \cdot r \cdot \cos\theta$

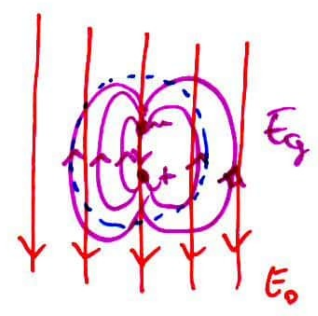
$0 = \varphi(a, \theta) = E_0 \cdot a \cdot \cos\theta + \varphi_g$

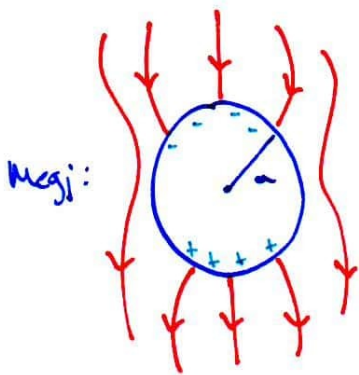
$\varphi_g(a, \theta) = -E_0 \cdot a \cdot \cos\theta$

\Rightarrow helyettesítő töltés: dipólus

$\frac{-P}{4\pi\epsilon} \cdot \frac{\cos\theta}{a^2} = E_0 \cdot a \cdot \cos\theta \Rightarrow P = -E_0 \cdot a^3 \cdot 4\pi\epsilon$

A két tér összege lesz a gömb előtt/ torzított ter





feltésszeítvétel (összeppeltes pe.)

Stacionárius állapotú felv

I. Alapösszefüggések

1) Maxwell: $\frac{\partial}{\partial t} = 0, \bar{J} \neq 0$

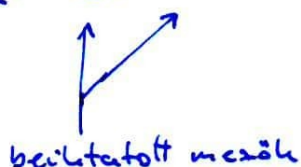
• $\text{rot } \bar{H} = \bar{J}$

$\text{div } \text{rot } \bar{H} = \text{div } \bar{J} = 0 \Rightarrow$ *maghasas egyenlet kiűzőbölése*

• $\text{div } \bar{J} = 0$

• $\text{rot } \bar{E} = 0$

• $\bar{J} = \sigma(\bar{E} + \bar{E}_b) + \bar{J}_b$



2) Laplace - Poisson :

$\text{rot } \bar{E} = 0 \Leftrightarrow \bar{E} = -\text{grad } \psi$

$\text{div}(\sigma(-\text{grad } \psi + \bar{E}_b) + \bar{J}_b) = 0$

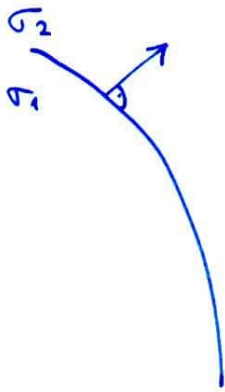
$\text{div}(\sigma \cdot \text{grad } \psi) = \text{div}(\sigma \bar{E}_b + \bar{J}_b)$: általános alak

Ha σ állandó :

$\Rightarrow \Delta \psi = \text{div}\left(\bar{E}_b + \frac{\bar{J}_b}{\sigma}\right)$: homogén térben

3) Hatarfeltetelek:

σ_1, σ_2 : különböző vezetőlepcsők



a) $E_{2t} = E_{1t}$: tangenciális komponensek



$$\phi_1 = \phi_2$$

b) $\vec{J}_{2n} = \vec{J}_{1n}$: normális komponensek



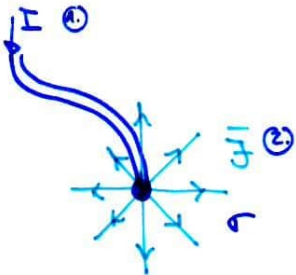
Ha nem lenne egyenlő \Rightarrow feltétlenül
halmozódna fel \Rightarrow lenne időbeli
változás.



$$\sigma_2 \cdot \frac{\partial \phi}{\partial n} = \sigma_1 \cdot \frac{\partial \phi}{\partial n} \quad (\vec{J} = \sigma \cdot \vec{E})$$

II. Egyzetű konfigurációk

1) Áram-pontforrás:



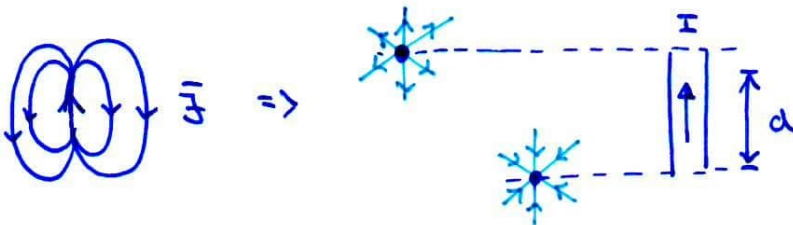
① \rightarrow ②

$$\vec{J}(r) = \frac{I}{4\pi r^2} \cdot \hat{e}_r$$

$$\vec{E}_r(r) = \frac{I}{4\pi \sigma r^2}$$

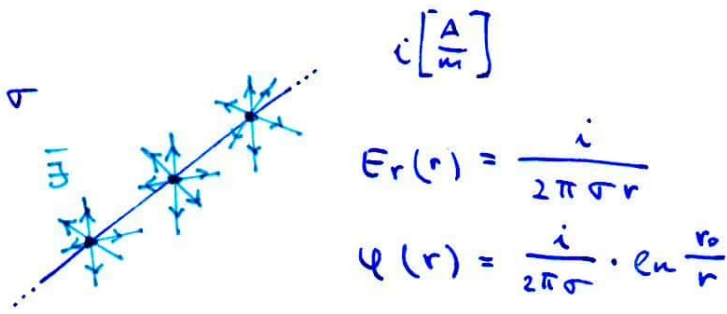
$$\phi(r) = \frac{I}{4\pi \sigma r}$$

2) Áramdipólus:



dipólusmomentum: $I \cdot d$

3) Áram- és a forrás:



$$E_r(r) = \frac{i}{2\pi\sigma r}$$

$$\varphi(r) = \frac{i}{2\pi\sigma} \cdot \ln \frac{r_0}{r}$$

III. Elektródarendszerek

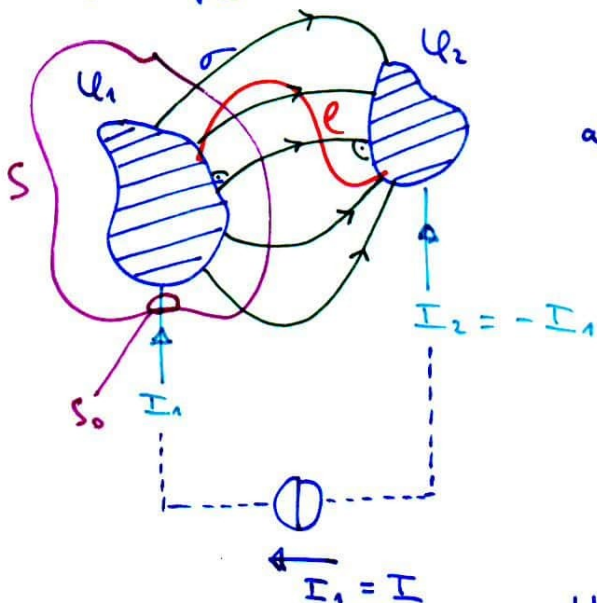
1) Az elektróda fogalma:

$$\sigma_{Au} = 6,3 \cdot 10^3 \frac{S}{m}$$

$$\sigma_{teflon} \approx 10^{-25} \frac{S}{m}$$

$$\frac{\sigma_{elektróda}}{\sigma_{közeg}} > \underline{1,2} \text{ nagyságrend (egy-hét)}$$

2) Elektródapár:



→ : áramvonalak

a) Lineáris közegre:

$$U_{12} = U_1 - U_2 \sim I_0$$

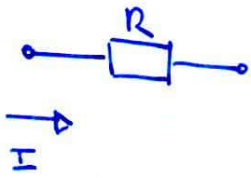
$$R = \frac{U}{I} [\Omega]$$

$$G = \frac{I}{U} [S]$$

$$I = \int_{S|S_0} \vec{J} d\vec{s} \rightarrow R = \frac{\int \vec{E} d\vec{e}}{\int_{S|S_0} \vec{E} \cdot d\vec{s}} \quad U_{12} = \int_e \vec{E} d\vec{e}$$

↻

b) Ellenőrizd a Joule-törvényt:



$$P = I^2 \cdot R \Rightarrow$$

$$R \triangleq \frac{P}{I^2}$$

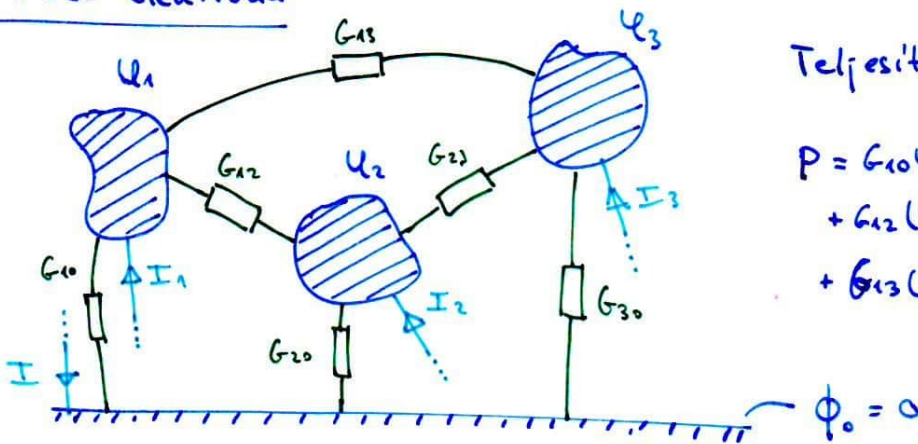
↳ disszipált teljesítmény

$$p = \vec{j} \cdot \vec{E} \left[\frac{W}{m^3} \right]$$

$$p = \frac{d^2}{\eta} \quad (\vec{E}_b = 0, \vec{j}_b = 0)$$

$$R = \frac{\int_V \frac{j^2}{\eta} dV}{I^2}$$

3) Több elektróda



Teljesítmény:

$$P = G_{10} U_1^2 + G_{20} U_2^2 + G_{30} U_3^2 + G_{12} (U_1 - U_2)^2 + G_{13} (U_1 - U_3)^2 + G_{23} (U_2 - U_3)^2$$

$$I = -(I_1 + I_2 + I_3)$$

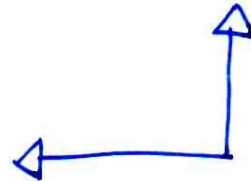
reakonditanciók:

$$I_1 = G_{10} U_1 + G_{12} (U_1 - U_2) + G_{13} (U_1 - U_3)$$

$$I_2 = G_{20} U_2 + G_{21} (U_2 - U_1) + G_{23} (U_2 - U_3)$$

$$I_3 = G_{30} U_3 + G_{31} (U_3 - U_1) + G_{32} (U_3 - U_2)$$

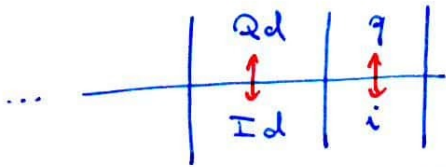
$$U_3 = U_3 - U_0 = U_3 - 0$$



IV. Elektrosztatika analógiaja

$$\begin{cases} \operatorname{div}(\epsilon \operatorname{grad} \varphi) = -\rho \\ \operatorname{div}(\sigma \operatorname{grad} \varphi) = \operatorname{div}(\sigma \vec{E}_b + \vec{J}_b) \end{cases} \longleftrightarrow \Delta \varphi = -\frac{\rho}{\epsilon}$$

elektroszt.	(φ, \vec{E})	ϵ	\vec{D}	ρ	C	rezkaptacitások	Q
áramlás	(φ, \vec{E})	σ	\vec{J}	$\operatorname{div}(\sigma \vec{E}_b + \vec{J}_b)$	G	rezkonduktanciák	I



spec.: homogén lineáris közeg:

$$G = C \quad \left| \quad \right. = C \cdot \frac{\sigma}{\epsilon}$$

$E \leftarrow \sigma$

: feladatmegoldásnak!

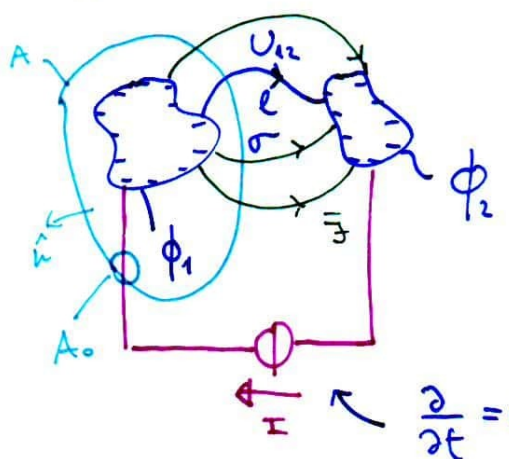
Tereh Gy.
15.10.15.

Stacionárius áramlás

$$\frac{\partial}{\partial t} = 0, \quad \vec{J} \neq 0, \quad \vec{J} = \sigma \cdot \vec{E}, \quad [\sigma] = \frac{S}{m}$$

$$\text{rot } \vec{E} = 0 \Rightarrow \vec{E} = -\text{grad } \varphi$$

$$\text{div } \vec{J} = 0$$



$$U_{12} = \int_e \vec{E} \cdot d\vec{l}$$

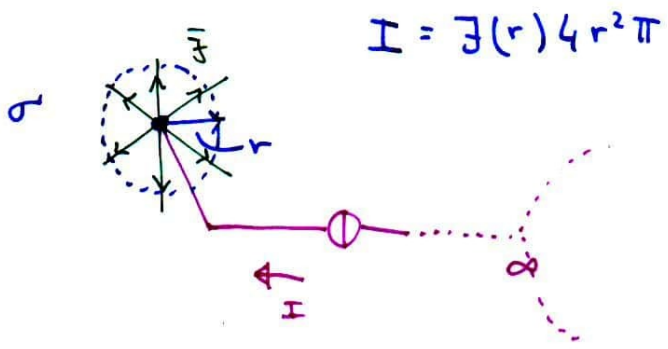
$\frac{\partial}{\partial t} = 0$ miatt

$$R \stackrel{\Delta}{=} \frac{U_{12}}{I}$$

$$\oint_A \vec{J} \cdot d\vec{A} = 0 = \int_{A \setminus A_0} \vec{J} \cdot d\vec{A} + \underbrace{\int_{A_0} \vec{J} \cdot d\vec{A}}_{-I}$$

-I ("-", mert \hat{n} wifele mutat)

Pontforrás:



$$I = J(r) 4r^2 \pi$$

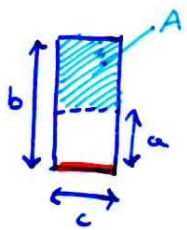
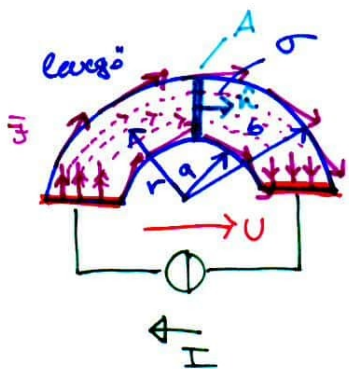
$$J(r) = \frac{I}{4\pi r^2}$$

$$E(r) = \frac{I}{4\pi \sigma} \cdot \frac{1}{r^2}$$

$$\varphi(r) = \frac{I}{4\pi \sigma} \cdot \frac{1}{r} \left(+ \frac{\varphi(\infty)}{0} \right)$$

Peldatár: 3.3 - 3.7

1 Ellenállás számítás: felgyűrű $R = ?$

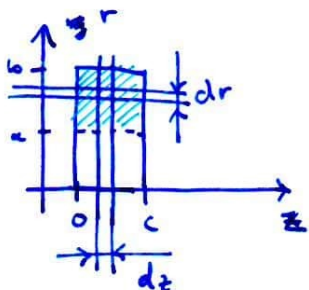


$$\vec{J} = \sigma \vec{E}$$

$$U = \int_e \vec{E} d\vec{e} = r \cdot \pi \cdot E(r) = r \pi \frac{J(r)}{\sigma}$$

$$\boxed{J(r) = \frac{U\sigma}{r\pi}} : \text{Aronsűrűség helyfüggő}$$

$$I = \int_A \vec{J} d\vec{A}$$

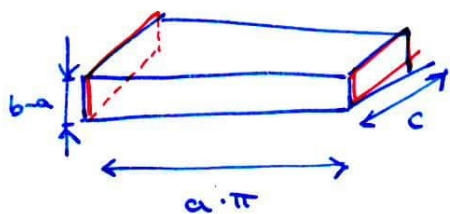


$$I = \int_A \vec{J} d\vec{A} = \int_{z=0}^b \int_{r=a}^c J(r) \cdot dr \cdot dz =$$

$$= c \cdot \frac{U\sigma}{\pi} \int_a^b \frac{1}{r} dr = c \frac{U\sigma}{\pi} \ln \frac{b}{a}$$

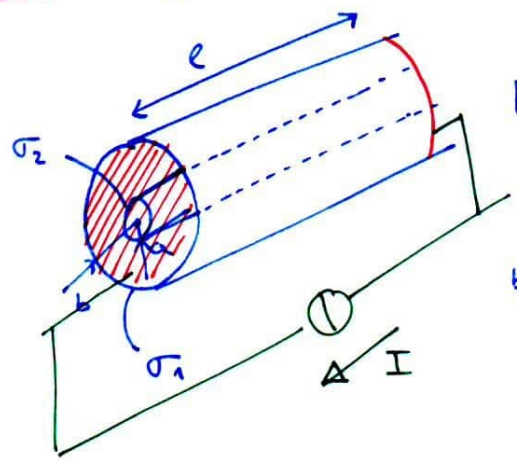
$$\Rightarrow \underline{\underline{R \hat{=} \frac{U}{I}}} = \underline{\underline{\frac{\pi}{c \cdot \sigma \cdot \ln \frac{b}{a}}}}$$

Közelítés: $b - a \ll a$

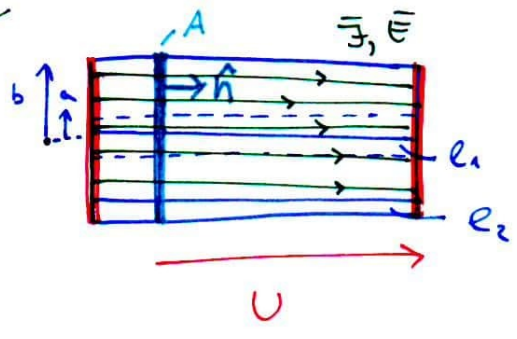


$$R = \frac{a \pi}{\sigma (b-a) \cdot c}$$

2. Rétvezet vezeték



$R' = \frac{R}{l}$: egység hosszra jutó ellenállás?



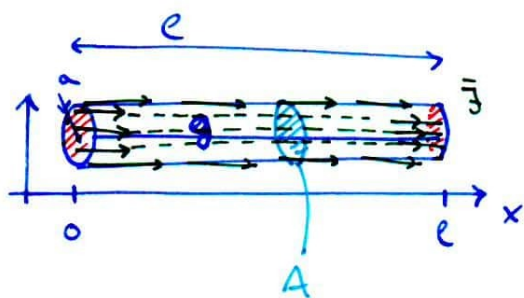
$$U = \int_{e_1} \vec{E} d\vec{l} = \int_{e_2} \vec{E} d\vec{l} \Rightarrow \vec{E} = \text{konst.} =$$

$$\begin{aligned} \text{er: } \vec{J}_1 &= \sigma_1 \vec{E}_a \\ \text{külső: } \vec{J}_2 &= \sigma_2 \cdot \vec{E} \end{aligned} \left. \vphantom{\begin{aligned} \text{er: } \vec{J}_1 &= \sigma_1 \vec{E}_a \\ \text{külső: } \vec{J}_2 &= \sigma_2 \cdot \vec{E} \end{aligned}} \right\} \rightarrow I = a^2 \pi \vec{J}_1 + (b^2 - a^2) \pi \vec{J}_2$$

$$R = \frac{U}{I} = \frac{l \cdot E_{(1)}}{a^2 \pi J_1 + (b^2 - a^2) \pi J_2} = \frac{l \cdot E}{a^2 \pi \sigma_1 \cdot E + (b^2 - a^2) \cdot \pi \sigma_2 E} = \frac{l}{a^2 \pi \sigma_1 + (b^2 - a^2) \pi \sigma_2}$$

$$R' = \frac{R}{l}$$

3 Váltakozó vezetékességű kábel



$$R = ?$$

$$\sigma(x) := \sigma_0 \cdot e^{-\alpha x} \leftarrow$$

$$\vec{J} = \text{const.} \Rightarrow \vec{E} \neq \text{const.}$$

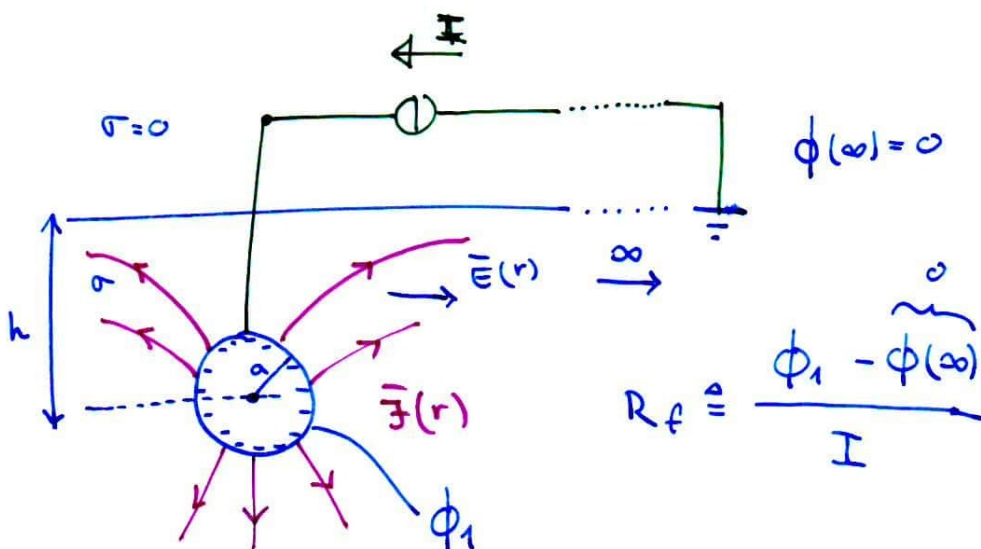
$$\vec{E} = \frac{\vec{J}}{\sigma(x)} \Rightarrow E(x) = \frac{J}{\sigma(x)}$$

$$U = \int_{\varphi} \vec{E} d\vec{q} = \int_0^l E(x) dx = \frac{J}{\sigma_0} \int_0^l \frac{dx}{e^{-\alpha x}} = \frac{J}{\sigma_0} \cdot \frac{e^{\alpha l} - 1}{\alpha}$$

$$I = J \cdot a^2 \pi \leftarrow \text{" } A \text{" - ből}$$

$$R = \frac{U}{I} = \frac{e^{\alpha l} - 1}{\sigma_0 a^2 \pi}$$

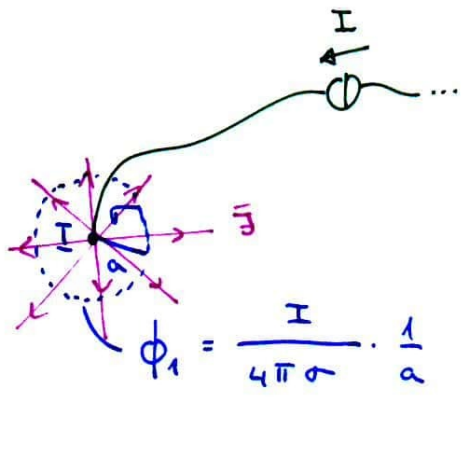
4 Földelési ellenállás



$$R_f \triangleq \frac{\phi_1 - \phi(\infty)}{I}$$



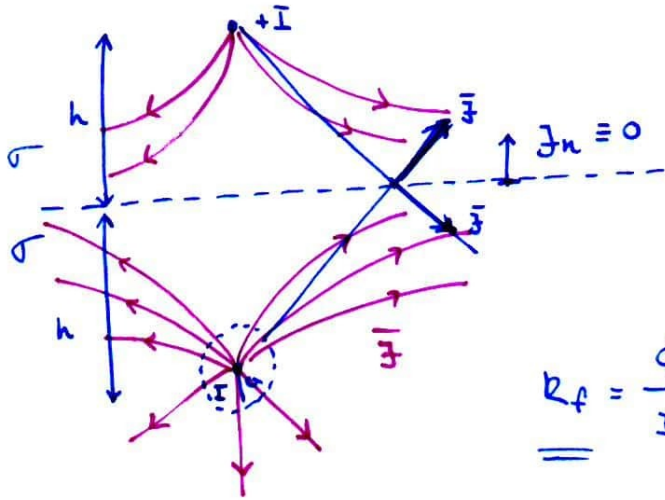
a) $h \gg a$



$$\phi_1 = \frac{I}{4\pi\sigma} \cdot \frac{1}{a} \Rightarrow R_f = \underline{\underline{\frac{1}{4\pi\sigma a}}}$$

b) $h \gg a$ (földfelszín hatása nem elhanyagolható)

Helyettesítő pontforrás



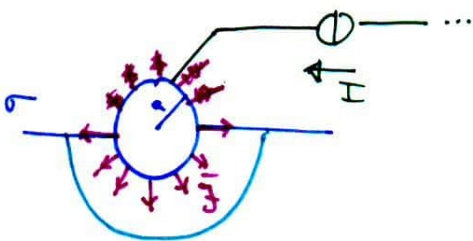
$$\phi_1 \approx \frac{I}{4\pi\sigma a} + \frac{I}{4\pi\sigma 2h} = \frac{I}{4\pi\sigma} \frac{2h+a}{2ha}$$

$$R_f = \frac{\phi_1}{I} = \frac{1}{4\pi\sigma} \frac{2h+a}{2ha} \xrightarrow{h \rightarrow \infty} \frac{1}{4\pi\sigma} \cdot \frac{1}{a}$$

c) $h \approx a$

↑
↔ ∇ zart alakú megoldás

d) $h = 0$



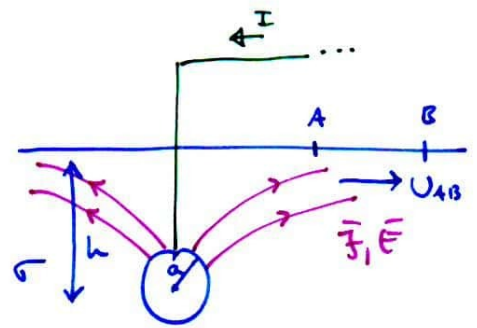
$$I = \int(r) \cdot \frac{4r^2\pi}{2}$$

↑
"D" miatt

$$R_f = 2 \cdot \frac{1}{4\pi\sigma \cdot a}$$

====

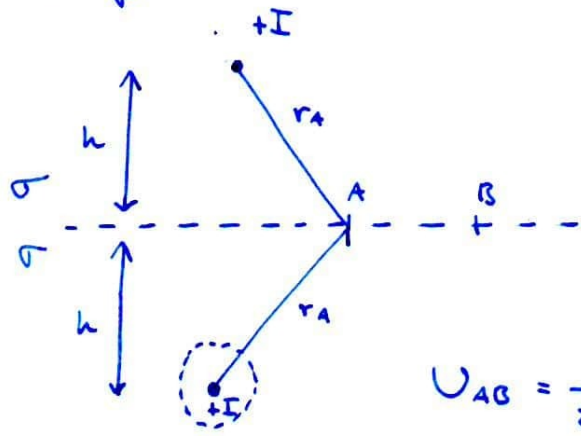
5) Lépésfeszültség



$U_{AB} = ?$

$U_{AB} = \int_A^B \vec{E} \cdot d\vec{l}$

$\vec{B} = \frac{\mu_0 I}{2\pi r}$



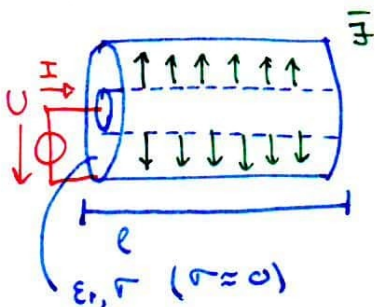
$\phi_A = 2 \cdot \frac{I}{4\pi\sigma} \cdot \frac{1}{r_A}$

$\phi_B = 2 \cdot \frac{I}{4\pi\sigma} \cdot \frac{1}{r_B}$

$U_{AB} = \phi_A - \phi_B$

$U_{AB} = \frac{I}{2\pi\sigma} \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$

6) Elektrosztatikus analógia



szivócső ellenállás = ?

$R_{sz} = ?$, C , ϵ_r , σ adott

$R_{sz} = \frac{U}{I}$ $R_{sz} = \frac{1}{G \cdot l}$

$\frac{C'}{G'} = \frac{\epsilon}{\sigma} = \frac{\epsilon_0 \cdot \epsilon_r}{\sigma}$

$G' = C' \cdot \frac{\sigma}{\epsilon_0 \cdot \epsilon_r}$

$R_{sz} = \frac{1}{G' \cdot l}$

Terdu E.
15.10.21.

Stacionárius áramok mágneses tere

$\frac{\partial}{\partial t} \equiv 0, \vec{J}$ helyi = állandó.

$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$

$rot \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

$rot \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$div \vec{B} = 0$

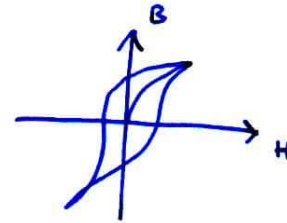
$div \vec{D} = \rho$

$\rightarrow rot \vec{H} = \vec{J}$

$div \vec{B} = 0$

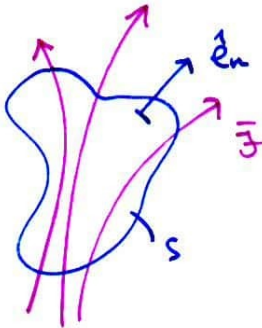
$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$

• Laplace-egyenlet: $\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$

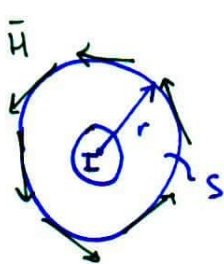


Miss terelés

$\int_S rot \vec{H} d\vec{S} = \int_S \vec{J} d\vec{S} \xrightarrow{\text{Stokes}} \oint_C \vec{H} d\vec{e} = \int_S \vec{J} d\vec{S} = I$



1) Egyenes vezetők által beltett mágneses tér:



$\oint_C \vec{H} d\vec{e} = \int_S \vec{J} d\vec{A} = I$

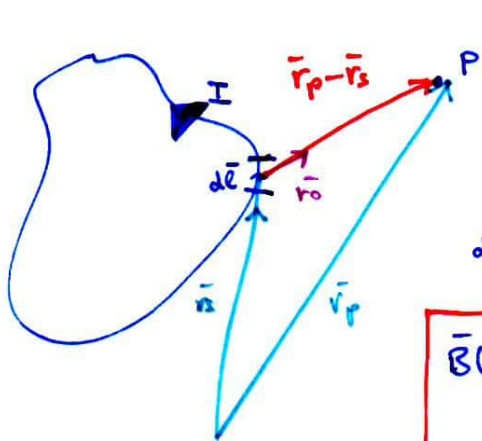
\vec{H} tangenciális. ← Mert $div \vec{B} = 0$

(Ha \vec{H} nem tangenciális lenne $\Rightarrow div \vec{B} \neq 0 \Rightarrow \nabla \cdot \vec{B} \neq 0$)

$2\pi H(r) r = I$

$H(r) = \frac{I}{2\pi r}$

2) • Vonalsterű áramok mágneses tere:



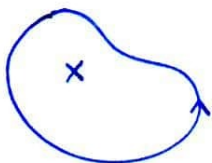
$$\frac{\vec{r}_p - \vec{r}_s}{|\vec{r}_p - \vec{r}_s|} = \vec{r}_0 = \text{egység}$$

$$d\vec{B}(\vec{r}_p) = \frac{\mu I}{4\pi} \cdot \frac{d\vec{l} \times \vec{r}_0}{|\vec{r}_p - \vec{r}_s|^2}$$

$$\vec{B}(\vec{r}_p) = \mu \frac{I}{4\pi} \oint \frac{d\vec{l}' \times (\vec{r}_p - \vec{r}')}{|\vec{r}_p - \vec{r}'|^2} \quad : \text{Biot-Savart}$$

egységvektor lesz

Következtetés:

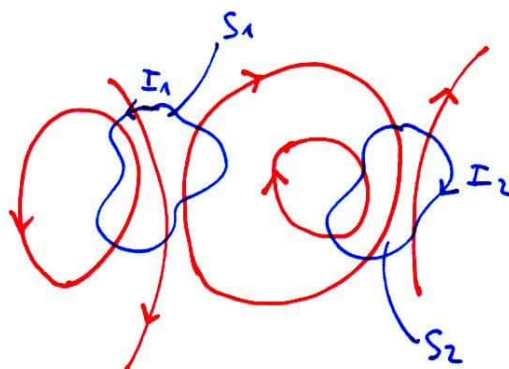
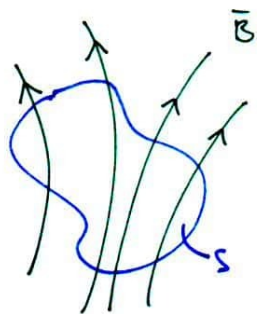


Síkban fekvő vezető mágneses tér
végtelen merőleges a vezető által
még határozott síkára.

3) • Mágneses fluxus, mágneses indukció

Vonalsterű áramok fluxusa

$$\Phi = \int_S \vec{B} \cdot d\vec{S}$$



$$\begin{cases} \Phi_1 = L_{11} I_1 + L_{12} I_2 \\ \Phi_2 = L_{21} I_1 + L_{22} I_2 \end{cases}$$

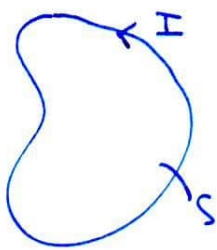
L_{11} : önműködés együttható

L_{22} : hátrösös indukciós együttható

$$L_{12} = L_{21}$$

$$\Phi_i = \sum_{j=1}^N L_{ij} I_j \quad i=1, \dots, N$$

Biot-Savart's ($W_M = \frac{1}{2} \sum_i \sum_j L_{ij} I_j I_i$)



$$W_M = \frac{1}{2} \mu_0 H^2 \quad (\text{levegőben})$$

$$W_M = \int_V w_M dV$$

$$\text{rot } \vec{H} = \vec{J}$$

$$\text{div } \vec{B} = 0 \xrightarrow{\text{def.}} \vec{B} \triangleq \text{rot } \vec{A} \Rightarrow \text{div } \vec{B} = 0 \quad \text{automatikusan teljesül}$$

azonosság: $\text{div}(\vec{H} \times \vec{A}) = \vec{A} \cdot \text{rot } \vec{H} - \vec{H} \cdot \text{rot } \vec{A}$

$$W_M = \frac{1}{2} \mu_0 H^2 = \frac{1}{2} \vec{H} \cdot \vec{B} = \frac{1}{2} \vec{H} \cdot \text{rot } \vec{A} \stackrel{\downarrow}{=} \frac{1}{2} \vec{A} \cdot \text{rot } \vec{H} - \frac{1}{2} \text{div}(\vec{H} \times \vec{A})$$

$$W_M = \int_V w_M dV = \int_V \frac{1}{2} \vec{A} \cdot \underbrace{\text{rot } \vec{H}}_{\vec{J}} dV - \int_V \frac{1}{2} \text{div}(\vec{H} \times \vec{A}) dV$$

$$\oint_{S_\infty} \frac{1}{2} \vec{H} \times \vec{A} dS$$

S_∞ : $\bigotimes^r \quad r \rightarrow \infty$

$$H \sim \frac{1}{r^2}, \quad A \sim \frac{1}{r}$$

$$W_M = \int_V \frac{1}{2} \vec{A} \cdot \vec{J} dV - \oint_{S_\infty} \frac{1}{2} \vec{H} \times \vec{A} dS \quad \text{⊖}$$

$$= \oint_e \frac{1}{2} \vec{A} \cdot \vec{I} d\vec{l} \quad \text{⊕}$$

↑
vonalmenti alakzatokra

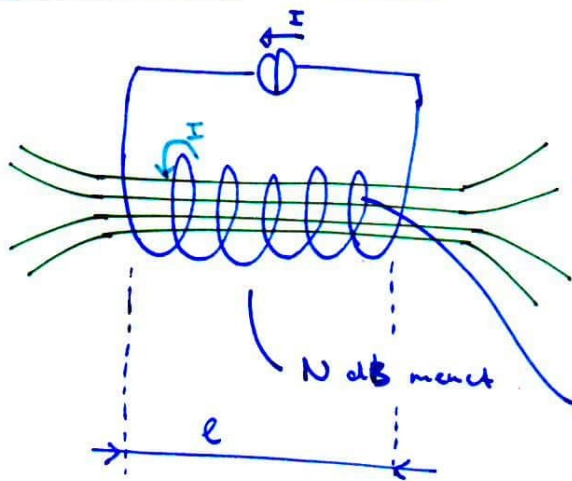
$= 0 \rightarrow$ mert $\vec{H}, \vec{A} \rightarrow \phi$

$$\text{⊕} \oint_e \frac{1}{2} \vec{A} \cdot \vec{I} d\vec{l} \stackrel{\uparrow \text{ Stokes}}{=} \int_S \frac{1}{2} \vec{B} \cdot \vec{I} dS = \vec{I} \cdot \frac{1}{2} \int_S \vec{B} dS = \frac{1}{2} \phi \cdot \vec{I} = W_M$$

Nóda hálót esetében:

$$W_M = \sum_{i=1}^N \frac{1}{2} \phi_i \cdot I_i = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N L_{ij} I_j I_i$$

Speciális eset: tekercs



$$e \cdot H = N \cdot I$$

$$H = \frac{N \cdot I}{e}$$

$$\phi = S_0 \cdot B = S_0 \cdot \mu \cdot \frac{N \cdot I}{e}$$

S_0 (tekercs keresztmetsete)

$$\Psi = N \cdot \phi = \frac{S_0 \mu N^2 I}{e} = L \cdot I \Rightarrow L = \frac{S_0 \mu N^2}{e}$$

↑ tekercs fluxus = (menetek) · (egy db menetfluxus)

4) Mágneses energia

$$w_M = \frac{1}{2} \vec{H} \cdot \vec{B} = \frac{1}{2} \mu \cdot H^2 = \frac{1}{2} \frac{B^2}{\mu}$$

↑ homogén, izotrop közegben

Teljes tér energiája:

$$W_M = \int_V \frac{1}{2} \vec{H} \cdot \vec{B} dV = \int_V \frac{1}{2} \mu \cdot H^2 dV$$

↑ homogén, izotrop

: közelethez alapja

$$W_M = \frac{1}{2} \sum_{i=1}^N \phi_i I_i$$

: valóság I_i -vel egyért. vezet. keretű csőben

$$\frac{1}{2} \sum_{i=1}^N \phi_i I_i = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N (L_{ij} I_j) I_i$$

↑ ϕ_i

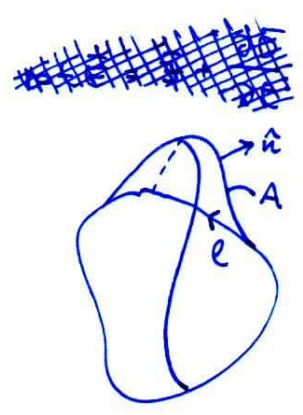
: távolhelei esetén

Terek E.
15.10.22.

Indukálási jelenségek

gyakorlatban: generátor, transzformátor

Faraday - törvény:



$$\text{rot } \vec{E} = - \frac{\partial \vec{B}}{\partial t} \Rightarrow \oint_l \vec{E} d\vec{l} = - \int_A \frac{\partial \vec{B}}{\partial t} d\vec{A}$$

Uygalmasi indukció

"álló" görbe: $\oint_l \vec{E} d\vec{l} = - \frac{d}{dt} \int_A \vec{B} d\vec{A} = - \frac{d\phi}{dt}$

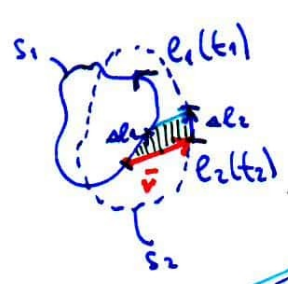
$\underbrace{\oint_l \vec{E} d\vec{l}}_{U_i(t)}$ $\underbrace{\int_A \vec{B} d\vec{A}}_{\phi \text{ fluxus}}$

indukált feszültség

$$U_i(t) = - \frac{d\phi}{dt}$$

Mozgási indukció

"mozgó" görbe: $\vec{B} = \vec{B}(\vec{r})$, \vec{B} időben állandó



$\Delta t = t_2 - t_1$
 $\Delta \phi = ?$

$\Delta S = \Delta l_1 (\vec{v} \cdot \Delta t \cdot \sin \alpha)$

$\Delta \vec{S} = (\vec{v} \cdot \Delta t) \times \Delta \vec{l}_1$

$\Delta \vec{S} \cdot \vec{B} = \vec{B} (\vec{v} \times \Delta \vec{l}_1) \cdot \Delta t = \Delta \vec{l}_1 (\vec{B} \times \vec{v}) \cdot \Delta t$

vektorszorzás
vektorok azonos irányú

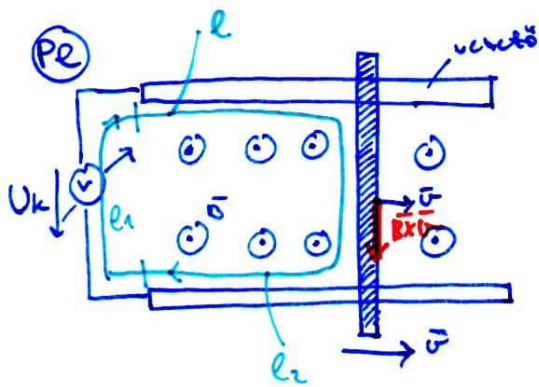
$\Delta \phi = \left(\oint_l d\vec{l} (\vec{B} \times \vec{v}) \right) \cdot \Delta t$

$$U_i(t) = - \frac{d\phi}{dt} = - \oint_l (\vec{B} \times \vec{v}) \cdot d\vec{l}$$

Fredő: $\vec{B} = \vec{B}(\vec{r}, t) + \ell$ mozog

$$u_i(t) = - \underbrace{\frac{d}{dt} \int_S \vec{B} d\vec{S}}_{\text{nyugalmi ind.}} - \underbrace{\oint_C (\vec{B} \times \vec{v}) d\vec{l}}_{\text{mozgási ind.}}$$

nyugalmi ind. mozgási ind.



\vec{B} homogén

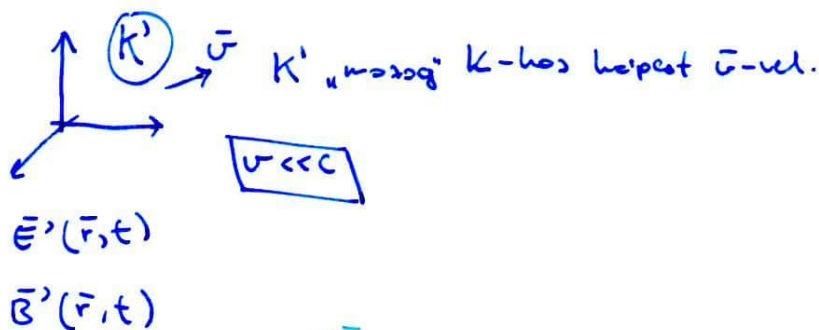
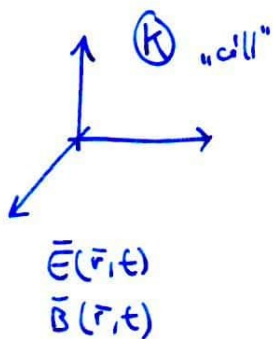
$$\ell = \ell_1 U \ell_2$$

$$u_i = - \oint_C (\vec{B} \times \vec{v}) d\vec{l} = -a \cdot B \cdot v$$

$$\oint_C \vec{E} d\vec{l} = \int_{\ell_1} \vec{E} d\ell + \underbrace{\int_{\ell_2} \vec{E} d\ell}_0 = -U_k$$

$$U_{ind} = -U_k$$

\vec{E} transformáció/és a mozgó vonatkoztatási rendszerekben



$$\vec{B} \equiv \vec{B}' \quad (v \ll c)$$

$$- \underbrace{\int_S \frac{\partial \vec{B}}{\partial t} d\vec{S} - \oint_C (\vec{B} \times \vec{v}) d\vec{l}}_{u_i \text{ K-ban}} \equiv \underbrace{\oint_C \vec{E}' d\ell}_{\text{K}'\text{-ben}}$$

ℓ görbe K' -ben áll.

u_i K-ban

K' -ben



$$\oint_e \vec{E} d\vec{e} = \oint_e (\vec{B} \times \vec{v}) d\vec{e} + \oint_e \vec{E}' d\vec{e}$$

$$\Rightarrow \vec{E} = \vec{B} \times \vec{v} + \vec{E}' \quad \text{-nek kell lenne}$$

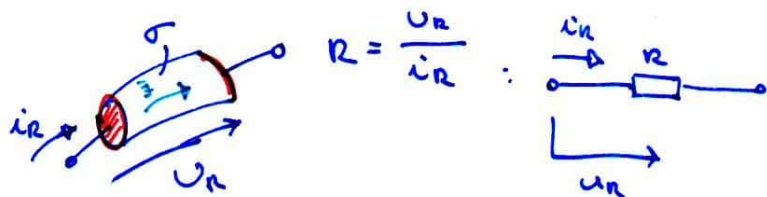
$$\Rightarrow \boxed{\vec{E}' \equiv \vec{E} + \vec{v} \times \vec{B}}$$

Koncentrált ~~paraméter~~ paraméterű hallózatok vizsgálata

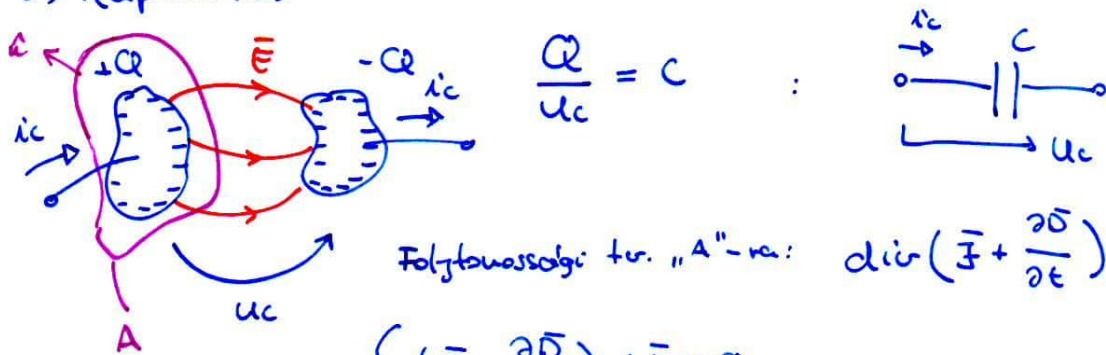
Kétpólusok leírásaitól + Kirchhoff-törvények

I. Kétpólusok

1) Ellenállás



2) Kapacitás



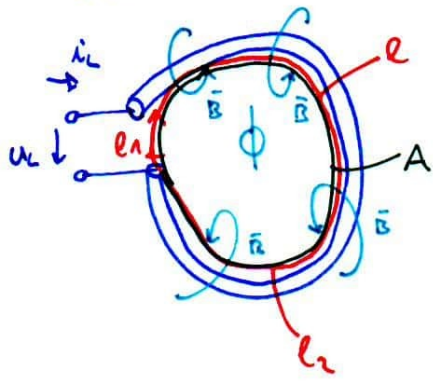
Folytonossági tör. "A"-ra: $\text{div}(\vec{J} + \frac{\partial \vec{D}}{\partial t}) = 0$

$$\oint_A (\vec{J} + \frac{\partial \vec{D}}{\partial t}) d\vec{A} = 0$$

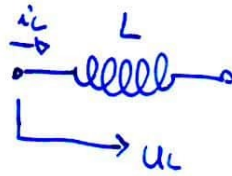
$$-i_C + \oint_A \frac{\partial \vec{D}}{\partial t} d\vec{A} = 0 \Rightarrow -i_C + \frac{dQ}{dt} = 0$$

$$\Rightarrow \boxed{i_C = C \cdot \frac{dU_C}{dt}}$$

3) Induktivitás



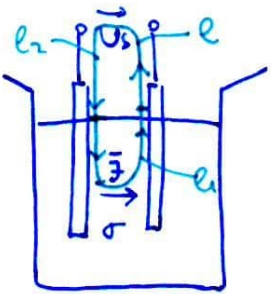
$$\oint_e \vec{E} d\vec{l} = - \frac{d\Phi}{dt}$$



$$\int_{e_1} \vec{E} d\vec{l} + \int_{e_2} \vec{E} d\vec{l} = - \frac{d\Phi}{dt}$$

$$u_L = - \left(- \frac{d\Phi}{dt} \right) \Rightarrow u_L = L \cdot \frac{di_L}{dt}$$

4.) Feszültségforrás

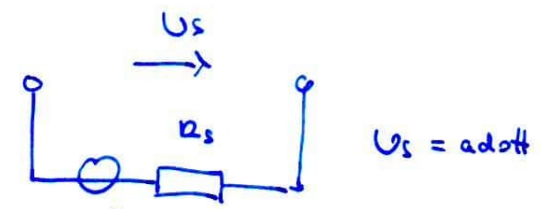


$$\vec{J} = \sigma(\vec{E} + \vec{E}_b) + \vec{J}_b, \quad \vec{J}_b := 0$$

$$\vec{E} = \frac{1}{\sigma} \vec{J} - \vec{E}_b$$

$$\oint_e \vec{E} d\vec{l} = 0$$

$$\int_{e_1} \vec{E} d\vec{l} + \int_{e_2} \vec{E} d\vec{l} = 0 \approx \int_{e_1} \left(\frac{1}{\sigma} \vec{J} - \vec{E}_b \right) d\vec{l}$$



ideális fesz.forr.: $\sigma \approx \infty$
 $\Rightarrow R_0 = 0$

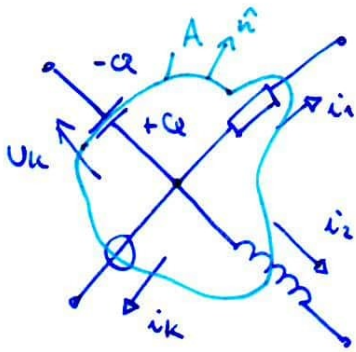
$$\int_{e_1} \left(\frac{1}{\sigma} \vec{J} - \vec{E}_b \right) d\vec{l} - U_0 = 0$$

$$U_0 = - \int_{e_1} \vec{E}_b d\vec{l} + \int_{e_1} \frac{1}{\sigma} \vec{J} d\vec{l}$$



II. Kirchhoff-törvények

1.) Csomóponti-törvény (töltésmegmaradás elve)



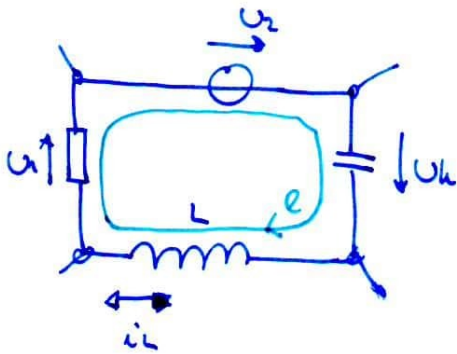
$$\oint_A (\vec{J} + \frac{\partial \vec{D}}{\partial t}) \cdot d\vec{A} = 0$$

$$\sum_{k=1}^K i_k + \underbrace{C \cdot \frac{dU_k}{dt}}_{i_c} = 0$$

tetszőleges csomópont:

$$\sum_{\text{összes}} i \equiv 0$$

2.) Körvonal-törvény



$$\oint_e \vec{E} \cdot d\vec{e} = - \underbrace{\frac{d\phi}{dt}}_{\frac{d}{dt}(L \cdot i)}$$

$$\sum_{k=1}^K U_k + L \cdot \frac{di_k}{dt} = 0$$

$$\textcircled{*} : \sum_{k=1}^K U_k$$

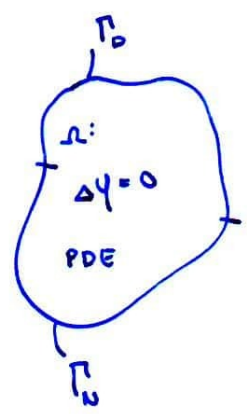
tetszőleges körvonal:

$$\sum_{\text{összes}} U_k \equiv 0$$

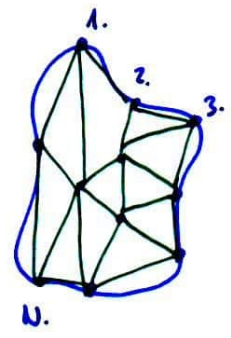
Számítógépes demonstráció

Vegyes elem-módszerrel \rightarrow

$\Delta\psi \equiv \text{div grad}\psi$

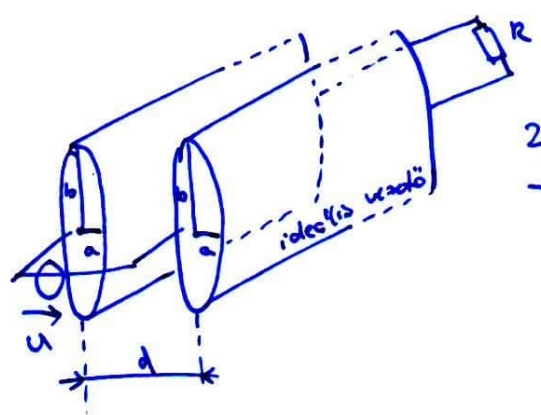


diszkrétizálás \rightarrow

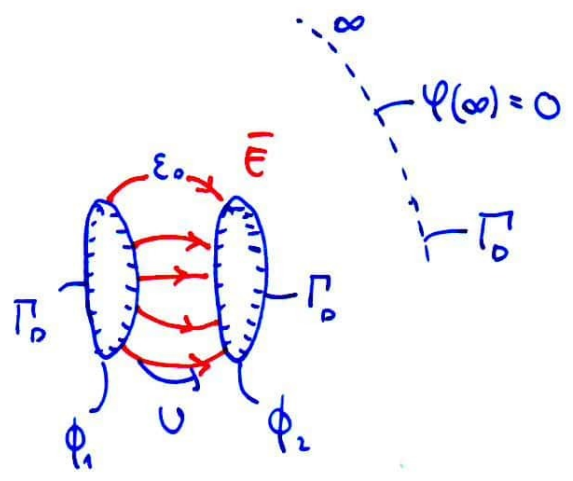


$\psi(x,y) = ? \rightarrow \psi_1, \psi_2, \dots, \psi_N = ?$: Lineáris algebraic egyenletrendszer

① Ellipszis alakú vezetők



2D metszet \rightarrow

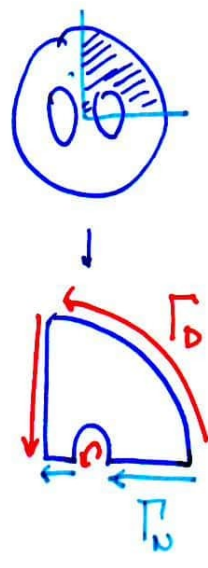


Hosszagszög és kapacitás?

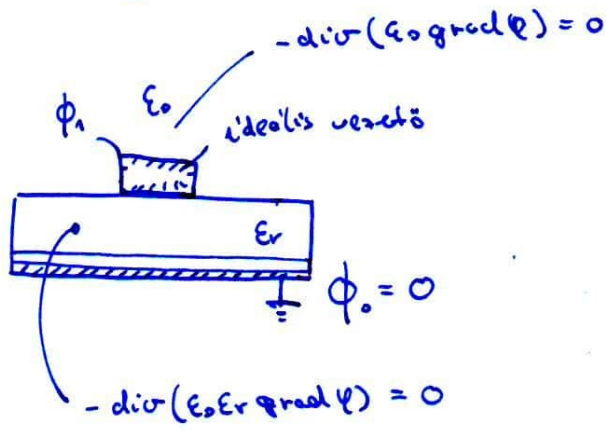
$W_E = \frac{1}{2} C U^2$

$\int w dV = W_E, w = \frac{1}{2} \epsilon_0 |\vec{E}|^2$

$\vec{E} = -\text{grad}\psi$



② Mikrostruktúra-vezetők



Tápvezetékek

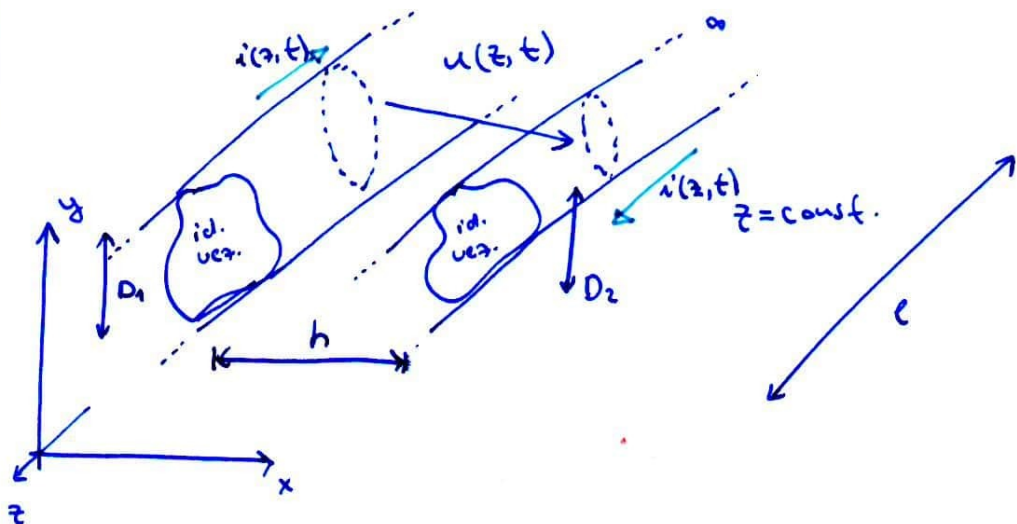
Elosztott paraméterű hálózat ⊕ 1D hullámterjedés

→ fész., áram

↳ $u = u(z, t)$

↑ hely-, időfüggés

$i = i(z, t)$



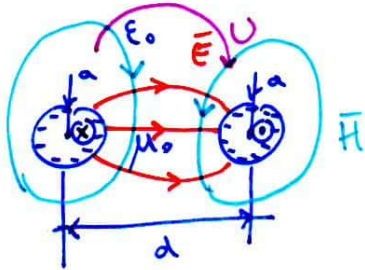
Hullámhossz: $\lambda = \frac{c}{f}$, $\lambda \gg D_1, D_2, h \rightarrow x, y$: statikus/stacionárius tér
 $c \gg \lambda \Rightarrow z$: hullám

pl.: $f = 50 \text{ Hz} \Rightarrow \lambda = \frac{3 \cdot 10^8 \frac{\text{m}}{\text{s}}}{50 \frac{1}{\text{s}}} = 6000 \text{ km}$

$f = 300 \text{ MHz} \Rightarrow \lambda \approx \frac{3 \cdot 10^8}{3 \cdot 10^8} = 1 \text{ m}$: összemérhető a kábel hosszával

Gyákerlatban:

- LECHER-vezeték:

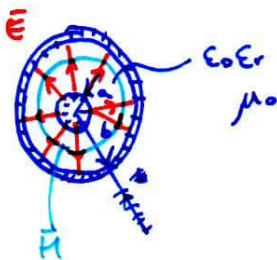


(köztes letekve $\epsilon_0 \cdot \epsilon_r$)

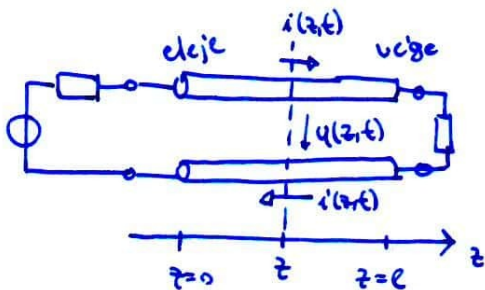
⊗: I befelé

⊙: I kifelé

- KOAXIÁLIS kábel:

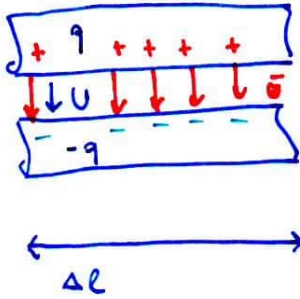


Sematikus ábra:



Vezetélpáraméterek

1.) Hosszegységére eső kapacitás: C'



$$\Delta Q = q \cdot \Delta l$$

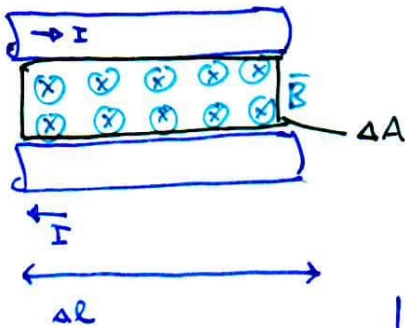
$$C_{\Delta l} = \frac{\Delta Q}{U} \Rightarrow C' = \frac{C_{\Delta l}}{\Delta l} = \frac{q}{U}$$

$$C' = \frac{C_{\Delta l}}{\Delta l} = \frac{q}{U}$$

$$\text{LECHER: } C' = \frac{\pi \epsilon_0 \epsilon_r}{\ln \frac{d}{a}}$$

$$\text{KOAXIÁLIS: } C' = \frac{2\pi \epsilon_0 \epsilon_r}{\ln \frac{b}{a}}$$

2.) Hosszegységére eső inductívitás: L'



fluxus:

$$\Phi_{\Delta A} = \int_{\Delta A} \vec{B} \cdot d\vec{A}$$

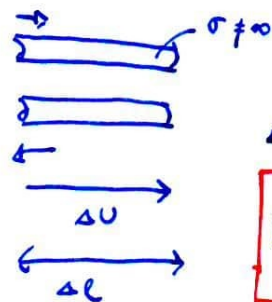
$$L_{\Delta A} = \frac{\Phi_{\Delta A}}{I} \Rightarrow L' = \frac{L_{\Delta A}}{\Delta l} = \frac{\Phi_{\Delta A}}{I \cdot \Delta l}$$

$$\text{LECHER: } L' = \frac{\mu_0}{\pi} \ln \frac{d}{a}$$

$$\text{KOAX: } L' = \frac{\mu_0}{2\pi} \ln \frac{b}{a}$$

$$\text{Bízonyítható: } L' \cdot C' \equiv \mu_0 (\epsilon_0 \cdot \epsilon_r)$$

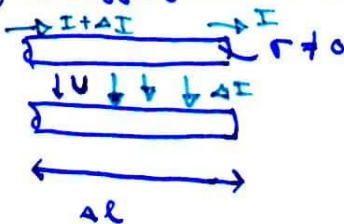
3.) Hosszegységére eső ellenállás:



$$R_{\Delta l} = \frac{\Delta U}{I}$$

$$R' = \frac{R_{\Delta l}}{\Delta l}$$

4.) Hosszegységére eső vezetési ΔI : szivárgási áram



$$G_{\Delta l} = \frac{\Delta I}{U} \Rightarrow G' = \frac{G_{\Delta l}}{\Delta l}$$

$$G' = \frac{G_{\Delta l}}{\Delta l}$$

Tereh Gy.
15.10.29.

Stacionárius áramlás: tér

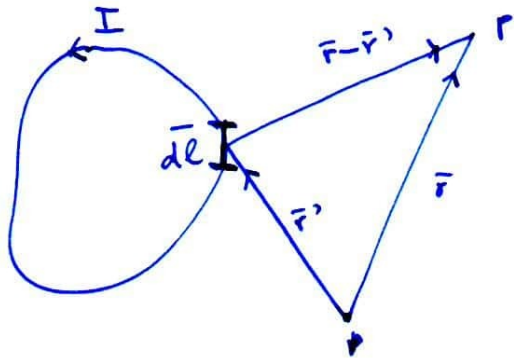
$$\operatorname{rot} \vec{H} = \vec{J}$$

$$\operatorname{div} \vec{B} = 0$$

$\vec{B} = \mu \vec{H}$ (lineáris, izotróp anyagok)

$$\text{Biot-Savart: } \vec{H}(\vec{r}) = \frac{I}{4\pi} \oint \frac{d\vec{\ell}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \quad \left(\oint, \text{ mert } \operatorname{div} \vec{B} = 0 \right)$$

$$d\vec{H}(\vec{r}) = \frac{I}{4\pi} \frac{d\vec{\ell}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$



$$\psi_1 = L_{11} I_1 + L_{12} I_2$$

$$\psi_2 = L_{21} I_1 + L_{22} I_2$$

$$L_{12} = L_{21}$$

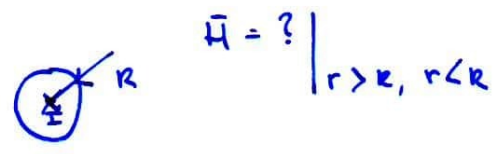
Indukció:

$$\operatorname{rot} \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{E} d\vec{\ell} = - \frac{\partial}{\partial t} \int \vec{B} d\vec{A}$$

$$u = - \frac{d\psi}{dt}$$

1) Végtelen hosszú vezeték tere



Szimmetria van. $\Rightarrow \vec{H}$ ellendős

I.

Diagram showing a circular Amperian loop of radius r (shaded) enclosing a wire with current I . The radius of the wire is R . The area of the loop is A .

$$\int_A \text{rot} \vec{H} d\vec{A} = \int_A \vec{J} d\vec{A}$$

$$\oint_C \vec{H} d\vec{e} = \int_A \vec{J} d\vec{A}$$

$$H_\varphi(r) 2\pi r = I$$

$$H_\varphi(r) = \frac{I}{2\pi r}, \quad \vec{H} = H_\varphi \cdot \vec{e}_\varphi$$

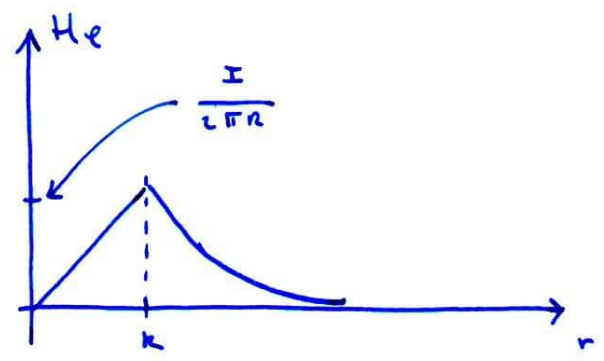
II.

Diagram showing a circular Amperian loop of radius r (shaded) outside a wire with current I . The radius of the wire is R .

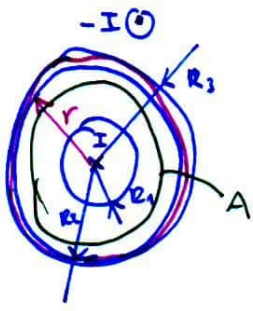
$$\oint_C \vec{H} d\vec{e} = \int_A \vec{J} d\vec{A}$$

$$H_\varphi(r) \cdot 2\pi r = \frac{I}{R^2 \pi} \cdot r^2 \pi$$

$$\Rightarrow H_\varphi(r) = \frac{I \cdot r}{R^2 2\pi}$$



2 Koaxialis hűvel, azonos fűtűeselyesítés



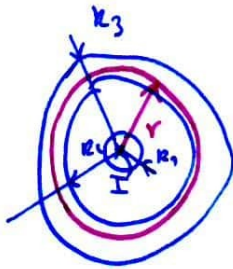
$$H_{\varphi} = \begin{cases} \frac{I \cdot r}{2\pi R_1^2}, & 0 < r < R_1 \\ \frac{I}{2\pi r}, & R_1 < r < R_2 \end{cases}$$

$$\oint_C \vec{H} \cdot d\vec{e} = \int_A \vec{J} \cdot d\vec{A} \Rightarrow H_{\varphi}(r) \cdot 2\pi r = I - \frac{I}{\pi(R_3^2 - R_2^2)} \cdot (r^2 - R_2^2) \cdot \pi$$

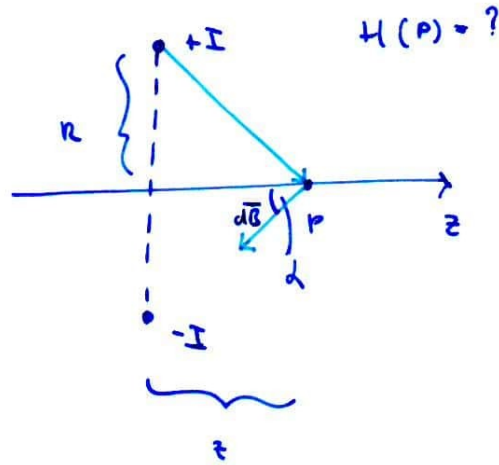
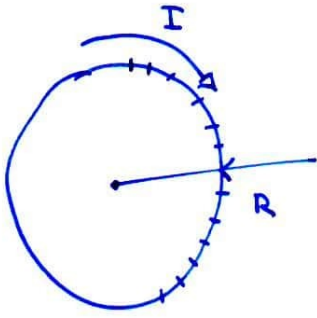
$$H_{\varphi}(r) \cdot 2\pi r = I \cdot \frac{R_3^2 - R_2^2 - r^2 + R_2^2}{R_3^2 - R_2^2}$$

$$H_{\varphi}(r) = I \frac{R_3^2 - r^2}{2\pi r (R_3^2 - R_2^2)}$$

$$R_2 < r < R_3$$



3] Velikougygyűrésű mágneses tere



$$\vec{B} = \frac{I\mu}{4\pi} \oint_e \frac{d\vec{\ell} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

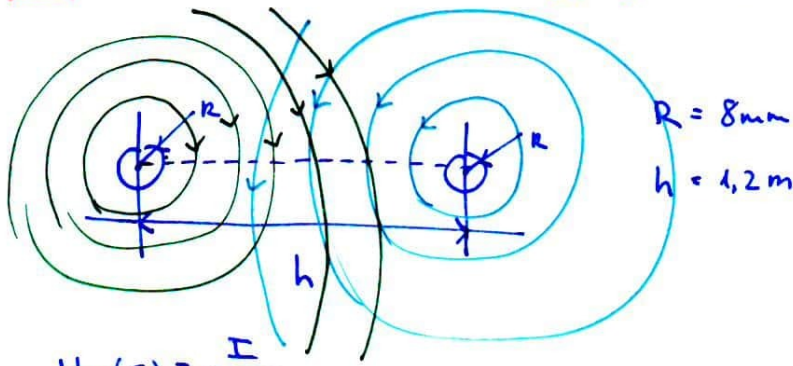
$$\vec{B}(z=0) = B_z \cdot \hat{e}_z \Rightarrow B_z = -\mu \cdot \frac{I}{4\pi} \frac{2\pi R}{R^2} = -\mu \cdot \frac{I}{2R}$$

$$d\vec{B} = \mu \frac{I}{4\pi} \frac{d\vec{\ell} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \quad \text{és} \quad \mu \frac{I}{4\pi} \frac{d\ell}{R^2 + z^2} = |d\vec{B}|$$

$$dB_z = \mu \frac{I}{4\pi} \frac{d\ell}{R^2 + z^2} \cdot \underbrace{\frac{R}{\sqrt{R^2 + z^2}}}_{\cos \alpha}$$

$$B_z = \oint_e B_z d\ell = \mu \cdot \frac{I}{4\pi} \cdot \frac{R}{(R^2 + z^2)^{\frac{3}{2}}} \cdot 2\pi R = \mu \frac{I}{2} \frac{R^2}{(R^2 + z^2)^{\frac{3}{2}}}$$

4 Lechner-vezeték hosszegységére eső induktivitása



$$H_{\varphi}(r) = \frac{I}{2\pi r}$$

$$\Psi = 2 \cdot \int_R^{h-R} \frac{I\mu}{2\pi r} dr = \mu \frac{I}{\pi} \ln \frac{h-R}{R}$$

$$\Psi = L \cdot I, \quad L' = \frac{L}{e}$$

$$\frac{\Psi}{e} = L' \cdot I \Rightarrow L'_k = \frac{\mu \cdot \ln \frac{h-R}{R}}{\pi} \quad L'_k: \text{külső induktivitás}$$

Belül:

$$H_{\varphi} = \frac{I \cdot r}{2\pi r^2}$$

$$W = \frac{1}{2} L \cdot I^2$$

$$W_b = \frac{1}{2} L_b \cdot I^2$$

$$W_b = e \int_0^R \frac{1}{2\mu} \left(\frac{I \cdot r}{2\pi r^2} \right)^2 \cdot 2\pi r \cdot dr = \frac{e \cdot I^2 \mu}{4\pi R^4} \int_0^R r^3 dr = \frac{e \cdot I^2 \mu}{4\pi R^4} \cdot \frac{R^4}{4} =$$

$$= e \cdot \mu \cdot \frac{I^2}{16\pi}$$

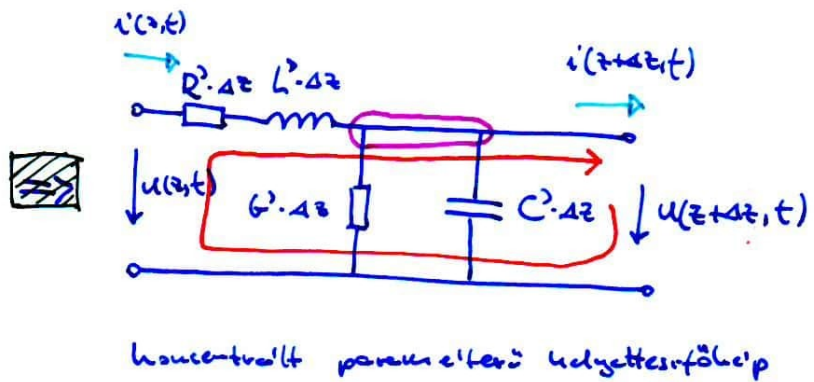
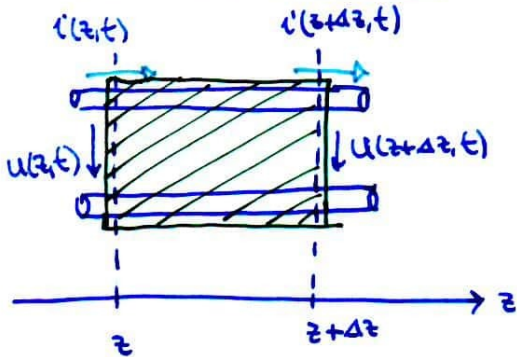
$$W_b = \frac{1}{2} L_b I^2 \Rightarrow L_b = e \cdot \mu \cdot \frac{1}{8\pi} \Rightarrow L'_b = \frac{\mu}{8\pi} ; \leftarrow \text{egys. vezeték esetén}$$

Távközlésteleki +

Vezető paraméterek: L', R', C', G'

$u(z, t), i(z, t) = ?$

Távúró egyenletek



koncentrált paraméterű helyettesítőábrák

Kirchhoff - tr.

I. $-u(z, t) + i(z, t) \cdot R' \Delta z + \frac{\partial i(z, t)}{\partial t} L' \Delta z + u(z + \Delta z, t) = 0$: \odot

II. $-i(z, t) + i(z + \Delta z, t) + u(z + \Delta z, t) \cdot G' \Delta z + \frac{\partial u(z + \Delta z, t)}{\partial t} C' \Delta z = 0$: \odot

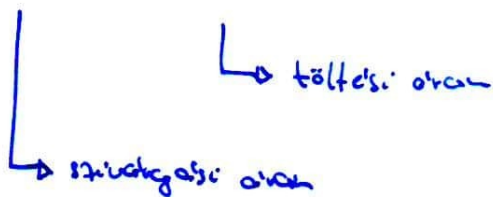
$\Delta z \rightarrow 0 \rightarrow \frac{u(z + \Delta z, t) - u(z, t)}{\Delta z} = \frac{\partial u(z, t)}{\partial z}$

I: $-\frac{\partial u(z, t)}{\partial z} = R' i(z, t) + L' \frac{\partial i(z, t)}{\partial t}$

II: $-\frac{\partial i(z, t)}{\partial z} = G' u(z, t) + C' \frac{\partial u(z, t)}{\partial t}$

2 változós elsőrendű

parciális differenciálegyenlet-rendszer





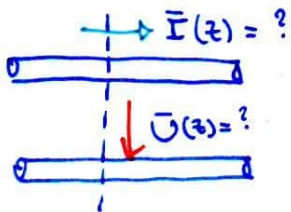
megoldásai:

- általában vektor
 - sinuszos időbeli változók (ω)
- } all.: Fourier-tr. }

$$u(z,t) \triangleq \hat{U}(z) \cdot \cos(\omega t + \varphi_u(z)) \equiv \text{Re} \left\{ \hat{U}(z) e^{j\varphi_u(z)} e^{j\omega t} \right\}$$

$$i(z,t) = \hat{I}(z) \cdot \cos(\omega t + \varphi_i(z)) \equiv \text{Re} \left\{ \hat{I}(z) e^{j\varphi_i(z)} e^{j\omega t} \right\}$$

komplex amplitúdó: $\bar{U}(z)$ (fázor)



$$\hat{I}(z) \equiv \text{Re} \left\{ \hat{I}(z) e^{j\varphi_i(z)} e^{j\omega t} \right\}$$

$\bar{I}(z)$

- $u(z,t) \rightarrow \bar{U}(z)$
- $i(z,t) \rightarrow \bar{I}(z)$
- $\frac{\partial u(z,t)}{\partial t} \rightarrow j\omega \bar{U}(z)$
- $\frac{\partial i(z,t)}{\partial t} \rightarrow j\omega \bar{I}(z)$
- $\frac{\partial u(z,t)}{\partial z} \rightarrow \frac{d\bar{U}(z)}{dz}$
- $\frac{\partial i(z,t)}{\partial z} \rightarrow \frac{d\bar{I}(z)}{dz}$

\Rightarrow Telvíró-egyenletek \Rightarrow

I. $-\frac{d\bar{U}(z)}{dz} = (R' + j\omega L') \bar{I}(z)$

II. $-\frac{d\bar{I}(z)}{dz} = (G' + j\omega C') \bar{U}(z)$

\rightarrow I. : $-\frac{d^2\bar{U}(z)}{dz^2} = (R' + j\omega L') \underbrace{(-G' - j\omega C')}_{-\frac{d\bar{I}(z)}{dz}} \bar{U}(z) \quad (\Rightarrow)$



$\Rightarrow \frac{d^2\bar{U}(z)}{dz^2} - \underbrace{(R' + j\omega L')(G' + j\omega C')}_{:= \gamma^2} \bar{U}(z) = 0$

I. : $\frac{d^2\bar{U}(z)}{dz^2} - \gamma^2 \bar{U}(z) = 0$

II. : $\frac{d^2\bar{I}(z)}{dz^2} - \gamma^2 \bar{I}(z) = 0$

Helmholtz-egyenletek
 köztületesen másodrendű diff. egy.

←

$$\left. \begin{aligned} \frac{d^2 \bar{U}(z)}{dz^2} - \gamma^2 \bar{U}(z) &= 0 \\ \frac{d^2 \bar{I}(z)}{dz^2} - \gamma^2 \bar{I}(z) &= 0 \end{aligned} \right\} \gamma^2 - \gamma^2 = 0 \rightarrow \gamma = \pm \gamma$$

A' (általános megoldás):

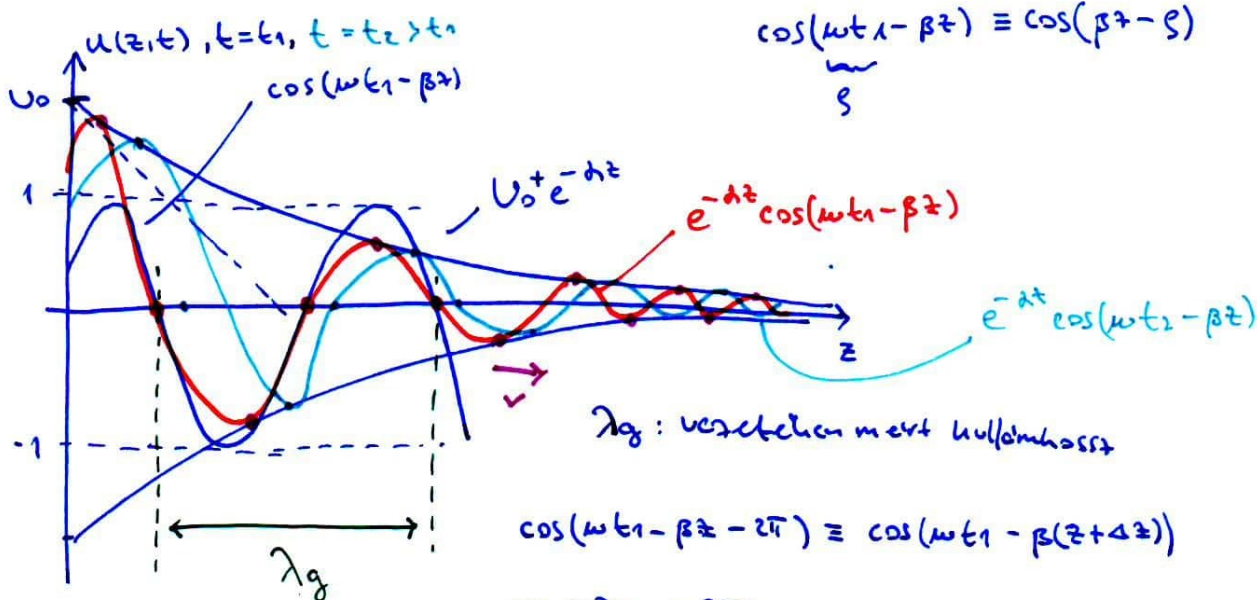
$$\left[\begin{aligned} \bar{U}(z) &= \underbrace{U_0^+ e^{-\gamma z}} + U_0^- e^{\gamma z} \\ \bar{I}(z) &= I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \end{aligned} \right. , \text{ ahol } \left. \begin{aligned} U_0^+, U_0^- \\ I_0^+, I_0^- \end{aligned} \right\} \text{ tetszőleges konstansok}$$

Megoldás értelmezése

$$u(z,t) = \underbrace{\operatorname{Re}\{U_0^+ e^{-\gamma z} e^{j\omega t}\}}_{u^+(z,t)} + \underbrace{\operatorname{Re}\{U_0^- e^{\gamma z} e^{j\omega t}\}}_{u^-(z,t)}$$

U_0^+ valódi, jelölés: $\gamma = \alpha + j\beta$

$$u^+(z,t) = U_0^+ \cdot e^{-\alpha z} \cdot \cos(\omega t - \beta z) \quad : \alpha, \beta > 0$$



$$\cos(\omega t_1 - \beta z) \equiv \cos(\beta z - \varphi)$$

λ_g : vezetékben mért hullámhossz

$$\cos(\omega t_1 - \beta z - 2\pi) \equiv \cos(\omega t_1 - \beta(z + \lambda_g))$$

$$\Rightarrow \beta \lambda_g = 2\pi$$

$$\Rightarrow \lambda_g = \frac{2\pi}{\beta}$$

" $+z$ " irányban haladó exponenciálisan csillapodó hullám hullámhossza

←

$$\omega \Rightarrow f = \frac{\omega}{2\pi} \rightarrow T = \frac{2\pi}{\omega}$$

$$v = \frac{\lambda f}{T} = \frac{2\pi}{\beta} \cdot f = \frac{\omega}{\beta}$$

$$v = \frac{\omega}{\beta} : \text{Hullám terjedési sebessége } \approx c$$

$$\alpha = \operatorname{Re}\{\gamma\} : \text{csillapítási tényező} \quad [\alpha] = \frac{1}{m}$$

$$\beta = \operatorname{Im}\{\gamma\} : \text{fázistényező} \quad [\beta] = \frac{1}{m}$$

Teljes megoldás:

$$u(z,t) = u^+(z,t) + u^-(z,t)$$

↳ "-z" irányba terjedő csill. hull.
 ↳ "+z" irányba terjedő csill. hull.

Feszültség és áram kapcsolata:

$$\bar{U}(z) = U_0^+ e^{-\gamma z}, \quad -\frac{d\bar{U}(z)}{dz} = (R' + j\omega L') \cdot \bar{I}(z)$$

$$\bar{I}(z) = -\frac{1}{R' + j\omega L'} \left(-\gamma U_0^+ e^{-\gamma z} + \gamma U_0^- e^{\gamma z} \right) \quad \textcircled{=}$$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} : \text{terjedési tényező}$$

$$\textcircled{=} \underbrace{\sqrt{\frac{G' + j\omega C'}{R' + j\omega L'}}}_{I_0^+} \cdot U_0^+ e^{-\gamma z} - \underbrace{\sqrt{\frac{G' + j\omega C'}{R' + j\omega L'}}}_{I_0^-} \cdot U_0^- e^{\gamma z}$$

↻

←

$$\underbrace{\frac{R' + j\omega L'}{G' + j\omega C'}}_{Z_0} = \frac{U_0^+}{I_0^+} = -\frac{U_0^-}{I_0^-}$$

Z_0 : hullámimpedancia

Def: $Z_0 \hat{=} \frac{U_0^+}{I_0^+} \hat{=} -\frac{U_0^-}{I_0^-}$

$A + z$ irányba terjedő fesz. és áram ^{komplex amplitúdójával}
 a hányadosa. _{hullám}

Ideális talvezeték (vesztésmentes)

• $R' = 0$

• $G' = 0$

• Terjedési sebesség: $v = \sqrt{(R' + j\omega L')(G' + j\omega C')} = j\omega \sqrt{L'C'}$ $\rightarrow \alpha = 0$
 $\beta = \omega \sqrt{L'C'}$

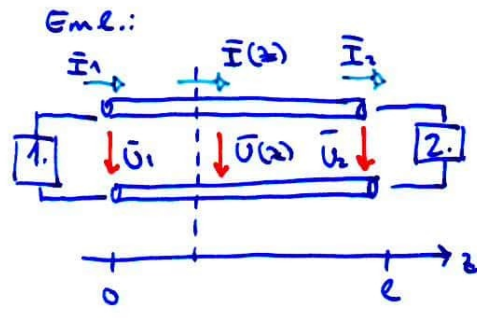
• Hullámimpedancia: $Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}}$: állandó valós konstans

• Fázissebesség: $v = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}$: frekvencia független

levezetési viszonyok: $L'C' = \mu_0(\epsilon_0 \epsilon_r)$

$$v = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\epsilon_r}} \cdot \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{c}{\sqrt{\epsilon_r}}$$

Levegőtelelési talvezet: $\epsilon_r = 1 \Rightarrow v = c$



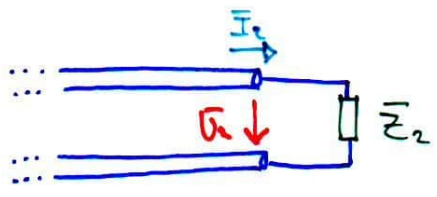
$$U(z) = U_0^+ e^{-\gamma z} + U_0^- e^{\gamma z}$$

$$I(z) = \frac{U_0^+}{Z_0} e^{-\gamma z} - \frac{U_0^-}{Z_0} e^{\gamma z}$$

Lezárt társvezeték

$$\left. \begin{aligned} \bar{U}_1 = \bar{U}(z=0), \quad \bar{U}_2 = \bar{U}(z=l) \\ \bar{I}_1 = \bar{I}(z=0), \quad \bar{I}_2 = \bar{I}(z=l) \end{aligned} \right\} \begin{aligned} &\text{adott: kétpólus karakterisztikája} \\ &\left. \begin{aligned} \bar{U}_1 = \bar{U}_1(\bar{I}_1) \\ \bar{U}_2 = \bar{U}_2(\bar{I}_2) \end{aligned} \right\} \Rightarrow U_0^+, U_0^- \text{ meghatározható} \end{aligned}$$

Reflexiók felgyeső a vezeték végén



Z_0, γ adott.
 $\bar{U}_2 = \bar{Z}_2 \cdot \bar{I}_2$

$$\bar{U}_2 = U_0^+ e^{-\gamma l} + U_0^- e^{\gamma l}$$

$$\bar{I}_2 = \frac{U_0^+}{Z_0} e^{-\gamma l} - \frac{U_0^-}{Z_0} e^{\gamma l}$$

$$\frac{1}{\bar{Z}_2} = \frac{U_0^+ e^{-\gamma l} + U_0^- e^{\gamma l}}{\frac{U_0^+}{Z_0} e^{-\gamma l} - \frac{U_0^-}{Z_0} e^{\gamma l}}$$

$$\frac{U_0^- e^{\gamma l}}{U_0^+ e^{-\gamma l}} = \frac{\bar{Z}_2 - Z_0}{\bar{Z}_2 + Z_0}$$

$$\bar{\Gamma}_2 \triangleq \frac{\bar{Z}_2 - Z_0}{\bar{Z}_2 + Z_0}$$

Reflexiók felgyeső a vezeték végén

$U_0^- e^{\gamma l}$: "-z" irányba haladó fesz. hullám komplex amplitúdója a vezeték végén

Passzív impedancia :

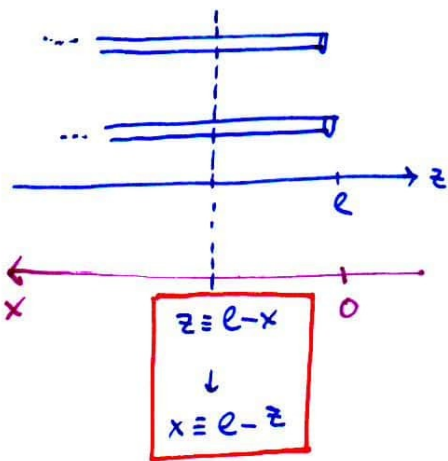
$$\bar{Z}_2 = R_2 + jX_2, \quad R_2 \geq 0, \quad X_2 > 0 \text{ és valós}$$

$$\bar{F}_2 = \frac{R_2 + jX_2 - Z_0}{R_2 + jX_2 + Z_0} = \frac{(R_2 - Z_0) + jX_2}{(R_2 + Z_0) + jX_2}$$

→ $|\bar{F}_2| \leq 1$ és $|\bar{F}_2| = 1 \iff R_2 = 0$: teljes visszaverődés

és $|\bar{F}_2| = 0 \iff R_2 = Z_0$: illesztett terhelés

Új koordináta-rendszer:



$$\bar{U}(x) = U_0^+ e^{-\gamma(l-x)} + U_0^- e^{\gamma(l-x)} =$$

$$= \underbrace{U_0^+ e^{-\gamma l}}_{\text{jobbra}} e^{\gamma x} + \underbrace{U_0^- e^{\gamma l}}_{\text{balra kezdő hullám}} e^{-\gamma x}$$

$$\downarrow$$

$$U_2^+$$

$$\downarrow$$

$$U_2^-$$

: vezeték végén definiezett komplex amplitúdók

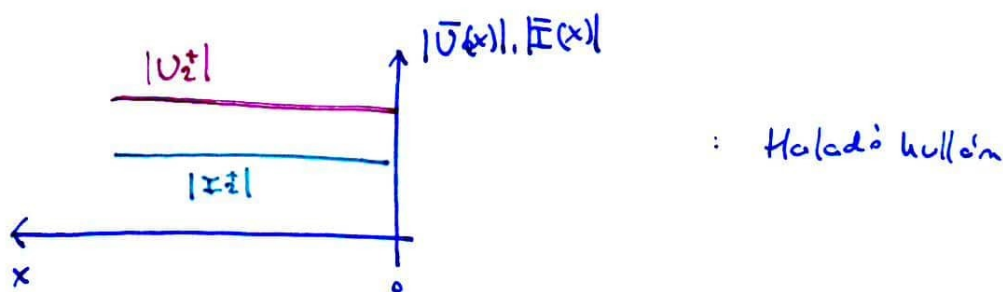
$$\bar{H}(x) = \frac{U_2^+}{Z_0} e^{\gamma x} - \frac{U_2^-}{Z_0} e^{-\gamma x}$$

Feszültség és áramerősség speciális lezárások mellett
ideális fémvezetőkön, x koord. rendszerben

a) I-lezárás: $Z_2 = Z_0 \Rightarrow r_2 = 0$

$$r_2 \triangleq \frac{U_2^-}{U_2^+} \Rightarrow \boxed{U_2^- = 0}$$

$$\Rightarrow \bar{U}(x) = U_2^+ e^{j\beta x} ; \bar{I}(x) = \frac{U_2^+}{Z_0} e^{j\beta x}$$

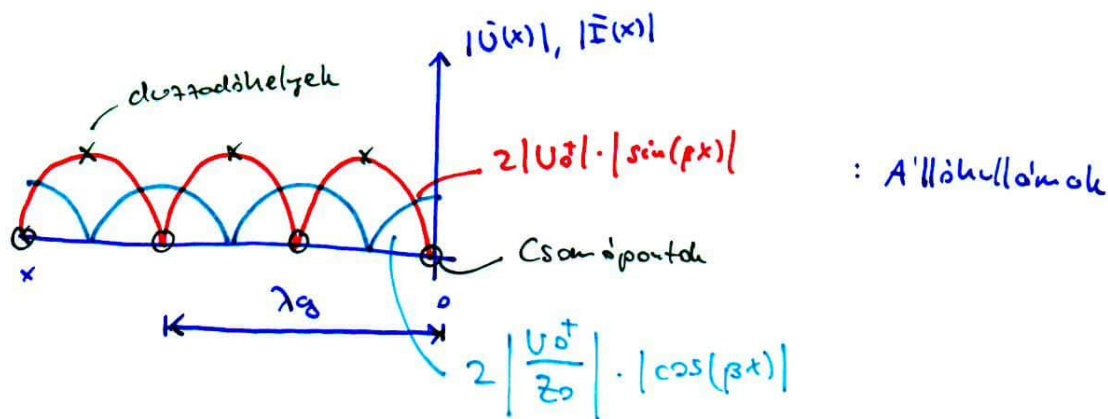


b) Rövidzár ($Z_2 = 0$) $\Rightarrow r_2 = -1$

$$\boxed{U_2^- = -U_2^+}$$

$$\Rightarrow \bar{U}(x) = U_2^+ (e^{j\beta x} - e^{-j\beta x}) = 2j U_2^+ \sin(\beta x)$$

$$\Rightarrow \bar{I}(x) = \frac{U_2^+}{Z_0} (e^{j\beta x} + e^{-j\beta x}) = 2 \cdot \frac{U_2^+}{Z_0} \cdot \cos(\beta x)$$



• \bar{U} és \bar{I} között $\frac{\lambda_g}{4}$ elcsúszás

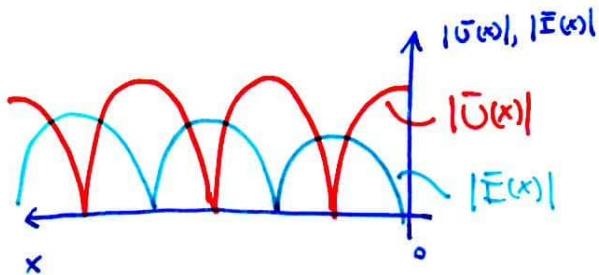
• Időben $\frac{T}{4}$ időnyi fáziseltérés adódik (90°)

c) Szabadság: $z_2 \rightarrow \infty \Rightarrow r_2 = 1$

$$\boxed{U_2^- = U_2^+}$$

$$\Rightarrow \bar{U}(x) = U_2^+ (e^{j\beta x} + e^{-j\beta x}) = 2U_2^+ \cos(\beta x)$$

$$\Rightarrow \bar{I}(x) = \frac{U_2^+}{Z_0} (e^{j\beta x} - e^{-j\beta x}) = 2j \frac{U_2^+}{Z_0} \sin(\beta x)$$



d) Reaktancia: $z_2 = jX_2$

$$r_2 = \frac{jX_2 - Z_0}{jX_2 + Z_0} = -\frac{Z_0 - jX_2}{Z_0 + jX_2} = e^{j\sigma} \rightarrow \boxed{U_2^- = e^{j\sigma} U_2^+}$$

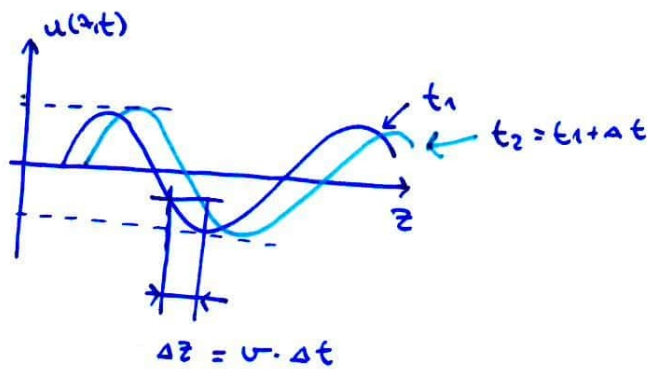
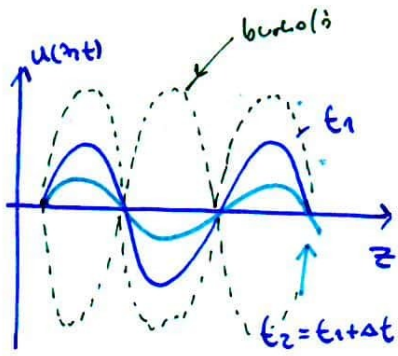
↑
komplex konjugáltak

$$\Rightarrow \bar{U}(x) = U_2^+ (e^{j\beta x} + e^{j\sigma} \cdot e^{-j\beta x}) = U_2^+ \cdot e^{j\frac{\sigma}{2}} (e^{j(\beta x - \frac{\sigma}{2})} + e^{-j(\beta x - \frac{\sigma}{2})}) =$$

$$= 2U_2^+ e^{j\frac{\sigma}{2}} \cdot \cos(\beta x - \frac{\sigma}{2}) : \text{állókültség (nem szelítet hatoóss teljesítményf)}$$

↑
vesztéselgmentes

A'ldöskullinn / heildöskullinn felgandi



- allí -

- heildí -

- Idealis ferulur: ($\gamma = i\beta$) lezatrois: $\bar{z}_2 \Rightarrow \underline{r_2} = \rho e^{i\sigma}$
 $0 \leq \rho \leq 1, -\frac{\pi}{2} \leq \sigma \leq \frac{\pi}{2}$

$$\bar{u}(x) = U_2^+ e^{i\beta x} + U_2^- e^{-i\beta x} \quad \text{⊕}$$

$$\uparrow$$

$$\underline{r_2} \cdot U_2^+$$

$$\text{⊕} \quad U_2^+ \left[e^{i\beta x} + \underline{\rho e^{i\sigma}} e^{-i\beta x} \right] = U_2^+ \left[\underbrace{e^{i\beta x} - \rho e^{i\beta x}}_{(1-\rho)e^{i\beta x}} + \underbrace{\rho e^{i\beta x} + \rho e^{i\sigma} e^{-i\beta x}}_{\rho e^{i\frac{\sigma}{2}} (e^{i(\beta x - \frac{\sigma}{2})} + e^{-i(\beta x - \frac{\sigma}{2})})} \right] \quad \text{⊕}$$

$$\text{⊕} \quad \underbrace{U_2^+ (1-\rho) e^{i\beta x}}_{\text{heildíh.}} + \underbrace{U_2^+ \rho e^{i\frac{\sigma}{2}} \cdot 2 \cos(\beta x - \frac{\sigma}{2})}_{\text{allíh.}}$$

Ha illisatés: $\rho = 0 \Rightarrow$ allískullinn = 0

Ha felpestærnis: $\rho = 1 \Rightarrow$ heildískullinn = 0

Terek E.
15. 11. 11.

Talvezete'lek +

ism.:

$$|U(x)| = |U_0| \cdot \underbrace{|1 + \rho e^{-j(2\beta x - \sigma)}|}_{q(x)}$$

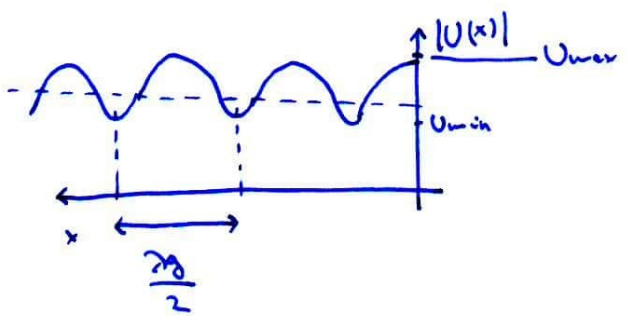
$$|q(x)| = \left| \underbrace{(1 + \rho \cos(-2\beta x + \sigma))}_{a(x)} + j \underbrace{\rho \sin(-2\beta x + \sigma)}_{b(x)} \right| = \sqrt{a^2(x) + b^2(x)} =$$

$$= \sqrt{1 + \rho^2 \cos^2(-2\beta x + \sigma) + 2\rho \cos(-2\beta x + \sigma) + \rho^2 \sin^2(-2\beta x + \sigma)} =$$

↓

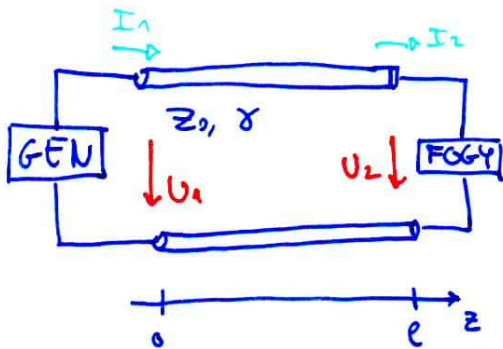
ρ^2

$$= \sqrt{1 + \rho^2 + 2\rho \cos(-2\beta x + \sigma)}$$



A távvezeték mint hálózat

- belső viselkedés nem érdekes
- generátor-terhelés hálózata



$$U(z) = U_0^+ e^{-\gamma z} + U_0^- e^{\gamma z}$$

$$I(z) = \frac{U_0^+}{Z_0} e^{-\gamma z} - \frac{U_0^-}{Z_0} e^{\gamma z}$$

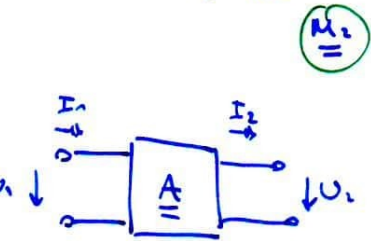
$$\begin{array}{l|l} U_1 = U(0) & U_2 = U(l) \\ I_1 = I(0) & I_2 = I(l) \end{array}$$

$$\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{Z_0} & -\frac{1}{Z_0} \end{pmatrix} \begin{pmatrix} U_0^+ \\ U_0^- \end{pmatrix} \quad \underline{M_1}$$

$$\begin{pmatrix} U_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} e^{-\gamma l} & e^{\gamma l} \\ \frac{e^{-\gamma l}}{Z_0} & -\frac{e^{\gamma l}}{Z_0} \end{pmatrix} \begin{pmatrix} U_0^+ \\ U_0^- \end{pmatrix} \Rightarrow \begin{pmatrix} U_0^+ \\ U_0^- \end{pmatrix} = \underline{M_2}^{-1} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}$$

\underline{A} távvezeték

$$\underline{A} = \underline{M_1} \cdot \underline{M_2}^{-1} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}$$



$$\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = \underline{A} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}$$

$$\underline{A} = \underline{M_1} \cdot \underline{M_2}^{-1} = \begin{pmatrix} \text{ch}(\gamma l) & Z_0 \cdot \text{sh}(\gamma l) \\ \frac{1}{Z_0} \cdot \text{sh}(\gamma l) & \text{ch}(\gamma l) \end{pmatrix}$$

$A_{11} \equiv A_{22} \Rightarrow$ szimmetrikus

$$\det \underline{A} = \text{ch}^2(\gamma l) - \text{sh}^2(\gamma l) \equiv 1$$

\Rightarrow reciprok hálózat

$$\text{ch}(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2}$$

$$\text{sh}(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2}$$

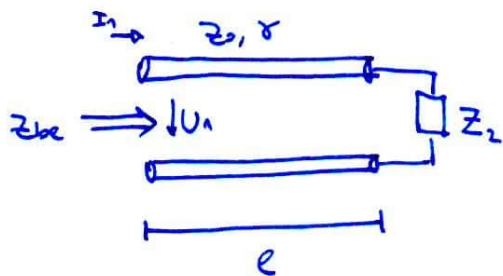
\Rightarrow Reciprok és szimmetrikus hálózat

← Idealis távwetelék esetén:

$$\begin{aligned} \gamma &= j\beta \\ \operatorname{ch}(j\beta l) &\equiv \cos(\beta l) \\ \operatorname{sh}(j\beta l) &\equiv j\sin(\beta l) \end{aligned} \Rightarrow A = \begin{pmatrix} \cos(\beta l) & jZ_0 \sin(\beta l) \\ -\frac{j}{Z_0} \sin(\beta l) & \cos(\beta l) \end{pmatrix}$$

← : $A_{11} \equiv A_{22}$
 $\det A = 1$ } \Rightarrow két független Lohc-paraméter.

Bemeneti impedancia:



$$Z_{be} \stackrel{\Delta}{=} \frac{U_1}{I_1} = ?$$

$$\frac{U_1}{I_1} = \frac{U_2 \operatorname{ch}(\gamma l) + I_2 Z_0 \operatorname{sh}(\gamma l)}{U_2 \cdot \frac{1}{Z_0} \operatorname{sh}(\gamma l) + I_2 \operatorname{ch}(\gamma l)} = \frac{Z_2 \cdot \operatorname{ch}(\gamma l) + Z_0 \cdot \operatorname{sh}(\gamma l)}{\frac{Z_2}{Z_0} \operatorname{sh}(\gamma l) + \operatorname{ch}(\gamma l)} = Z_0 \frac{Z_2 + Z_0 \operatorname{th}(\gamma l)}{Z_0 + Z_2 \operatorname{th}(\gamma l)}$$

Idealis távwetelék:

$$\gamma = j\beta \rightarrow \operatorname{th}(\gamma l) \equiv j \operatorname{tg}(\beta l)$$

$$Z_{be} = Z_0 \frac{Z_2 + jZ_0 \operatorname{tg}(\beta l)}{Z_0 + jZ_2 \operatorname{tg}(\beta l)}$$

pe

① Ideális: $Z_2 = Z_0$ (illesztett terhelés)

$$\Rightarrow Z_{be} = Z_0$$

② Ideális: $Z_2 = jX$ (tiszta reaktív terhelés: $X > 0 \Rightarrow$ teheres)

$$Z_{be} = Z_0 \frac{jX + jZ_0 \cdot \operatorname{tg}(\beta l)}{Z_0 - X \operatorname{tg}(\beta l)} \quad : \text{Képtetes!}$$

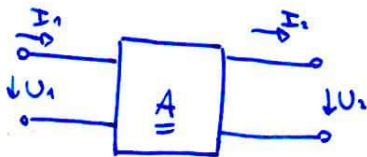
a) $l = \lambda g \Rightarrow \beta l = 2\pi \rightarrow \operatorname{tg} 2\pi = 0$

$$\Rightarrow Z_{be} = jX$$

b) $l = \frac{\lambda g}{4} \Rightarrow \beta l = \frac{\pi}{2} \rightarrow \operatorname{tg} \frac{\pi}{2} \rightarrow \infty$

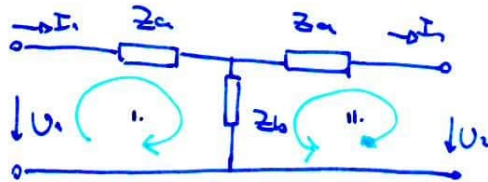
$$\Rightarrow Z_{be} = Z_0 \frac{\underbrace{Z_2 \cos(\beta l)}_0 + j \underbrace{Z_0 \sin(\beta l)}_1}{\underbrace{Z_0 \cos(\beta l)}_0 + j \underbrace{Z_2 \sin(\beta l)}_1} = Z_0 \frac{j Z_0}{j X} = -j \frac{Z_0^2}{X}$$

Távvezeték helyettesítőhálói



egyszerű, koncentrált paraméterű helyettesítőháló

① T-háló

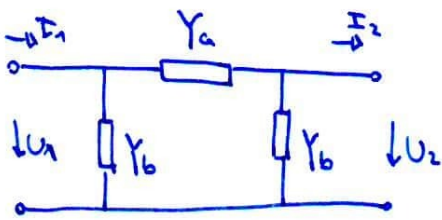


$$\left. \begin{aligned} U_1 &= A_{11} U_2 + A_{12} I_2 \\ I_1 &= A_{21} U_2 + A_{22} I_2 \end{aligned} \right\}$$

$$\left. \begin{aligned} \text{I.} : -U_1 + I_1 Z_a + (I_1 - I_2) Z_b &= 0 \\ \text{II.} : -U_2 + Z_a (-I_2) + Z_b (I_1 - I_2) &= 0 \end{aligned} \right\} \begin{aligned} Z_a &= Z_0 \frac{\operatorname{ch}(\beta l) - 1}{\operatorname{sh}(\beta l)} \\ Z_b &= Z_0 \frac{1}{\operatorname{sh}(\beta l)} \end{aligned}$$

\Rightarrow Egy adott n -u m ühédikh!

② Π -tag:

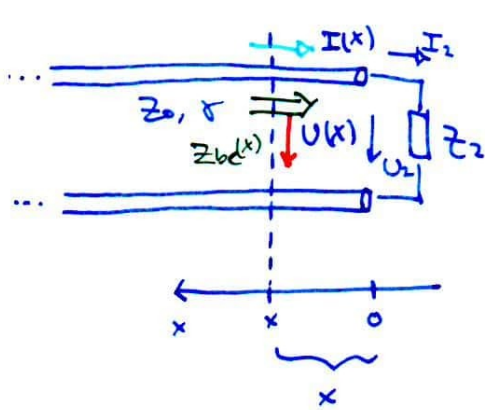


$$Y_a = \dots$$

$$Y_b = \dots$$

Egy adott ω -n meghatározni.

Bemeneti impedancia és reflexió tényező, mint a hely függvénye



$$\begin{pmatrix} U(x) \\ I(x) \end{pmatrix} = \begin{pmatrix} \text{ch}(\gamma x) & Z_0 \cdot \text{sh}(\gamma x) \\ \frac{1}{Z_0} \text{sh}(\gamma x) & \text{ch}(\gamma x) \end{pmatrix} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}$$

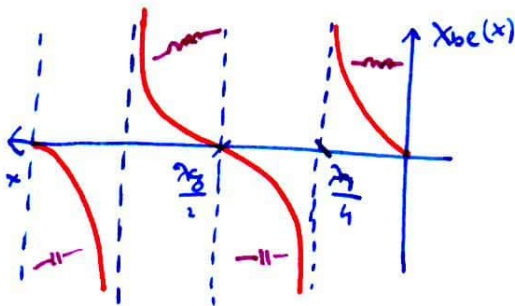
$$Z_{be}(x) \triangleq \frac{U(x)}{I(x)} = Z_0 \frac{Z_2 + Z_0 \tanh(\gamma x)}{Z_0 + Z_2 \tanh(\gamma x)}$$

pe id. TV. r_z-val leterve: $Z_2 = 0$

$$\hookrightarrow \gamma = j\beta$$

$$Z_{be}(x) = Z_0 \frac{0 + jZ_0 \tanh(j\beta x)}{Z_0 + 0} = \underbrace{jZ_0 \tanh(j\beta x)}_{X_{be}(x)} : \text{frissítő kapacitás}$$

\Leftarrow r_z-nem disszipatív (telenszem)



R_2 $Z_2 = R_2$ resistiv

$$Z_{be}(x) = Z_0 \frac{R_2 + jZ_0 \tan(\beta x)}{Z_0 + jR_2 \tan(\beta x)} = R_{be}(x) + jX_{be}(x)$$

$$\min\left(R_2, \frac{Z_0^2}{R_2}\right) \leq R_{be}(x) \leq \max\left(R_2, \frac{Z_0^2}{R_2}\right)$$

$$r_2 \hat{=} \frac{U_2^-}{U_2^+}$$

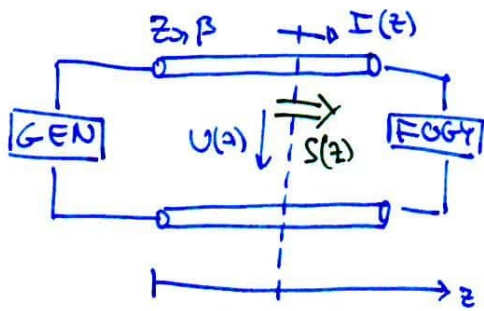
$$r(x) \hat{=} \frac{U_2^- e^{-\gamma x}}{U_2^+ e^{\gamma x}} \quad : \text{ Helmholtz-Ges. } (U(x) = U_2^+ e^{\gamma x} + U_2^- e^{-\gamma x})$$

$$\hookrightarrow r = \frac{Z_{be}(x) - Z_0}{Z_{be}(x) + Z_0}$$

$$\text{Idealis: } \gamma = j\beta \rightarrow r_0(x) = \frac{U_2^-}{U_2^+} e^{-2j\beta x} \Rightarrow$$

$$\Rightarrow |r_2(x)| = |r_2| = \text{konstant.}$$

Teilleistung eines verlustlosen idealen Transmissionselements



$$S(z) = \frac{1}{2} U(z) I^*(z) \quad (\ominus)$$

$$U(z) = U_0^+ e^{-j\beta z} + U_0^- e^{j\beta z}$$

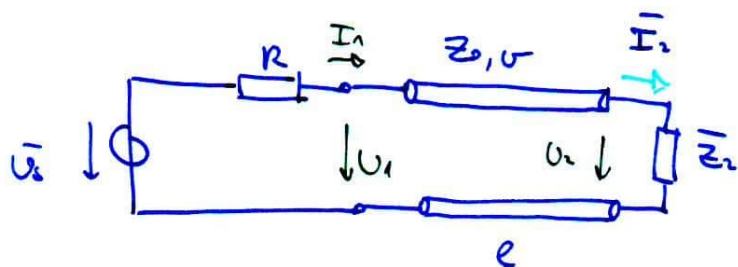
$$I(z) = \frac{U_0^+}{Z_0} e^{-j\beta z} - \frac{U_0^-}{Z_0} e^{j\beta z}$$

$$\ominus \frac{1}{2} \left[\underbrace{\frac{|U_0^+|^2}{Z_0} - \frac{|U_0^-|^2}{Z_0}}_{\text{verlusts}} - \underbrace{\frac{U_0^+ U_0^{-*}}{Z_0} e^{-2j\beta z} + \frac{U_0^- U_0^{+*}}{Z_0} e^{2j\beta z}}_{\text{eristation verlustlos}} \right]$$

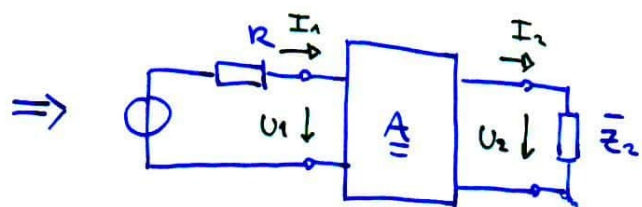
$$\Rightarrow P(z) = \operatorname{Re}\{S(z)\} = \frac{|U_0^+|^2 - |U_0^-|^2}{2Z_0} \quad (\ominus)$$

$$\ominus |U_0^+|^2 \cdot \frac{1}{2Z_0} \left(1 - \underbrace{|r(x)|^2}_{\text{verlustlos}} \right) \Rightarrow \text{verlustfrei}$$

1 Látványjellegű jel



$l = 1 \text{ km}$
 $Z_0 = 50 \Omega$
 $v = 2,2 \cdot 10^8 \frac{\text{m}}{\text{s}}$
 $\omega = 2\pi \cdot 10^5 \frac{1}{\text{s}}$
 $R = 10 \Omega$
 $Z_L = (50 + j50) \Omega$
 $\bar{U}_s = 100 \text{ V}$
 $\bar{I}_2 = ?$



$\rightarrow \begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = \underline{A} \begin{pmatrix} U_2 \\ I_2 \end{pmatrix}$

Ⓐ, Ⓑ

$U_1 = U_s - R I_1$

Ⓒ

$U_2 = Z_L \cdot I_2$

Ⓓ

4 függ. egy.

id. TV:

$\underline{A} = \begin{pmatrix} \cos \beta l & j \sin \beta l \\ j \sin \beta l & \cos \beta l \end{pmatrix}$

$\beta = \frac{\omega}{v} = \frac{2\pi \cdot 10^5}{2,2 \cdot 10^8} \frac{1}{\text{m}} = \dots \beta \checkmark$

$U_1: \text{III} \rightarrow \text{I}$

$U_2: \text{IV} \rightarrow \text{I, II}$

I., II. : $I_1, I_2 \checkmark$

$\Rightarrow \underline{A} = \begin{pmatrix} -0,9995 & j14,09 \Omega \\ j5,635 \text{ mS} & -0,9995 \end{pmatrix}$

\Rightarrow

$\underline{I}_2 = 1,473 e^{j2,666} \text{ A}$

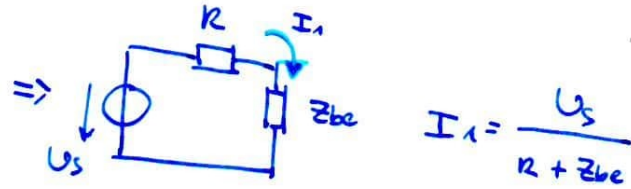
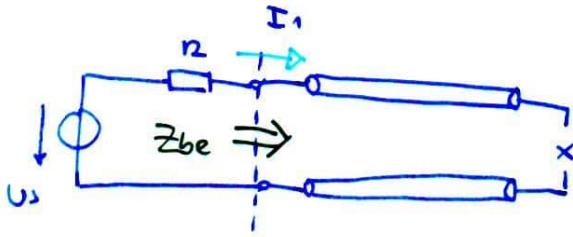
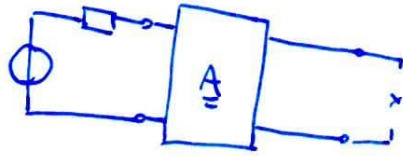
\Rightarrow

$i_2(t) = 1,473 \cdot \cos(\omega t + 2,666) \text{ A}$

2. Bemeneti impedancia

Előző feladat: $Z_2 \rightarrow \infty$

$\bar{I}_1 = ?$



$$I_1 = \frac{U_s}{R + Z_{be}}$$

$$\underline{Z_{be}} = Z_0 \frac{Z_2 + Z_0 j \tan \beta l}{Z_0 + Z_2 j \tan \beta l} \xrightarrow{Z_2 \rightarrow \infty} Z_0 \frac{1}{j \tan \beta l} = \underline{j 170,3 \Omega} \quad (\Rightarrow)$$

Legkiseb hűtés: id. TV.-en nem disszipálódik teljesítmény.

$$\Rightarrow I_1 = \frac{100}{10 + j 170,3} = 0,568 e^{-j 1,512} \text{ A}$$

3. Ferris hűtés és meddő teljesítménye az előző példában

$\bar{S} = \frac{1}{2} U_s \cdot \bar{I}_1^*$ \Rightarrow CSAK, HA A FESZ. ÉS A JÁRÓ ÁRAM EGY IRÁNYBA FOLYIK!!



$$\Rightarrow \bar{S} = \frac{1}{2} U_s (-\bar{I}_1^*)$$

$$\Rightarrow \underline{\bar{S}} = -\frac{1}{2} U_s \cdot \bar{I}_1^* = (-1,72 - j 29,3) \text{ VA}$$

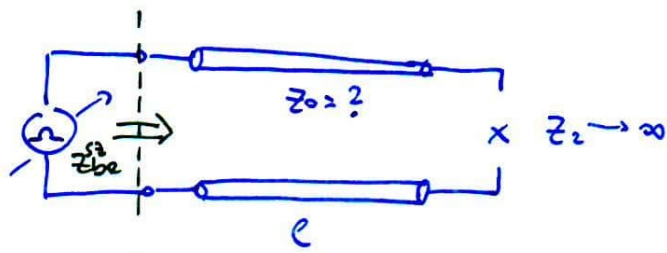
$$\underline{P} = \text{Re}\{\bar{S}\} = -1,72 \text{ W} < 0 \Rightarrow \text{TERMEL}$$

$$\underline{Q} = \text{Im}\{\bar{S}\} = -29,3 \text{ var}$$

$$P_R = \frac{1}{2} R |\bar{I}_1|^2 = \dots = 1,72 \text{ W}$$

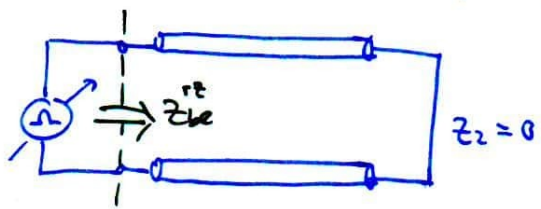
$$Q_{TV} = \frac{1}{2} \text{Im}\{Z_{be}\} \cdot |\bar{I}_1|^2 = \dots = 29,3 \text{ var}$$

4) Nullcímimpedancia mérésre n -mérővel



$$Z_{be}^{st} = Z_0 \frac{1}{j \tan \beta l}$$

$$Z_{be}^{rt} = Z_0 \frac{Z_2 + j Z_0 \tan \beta l}{Z_0 + j Z_2 \tan \beta l} = j Z_0 \tan \beta l$$



$$Z_0^2 = Z_{be}^{st} \cdot Z_{be}^{rt}$$

$$\omega = ? \leftarrow \frac{\tan \beta l}{\tan \beta l} = 1$$

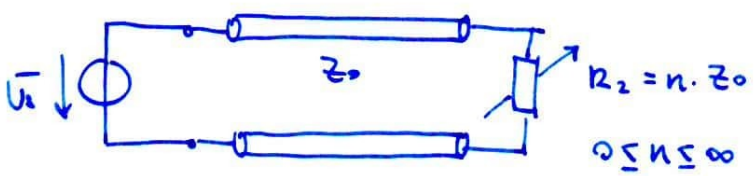
$$\beta = \frac{k}{v} = \frac{2\pi f}{v} \quad ; \quad \begin{aligned} v &\sim 10^8 \frac{m}{s} \\ l &\sim 10m \\ f &\sim 100Hz \end{aligned}$$

$$\left. \begin{aligned} \beta l &\sim 10^{-5} \\ &\downarrow \\ \tan \beta l &\sim 10^{-5} \Rightarrow \end{aligned} \right\}$$

$$\Rightarrow \frac{\sim 10^{-5}}{\sim 10^{-5}} = 1$$

Ha $\lambda \sim l \Rightarrow$ lehet jó mérést végezni

5 Allöhullámarány

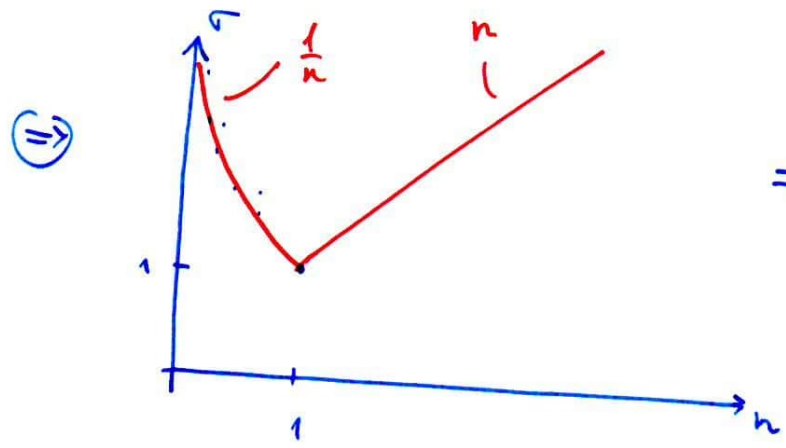


$\sigma(n) = ?$

$\sigma \triangleq \frac{U_{max}}{U_{min}} = \frac{1 + |r_2|}{1 - |r_2|}$

$r_2 = \frac{R_2 - Z_0}{R_2 + Z_0} = \frac{n - 1}{n + 1} \rightarrow |r_2| \begin{cases} \frac{n-1}{n+1}, & n \geq 1 \\ \frac{1-n}{n+1}, & n < 1 \end{cases}$

$\Rightarrow \sigma = \frac{1 + |r_2|}{1 - |r_2|} \begin{cases} \frac{1 + \frac{n-1}{n+1}}{1 - \frac{n-1}{n+1}} = n \\ 1 + \frac{1-n}{n+1} \\ \frac{1 + \frac{1-n}{n+1}}{1 - \frac{1-n}{n+1}} = \frac{1}{n} \end{cases}$



$\Rightarrow \sigma \geq 1$

6 Flötsbö | : $n = 0, 2$, l adott

$$|U(z)| = \frac{1}{2} |U_0^+| \quad \leftarrow z = \frac{z}{2} \text{ (haq teljesül)}$$

$$U(z) = \underbrace{U_0^+ e^{-j\beta z}}_{+z} + \underbrace{U_0^- e^{j\beta z}}_{-z \text{ irány}}$$

$$|U(z)| = |U_0^+ e^{-j\beta z} + U_0^- e^{j\beta z}| = \frac{1}{2} |U_0^+|$$

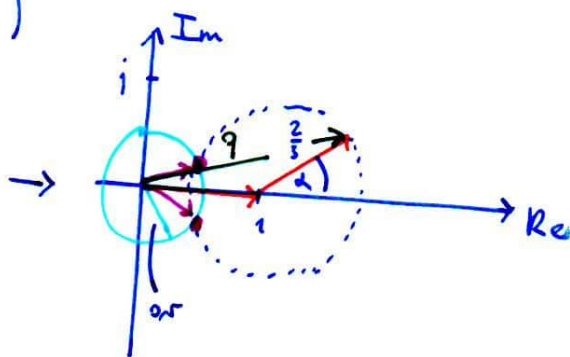
$$r_2 \triangleq \frac{U_0^- e^{j\beta l}}{U_0^+ e^{-j\beta l}} = \frac{R_2 - z_0}{R_2 + z_0} = \frac{0,2 - 1}{0,2 + 1} = -\frac{2}{3} \quad (\Rightarrow)$$

$$\Rightarrow U_0^- = U_0^+ e^{-j2\beta l} \cdot \left(-\frac{2}{3}\right)$$

$$\Rightarrow |U_0^+| \cdot \underbrace{\left| e^{-j2\beta z} - \frac{2}{3} e^{j\beta z} \cdot e^{-j2\beta l} \right|}_{p(z)} = \frac{1}{2} |U_0^+|$$

$$p(z) = e^{-j\beta z} \left(1 - \frac{2}{3} e^{+j2(\beta z - \beta l)} \right)$$

$$p(z) = \underbrace{\left| 1 \pm \frac{2}{3} e^{2j\beta(z-l) + j\pi} \right|}_{q(z)}$$



$$\alpha = 2\beta(z-l) + \pi$$

$$\Rightarrow \cos^{-1}(\dots) \Rightarrow 2\beta(z-l) + \pi = \dots + 2k\pi$$

• : ahol teljesül a feltétel

○ : 0,5 sugari kör \Rightarrow ahol metax

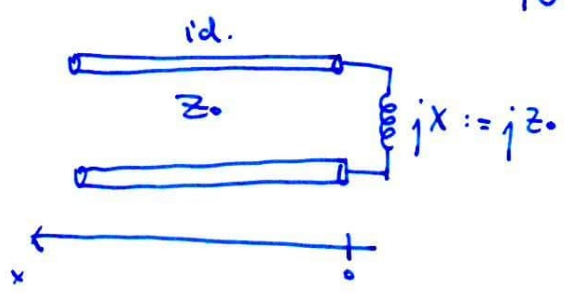
○ - t, ott teljesül az egyenlet.

$$q = 1 + \frac{2}{3} \cos(2\beta(z-l) + \pi) + j \frac{2}{3} \sin(2\beta(z-l) + \pi)$$

$$|q| = \sqrt{\left(1 + \frac{2}{3} \cos(\dots)\right)^2 + \left(\frac{2}{3} \sin(\dots)\right)^2} = \sqrt{1 + \frac{4}{3} \cos(\dots) + \frac{4}{9}} \quad (\Rightarrow)$$

7 Feladat lejárát felvételk

$|U(x)| = ?$



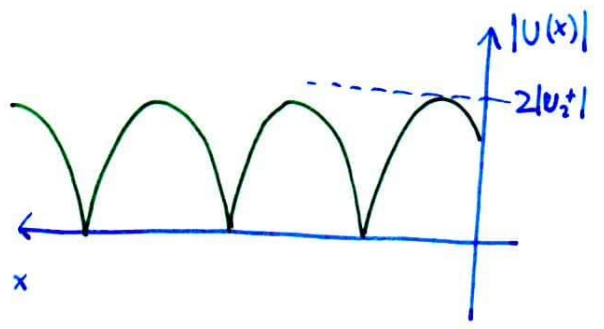
$U(x) = U_2^+ e^{j\beta x} + U_2^- e^{-j\beta x} \Leftrightarrow$

$\frac{U_2^-}{U_2^+} = r_2 = \frac{jz_0 - z_0}{jz_0 + z_0} = \frac{j-1}{j+1} \Rightarrow r_2 = e^{j\frac{\pi}{2}} = j \rightarrow$

$|r_2| = 1$

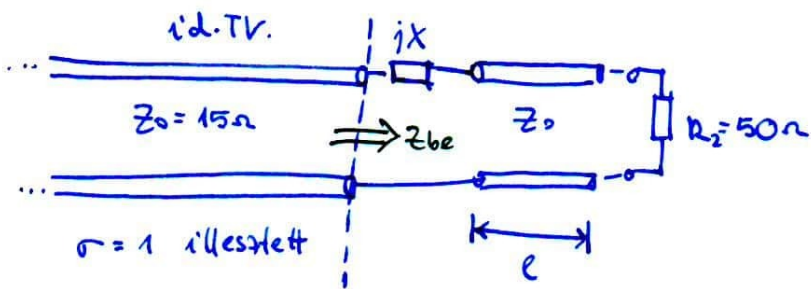
$\Leftrightarrow U_2^+ (e^{j\beta x} + e^{j\frac{\pi}{2}} e^{-j\beta x}) = U_2^+ e^{j\frac{\pi}{4}} (e^{j(\beta x - \frac{\pi}{4})} + e^{-j(\beta x - \frac{\pi}{4})}) \Leftrightarrow$

$\Rightarrow |U(x)| = |U_2^+| \cdot 2 \cos(\beta x - \frac{\pi}{4})$



• $\frac{\pi}{4}$ - szög eltolás $\cos(\dots)$

8. Illesztés. adott $w-n \Rightarrow \beta = \frac{w}{v}$ adott



$jX = ?$, $l = ?$: $Z_{be} = Z_0$ legyen!

$$Z_{be} = jX + Z_0 \frac{R_L + jZ_0 \tan \beta l}{Z_0 + jR_L \tan \beta l} := Z_0$$

$$\text{Re: } \text{Re} \left\{ Z_0 \frac{R_L + jZ_0 \tan \beta l}{Z_0 + jR_L \tan \beta l} \right\} := Z_0 \quad \text{(I)}$$

$$\text{Im: } X + \text{Im} \left\{ Z_0 \frac{\dots}{\dots} \right\} := 0 \quad \text{(II)}$$

$$\text{(I)} \rightarrow \tan \beta l = \dots \rightarrow \beta l = \dots \rightarrow l = \dots \checkmark$$

$$l \rightarrow \text{(II)} \rightarrow X = \dots \checkmark$$

Elektromágneses hullámok

Maxwell:

mághatározó szerepe

I. $\text{rot } \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$

$\vec{D} = \epsilon \vec{E}$

$\frac{\partial}{\partial t}$ nem hanyagolható el.

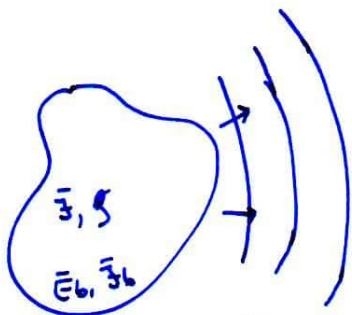
II. $\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{B} = \mu \vec{H}$

III. $\text{div } \vec{D} = \rho$

$\vec{j} = \sigma(\vec{E} + \vec{E}_b) + \vec{j}_b$

IV. $\text{div } \vec{D} = \rho$



ferroismentes test

homogén közeg, anyagtulajdonság: ϵ, μ, σ

Alapegyenletek

I. $\text{rot rot } \vec{H} = \sigma \cdot \text{rot } \vec{E} + \epsilon \frac{\partial \text{rot } \vec{E}}{\partial t} \stackrel{\text{II.}}{=} -\mu \cdot \sigma \frac{\partial \vec{H}}{\partial t} - \epsilon \mu \frac{\partial^2 \vec{H}}{\partial t^2}$

II. $\text{rot rot } \vec{E} = -\mu \frac{\partial \text{rot } \vec{H}}{\partial t} \stackrel{\text{I.}}{=} -\mu \sigma \frac{\partial \vec{E}}{\partial t} - \epsilon \mu \frac{\partial^2 \vec{E}}{\partial t^2}$

$\text{rot rot } \vec{x} = \text{grad div } \vec{x} - \Delta \vec{x} : \text{azonosság} \Leftrightarrow$

$\Leftrightarrow \Delta \vec{E} - \mu \sigma \frac{\partial \vec{E}}{\partial t} - \epsilon \mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0$

$\Delta \vec{H} - \mu \sigma \frac{\partial \vec{H}}{\partial t} - \epsilon \mu \frac{\partial^2 \vec{H}}{\partial t^2} = 0$

Hullámegyenletek
homogén (nincs gerjesztés)
parciális diff. egyenletek
(metódrendő)

Megj.:

$$\Delta \vec{E} \triangleq \text{graddiv } \vec{E} - \text{rotrot } \vec{E}$$

$$\vec{E}(x, y, z, t) \Rightarrow \Delta \vec{E} = \hat{e}_x \Delta E_x + \hat{e}_y \Delta E_y + \hat{e}_z \Delta E_z$$

||

$$\hat{e}_x E_x(x, y, z, t) + \hat{e}_y E_y(x, y, z, t) + \hat{e}_z E_z(x, y, z, t)$$

$$\Delta E_x(x, y, z, t) = \frac{\partial^2}{\partial x^2} E_x + \frac{\partial^2}{\partial y^2} E_x + \frac{\partial^2}{\partial z^2} E_x$$

"Legegyszerűbb" megoldás : a síkhullám

$$\vec{E}(x, y, z, t) \triangleq \hat{e}_x E_x(z, t) \quad : \quad z \text{ irányba terjed}$$

Hullámegyenlet:

$$\Delta \vec{E} = \hat{e}_x \Delta E_x = \hat{e}_x \frac{\partial^2}{\partial z^2} E_x(z, t)$$

$$\frac{\partial}{\partial t} \vec{E} = \hat{e}_x \frac{\partial}{\partial t} E_x(z, t)$$

$$\frac{\partial^2}{\partial t^2} \vec{E} = \hat{e}_x \frac{\partial^2}{\partial t^2} E_x(z, t)$$

$$\frac{\partial^2 E_x}{\partial t^2} - \mu \sigma \frac{\partial E_x}{\partial t} - \mu \epsilon \frac{\partial^2 E_x}{\partial t^2} = 0$$

$[\sigma = 0 : \text{ideális szigetelő}]$

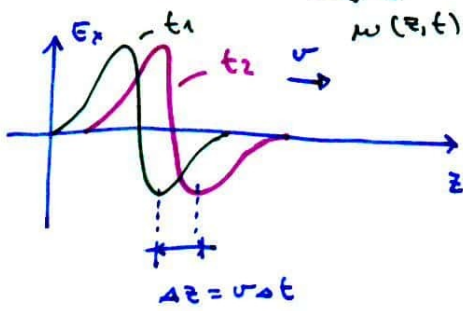
$(\sigma = \infty : \text{ideális vezető})$

σ "kicsi" : veszteséges szigetelő

σ "nagy" : veszteséges vezető

Terjedő hullám-e?

$$E_x(z, t) \stackrel{?}{=} E_x(z - vt)$$



$$\frac{\partial}{\partial z} E_x(\omega(z, t)) = E_x' \frac{\partial \omega}{\partial z} \quad 1$$

$$\frac{\partial}{\partial t} E_x(\omega(z, t)) = E_x' \frac{\partial \omega}{\partial t} \quad \rightarrow v$$

$$\frac{\partial^2 E_x}{\partial z^2} - \mu \epsilon \frac{\partial^2 E_x}{\partial t^2} = \left(\frac{\partial}{\partial z} - \sqrt{\mu \epsilon} \frac{\partial}{\partial t} \right) \left(\frac{\partial}{\partial z} + \sqrt{\mu \epsilon} \frac{\partial}{\partial t} \right) E_x = 0$$

$$\frac{\partial E_x}{\partial z} \pm \sqrt{\mu \epsilon} \frac{\partial E_x}{\partial t} = 0$$

$$\Rightarrow E_x' \pm v \sqrt{\mu \epsilon} E_x' = 0$$

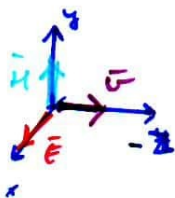
$$(1 \pm v \sqrt{\mu \epsilon}) = 0 \quad \left. \begin{array}{l} v = \frac{1}{\sqrt{\mu \epsilon}} \\ v = -\frac{1}{\sqrt{\mu \epsilon}} \end{array} \right\} \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = 3 \cdot 10^8 \frac{m}{s}$$

$$\vec{E} = \hat{e}_x E_x(z - vt) \rightarrow \vec{H} = ?$$

$$\text{rot } \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \rightarrow \frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu} \text{rot } \vec{E}$$

$$-\frac{1}{\mu} \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(z-vt) & 0 & 0 \end{vmatrix} = -\frac{1}{\mu} \frac{\partial}{\partial z} E_x(z-vt) \hat{e}_y = \boxed{+\frac{\sqrt{\mu \epsilon}}{\mu} \frac{\partial}{\partial t} E_x(z-vt) \hat{e}_y} = \frac{1}{v} \frac{\partial}{\partial t} E_x(z-vt) \hat{e}_y = \vec{H}(z, t)$$

$$\vec{H}(z, t) = \hat{e}_y \sqrt{\frac{\epsilon}{\mu}} E_x(z - vt) = H_y(z - vt)$$



Def: Síkhullám: a terjedés iránya \perp síkban \vec{E} és \vec{H} helyfüggetlen.

• $\vec{E} \perp \vec{H}$

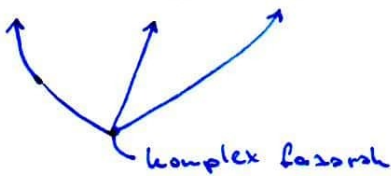
• $\vec{E} \perp \vec{v}$ és $\vec{H} \perp \vec{v}$

Időben szinuszos vektortér

$$\vec{E}(x, y, z, t) \equiv \text{Re} \left\{ \underbrace{\tilde{\vec{E}}(x, y, z)}_{\text{komplex rádfüggvény}} e^{j\omega t} \right\}$$

komplex rádfüggvény

$\vec{E} \approx \tilde{E}_x \hat{e}_x + \tilde{E}_y \hat{e}_y + \tilde{E}_z \hat{e}_z$: komplex vektor



$$\Delta \vec{E} - \mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \xrightarrow{\frac{\partial}{\partial t} \rightarrow j\omega} \text{Helmholtz-egyenlet}$$

$$\Delta \tilde{\vec{E}} - j\omega\sigma\mu \tilde{\vec{E}} + \omega^2\mu\epsilon \tilde{\vec{E}} = 0$$

$$\underbrace{\Delta \tilde{\vec{E}} + (\omega^2\mu\epsilon - j\omega\mu\sigma)}_{-\gamma^2} \tilde{\vec{E}} = 0 \quad | \quad \gamma: \text{terjedési együttható}$$

Síkhullámok:

$$\frac{d^2 \tilde{E}_x}{dz^2} - \gamma^2 \tilde{E}_x = 0$$

$$\left(\text{TV: } \frac{d^2 U}{dz^2} - \gamma^2 U = 0 \right)$$

általános megoldás:

$$\tilde{E}_x(z) = E_{x0}^+ e^{-\gamma z} + E_{x0}^- e^{\gamma z}$$

$$\tilde{H}_y(z) = H_{y0}^+ e^{-\gamma z} + H_{y0}^- e^{\gamma z}$$

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon} \quad \rightarrow \quad \begin{cases} \gamma = j\omega\sqrt{\mu\epsilon} \ominus \text{(ideális szigetelő)} \\ \ominus j\beta \end{cases}$$

$$\rightarrow \gamma = \alpha + j\beta \quad \text{(vesztékes szigetelő)}$$

$$\rightarrow \gamma = \alpha + j\beta \quad (\alpha = \beta) \quad \text{(vesztéktelen)}$$

α : csillapítási tényező

β : fázistényező

11. $\text{rot} \tilde{\mathbf{E}} = -j\omega \tilde{\mathbf{H}}$

$$\tilde{\mathbf{H}} = -\frac{1}{j\omega\mu} \text{rot} \tilde{\mathbf{E}} = -\frac{1}{j\omega\mu} \left[\begin{array}{ccc|c} \hat{e}_x & \hat{e}_y & \hat{e}_z & \\ 0 & 0 & \frac{\partial}{\partial z} & \\ \hline E_{x0}^+ e^{-\gamma z} & 0 & 0 & \\ \hline E_{x0}^- e^{\gamma z} & 0 & 0 & \end{array} \right] =$$

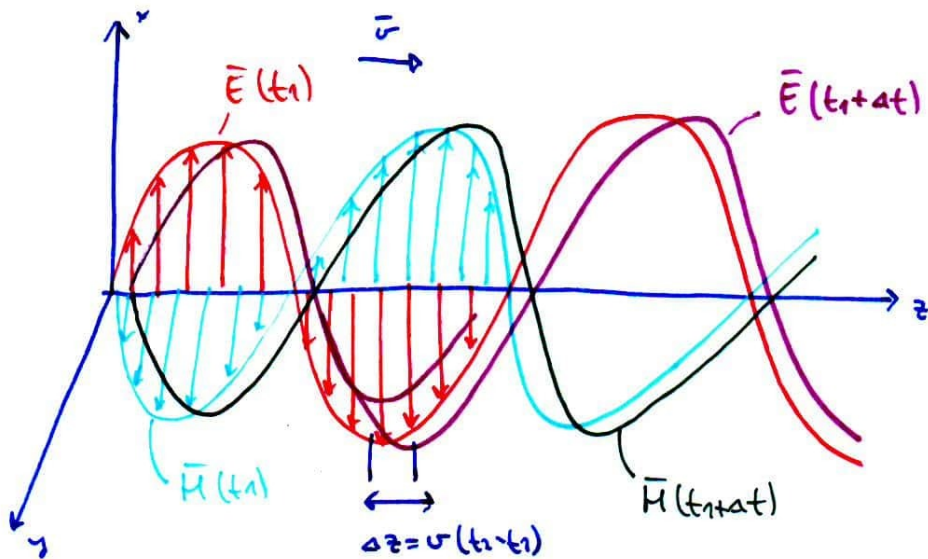
$$= \frac{1}{j\omega\mu} \left(\gamma E_{x0}^+ e^{-\gamma z} - \gamma E_{x0}^- e^{\gamma z} \right) \hat{e}_y$$

$$\frac{j\omega\mu}{\gamma} \triangleq Z_0 : \text{ hullámimpedancia} \Rightarrow Z_0 \triangleq \frac{E_{x0}^+}{H_{y0}^+} = -\frac{E_{x0}^-}{H_{y0}^-} \quad (\Rightarrow)$$

$$H_y(z) = \underbrace{\frac{E_{x0}^+}{Z_0}}_{H_{y0}^+} e^{-\gamma z} + \underbrace{-\frac{E_{x0}^-}{Z_0}}_{H_{y0}^-} e^{\gamma z}$$

$$\Rightarrow Z_0 = \frac{j\omega\mu}{\sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon}}$$

$Z_0 = \sqrt{\frac{\mu}{\epsilon}} : \text{ tértől valóbs levegő}$
 $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega : \text{ vákuum hullámimpedancia}$
 $Z_0 \in \mathbb{C}$



$$\tilde{E}_x(z) = E_{x0}^+ e^{-j\beta z}$$

$$\tilde{H}_y(z) = \frac{E_{x0}^+}{Z_0} e^{-j\beta z}$$

$$\lambda = T \cdot v = \frac{v}{f}$$

$$\lambda = \frac{2\pi}{\beta}$$

hullámhossz

Elektromágneses hullámok ⊕

• Szinuszos időfüggés → komplex amplitúdák

• Síkhullám $\rightarrow \mathbb{E} = \hat{e}_x E_x, \vec{H} = \hat{e}_y H_y$

$$\tilde{E}_x(z) = E_{x0}^+ e^{-\gamma z} + E_{x0}^- e^{\gamma z}$$

$$\tilde{H}_y(z) = \frac{E_{x0}^+}{Z_0} e^{-\gamma z} - \frac{E_{x0}^-}{Z_0} e^{\gamma z}$$

Teljesítményáramlás síkhullámban

Poynting-vektor: $\vec{S} = \vec{E} \times \vec{H}$: valós, időfüggő

$$E_x(z,t) = \hat{E}_x(z) \cos(\omega t + \beta_E(z))$$

$$H_y(z,t) = \hat{H}_y(z) \cdot \cos(\omega t + \beta_H(z))$$

⇒ $E \times H$: z irányú lesz.

$$S_z(t) = E_x(z,t) \cdot H_y(z,t) = \hat{E}_x(z) \hat{H}_y(z) \cos(\omega t + \beta_E(z)) \cdot \cos(\omega t + \beta_H(z)) =$$

$$= \frac{1}{2} \hat{E}_x(z) \hat{H}_y(z) \left[\underbrace{\cos(2\omega t + \beta_E + \beta_H)}_{\text{időfüggő}} + \underbrace{\cos(\beta_E - \beta_H)}_{\text{időfüggetlen}} \right]$$

$$S_z^{\text{átl.}}(z) = \frac{1}{2} \hat{E}_x \hat{H}_y \cdot \cos(\beta_E - \beta_H)$$

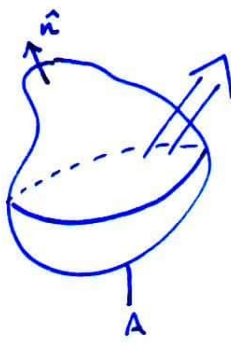
→ közepértékű 0.

$$S_z^{\text{átl.}}(z) = \text{Re} \left\{ \frac{1}{2} \tilde{E}_x \tilde{H}_y^* \right\}$$

↑
komplex Poynting-vektor „z” irányú rendszerje

Ált.: Komplex P.-vektor:

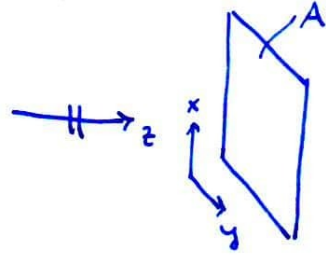
$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^*$$



$$P_s = \oint_A \operatorname{Re}\{\vec{S}\} d\bar{A}$$

sugárzó határos teljesítmény

Síkhullám:



$$P_s = A \cdot \operatorname{Re}\{\vec{S}_z\}$$

↑ komplex P.-vektor

Síkhullám vezetőkben

Hullámparaméterek: $\sigma \gg \omega \cdot \epsilon$ (rot $\vec{H} = \vec{J} + j\omega\vec{D} = (\sigma + j\omega\epsilon)\vec{E}$)
 ↑ elanyagolható

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon} = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

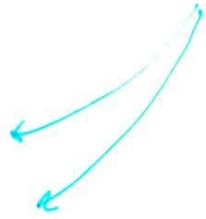
$$\Rightarrow \gamma = \sqrt{j\omega\mu\sigma} \quad \ominus \quad \ominus \alpha + j\beta \quad ; \quad \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$Z_0 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \Rightarrow Z_0 = \sqrt{\frac{j\omega\mu}{\sigma}} = (1+j) \sqrt{\frac{\omega\mu}{2\sigma}}$$

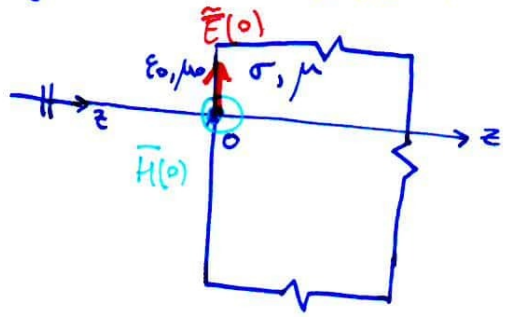
pl.: réz: $\sigma = 57 \frac{MS}{m}$

$f = 500 \text{ kHz} \rightarrow |Z_0| = 0,26 \text{ m}\Omega$

$f = 5 \text{ kHz} \rightarrow |Z_0| = 0,026 \text{ m}\Omega$



• Végtelek közötti felület:



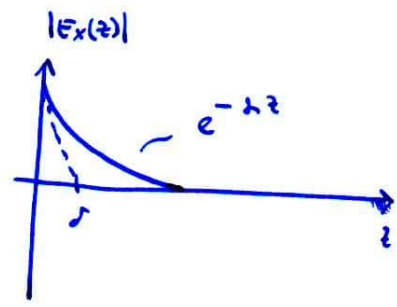
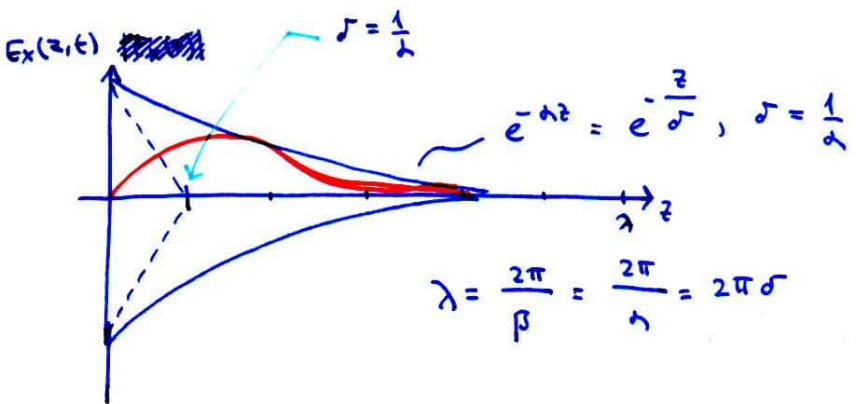
$$\bar{E}_x = E_{x0}^+ e^{-\alpha z} + E_{x0}^- e^{\alpha z}$$

+z irány nem alakul ki

$$\bar{H}_y(z) = \frac{E_{x0}^+}{z_0} e^{-\alpha z} = H_{y0}^+$$

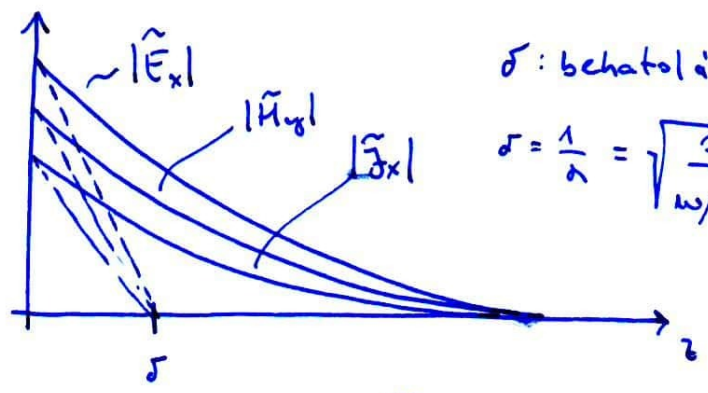
$$E_x(z) = E_x(0) \cdot e^{-\alpha z} \cdot e^{j\beta z}$$

$$|E_x(z)| = |E_x(0)| \cdot e^{-\alpha z} \cdot 1$$



$$|\tilde{H}_y(z)| = \frac{|\tilde{E}_x(z)|}{|z_0|}$$

$$\tilde{J} = \sigma \tilde{E} \rightarrow \tilde{J}_x(z) = \sigma \tilde{E}_x(z)$$



δ : behatólási mélység (skin-leadés)

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \sigma}}$$

pl.: réz: $\sigma = 57 \frac{MS}{m}$

f	δ (mm)
50 Hz	9,43
5000 Hz	0,943
500 kHz	0,0943

- $z=0$ sík "A" keresztmetszetén átváramló hatósos teljesítmény:
(felületen disszipalációs teljesítmény)

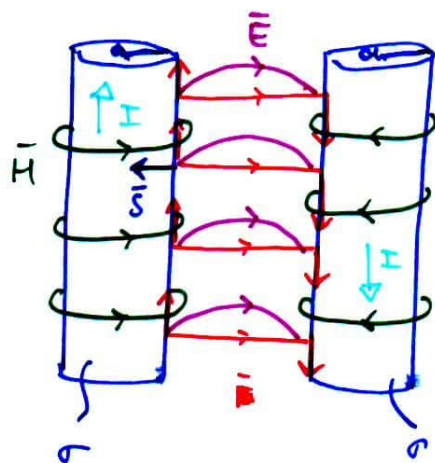
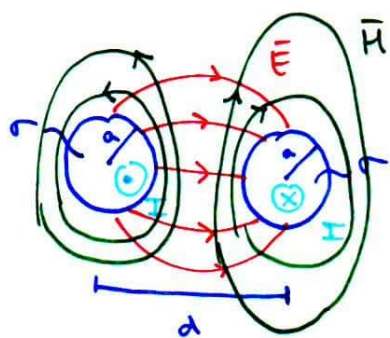
$$\boxed{\bar{S} = \frac{1}{2} \bar{E} \times \bar{H}^* = \frac{1}{2} (\hat{e}_x E_x) \times (\hat{e}_y H_y)^* = \frac{1}{2} \hat{e}_z z_0 |H_y|^2}$$

$$\boxed{P_A = \text{Re}\{S_z\} \cdot A = \frac{1}{2} \text{Re}\{z_0\} |H_y|^2 \cdot A = \frac{1}{2} \sqrt{\frac{\omega\mu}{2\sigma}} |H_y|^2 \cdot A}$$

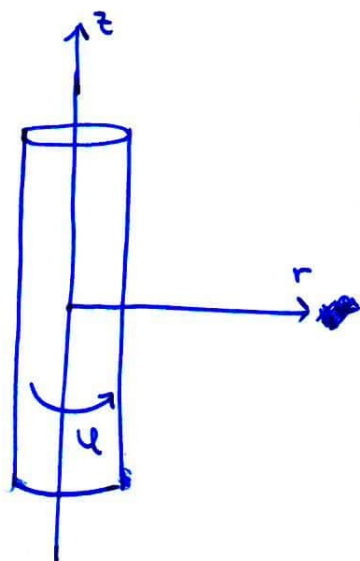
$\hookrightarrow z_0 = -\sqrt{\frac{j\omega\mu}{\sigma}}$

Hengeres vezető váltóáramú ellenállása

Lecher-vezeték:



$d \gg a$

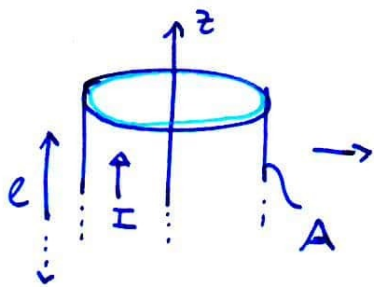


$$\bar{H} = \hat{e}_\varphi H_\varphi$$

$$\bar{E} = \hat{e}_z E_z (+\hat{e}_\varphi + \dots + \hat{e}_\varphi)$$

$$\bar{S} = \frac{1}{2} \bar{E} \times \bar{H}^*$$

$$S_r = -\frac{1}{2} E_z H_\varphi^*$$



i fcs. inkes nagyon közel közel
az EM hullám.

~~Amper-tör.~~

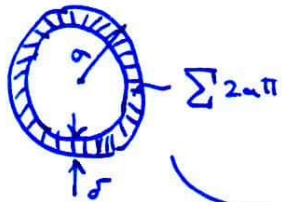
a) $\delta \ll a$

Amper-tör.

$$P_A = \frac{1}{2} \sqrt{\frac{\mu_0}{2\sigma}} \cdot \left(\frac{|I|}{2\pi a}\right)^2 \cdot l 2\pi a = \frac{1}{2} |I|^2 \left[\frac{1}{\sigma} \frac{l}{2\pi a} \right]$$

váltakozó ellenállás
"l" hosszú, "a" sugarú,
 σ fajlagos vez. kép.
hosszra vezetősé

Egyenáramú: $R_0 = \frac{l}{\sigma a^2 \pi}$



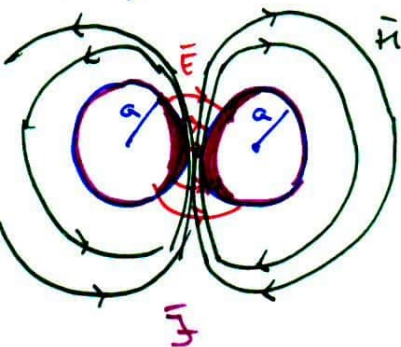
Egyenáramú alakra az ellenállás, mint váltóáramú a tömör hengerre.

b) $\delta \gg a$

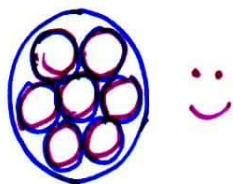
$$\delta = \sqrt{\frac{2}{\mu_0 \sigma}} \Rightarrow R \approx R_0 \text{ (egyenáramú)}$$

c) $\delta \approx a$: bonyolult eset

d) $\delta \gg a$



hővesztési hatás



Síkhullám - terjedésének analógia

TV.

SH. ($\rightarrow z$)

$$\frac{d^2 U}{dz^2} - \gamma^2 U = 0$$

$$\frac{d^2 E_x}{dz^2} - \gamma^2 E_x = 0$$

$$U(z) = U_0^+ e^{-\gamma z} + U_0^- e^{\gamma z}$$

$$E_x(z) = E_{x0}^+ e^{-\gamma z} + E_{x0}^- e^{\gamma z}$$

$$I(z) = \frac{U_0^+}{Z_0} e^{-\gamma z} - \frac{U_0^-}{Z_0} e^{\gamma z}$$

$$H_y(z) = \frac{E_{x0}^+}{Z_0} e^{-\gamma z} - \frac{E_{x0}^-}{Z_0} e^{\gamma z}$$

$$\gamma = \sqrt{(j\omega L' + R')(j\omega C' + G')}$$

$$\gamma = \sqrt{j\omega\mu(j\omega\epsilon + \sigma)}$$

$$Z_0 = \sqrt{\frac{j\omega L' + R'}{j\omega C' + G'}}$$

$$Z_0 = \sqrt{\frac{j\omega\mu}{j\omega\epsilon + \sigma}}$$

ideális TV.: $\gamma = j\omega\sqrt{LC'}$

ideális sírhullám: $\sigma = 0$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

$$\gamma = j\omega\sqrt{\mu\epsilon}$$

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}$$

TV	L'	C'	R'	G'	U	I	γ	Z_0	... szétválasztott mennyiségek
SH	μ	ϵ	σ	σ	E_x	H_y	γ	Z_0	... n

TV./hálózat tetjellemező hullámprop.

\ominus^* oka: TV.-re a telvírőegyenlet: $-\frac{dU}{dz} = j\omega L' \cdot I + R' \cdot I$

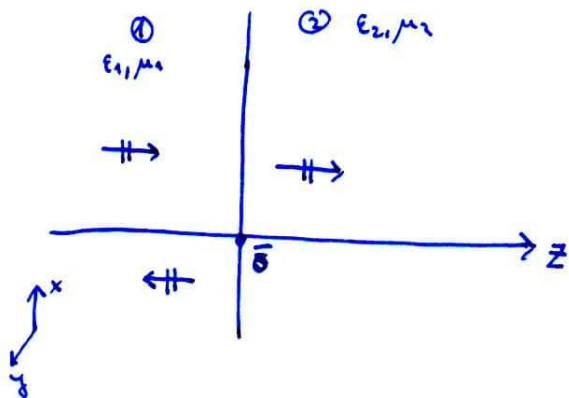
$$-\frac{dI}{dz} = j\omega C' \cdot U + G' \cdot U$$

SH.-ra: $\text{rot } \vec{E} = -j\omega\mu \vec{H} + \vec{J}$: Maxwell-egy. a szétválasztás miatt
 $\text{rot } \vec{H} = j\omega\epsilon \vec{E} + \vec{J}$ (\vec{J} -igényes elem) R' -vel nincs potja.

Síkhullókn vesélkedése közeghatáron

(S.H. megrölegesen csak az sík hatorfelületre)

I. Reflexió vágatlan felület határvonal



$$1.: E_x(z) = \boxed{E_{x01}^+} e^{-\gamma_1 z} + \boxed{E_{x01}^-} e^{\gamma_1 z}$$
$$H_y(z) = \frac{E_{x01}^+}{Z_{01}} e^{-\gamma_1 z} - \frac{E_{x01}^-}{Z_{01}} e^{\gamma_1 z}$$

□ : 3 ráfordónk van

$$2.: E_x(z) = \boxed{E_{x02}^+} e^{-\gamma_2 z}$$
$$H_y(z) = \frac{E_{x02}^+}{Z_{02}} e^{-\gamma_2 z}$$

Ha pl.: E_{x01}^+ adott \Rightarrow 2 ismeretlen: $E_{x02}^+, E_{x01}^- = ?$

\Rightarrow két ismeretlenre 2 folytonossági egyenlet E_x -re és H_y -ra

$$\left. \begin{array}{l} \text{I. } E_{x01}^+ + E_{x01}^- = E_{x02}^+ \\ \text{II. } \frac{E_{x01}^+}{Z_{01}} - \frac{E_{x01}^-}{Z_{01}} = \frac{E_{x02}^+}{Z_{02}} \end{array} \right\}$$

Reflexiók tényező ($z=0$):

$$r(0) = \frac{E_{x01}^-}{E_{x01}^+} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

Feladatolábbon tipikusán: $\mu_1 = \mu_2 = \mu_0$

$$\epsilon = \epsilon_r \epsilon_0$$

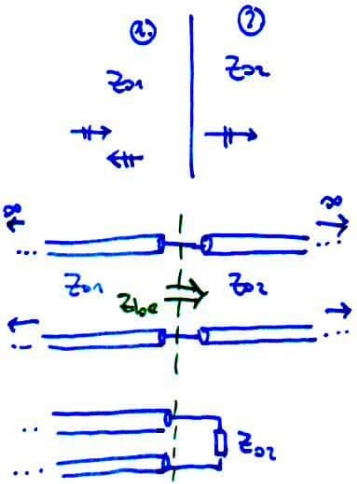
id. szig.: $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}} = \frac{1}{\sqrt{\epsilon_r}} \cdot 377 \Omega$

$$\gamma = j\omega \sqrt{\mu_0 \epsilon_0} \sqrt{\epsilon_r} = j\omega \sqrt{\epsilon_r} \cdot \frac{1}{c}$$

$\underbrace{\hspace{2cm}}$
 β fázistényező

$$\Rightarrow v = \frac{c}{\sqrt{\epsilon_r}}, \quad \sqrt{\epsilon_r} = n: \text{ törésmutató}$$

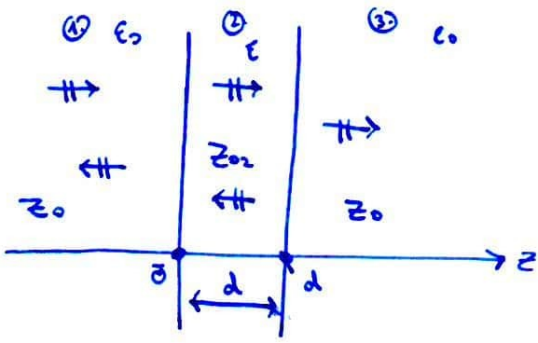
TV. modell :



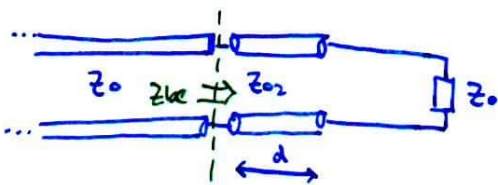
$$\left. \begin{aligned} U(z) &= U_{02}^+ e^{-\gamma z} \\ I(z) &= \frac{U_{02}^+}{Z_0} e^{-\gamma z} \end{aligned} \right\} Z_{bc} = \frac{U(z)}{I(z)} = Z_{02}$$

$$r(0) = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

II. Reflexió végtelek levezetésekor :



$$r(0) = ?$$



$$r(0) = \frac{Z_{bc} - Z_0}{Z_{bc} + Z_0}$$

$$Z_{bc} = Z_{02} \frac{Z_0 + j Z_{02} \tan \beta_2 d}{Z_{02} + j Z_0 \tan \beta_2 d}$$

$$Z_{02} = \frac{Z_0}{\sqrt{\epsilon_r}} \quad \beta_2 = \frac{\omega}{v} = \frac{\omega}{c \cdot \sqrt{\epsilon_r}}$$

Korlatok:

- csak mérőleges beesés sík határfelületén
- E_x -szel jellemezhető
- lineárisan polarizált síkhullám

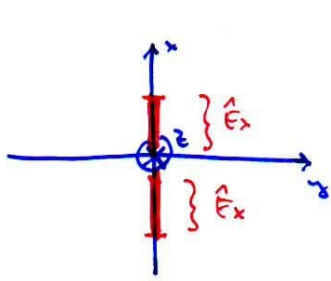
SH. → polarizációja

$$E_1(z,t) = \hat{e}_x E_x(z,t) = \hat{e}_x \cdot \hat{E}_x \cdot \cos(\omega t + \beta z) : SH$$

$$E_2(z,t) = \hat{e}_y E_y(z,t) = \hat{e}_y \cdot \hat{E}_y \cdot \cos(\omega t + \beta z) : SH$$

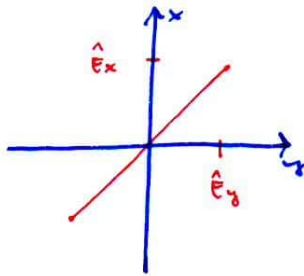
$E_1 + E_2$ is SH. lesz.

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$



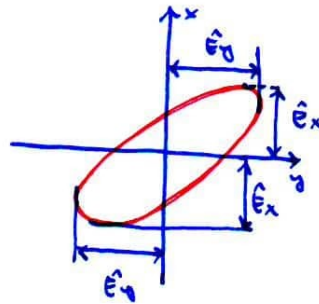
$$\hat{E}_y = 0$$

Lineáris pol.
x irányú



$$\beta_x = \beta_y$$

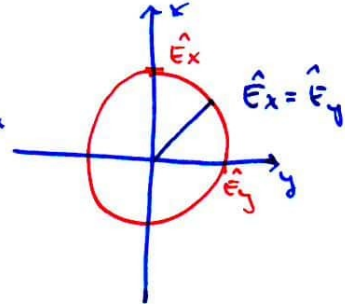
Lineáris pol.
átfordult irányú



$$\beta_x \neq \beta_y$$

$$\hat{E}_x \neq \hat{E}_y$$

Elliptikus polarizáció



$$\hat{E}_x = \hat{E}_y$$

$$\beta_x - \beta_y = \frac{\pi}{2}$$

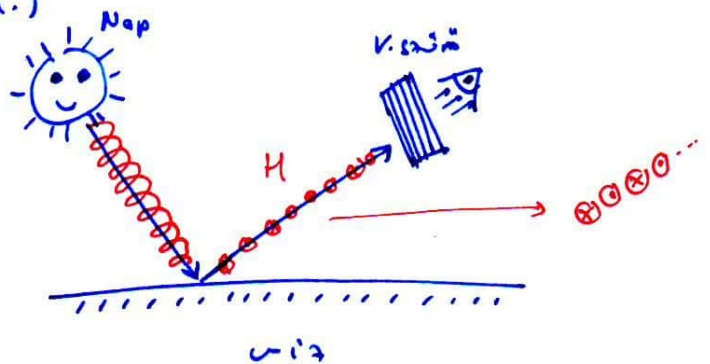
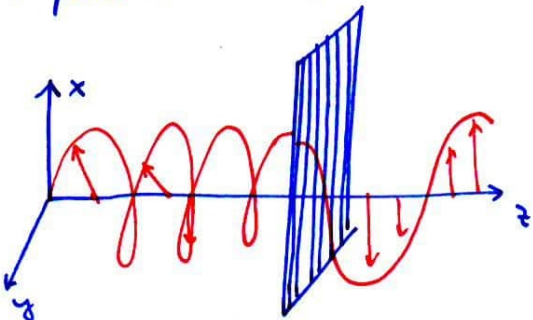
Cirkuláris polarizáció

↳ vízszintes = horizontális

Függőleges = vertikális

Alkalmazások:

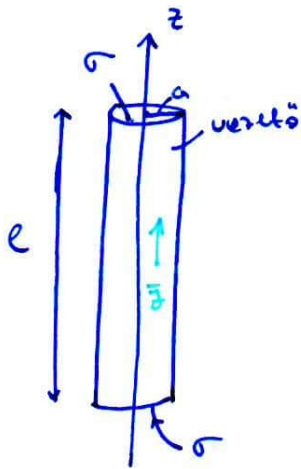
- különböző csatornák (horiz./vertik.)
- polarizáció vert. szűrő



Hullámok helyese - a Hertz-dipólus

eddig: forrasmentes közegben a hullámok terjedése

most: források helyese: $\vec{J}, \rho \rightarrow \vec{A}, \varphi \rightarrow \vec{E}, \vec{B}$
 potenciálak



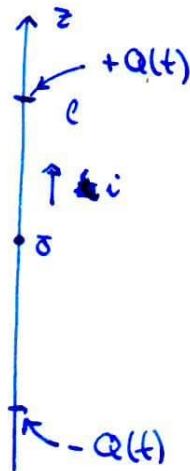
$$\vec{J} = \hat{e}_z \cdot \vec{J} \cdot \cos(\omega t)$$

$$\text{div } \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\vec{r} = \frac{\vec{J}}{\omega} \cdot \sin(\omega t)$$

$$a \ll l$$

\Rightarrow

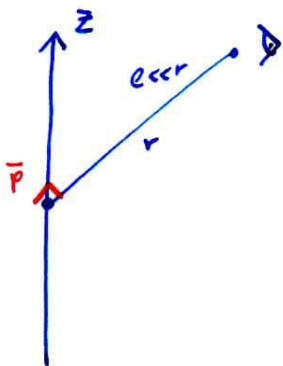


\Rightarrow

$$i(t) = a^2 \pi \cdot \vec{J} \cdot \cos(\omega t)$$

helyfüggetlen!

\Rightarrow megfigyelő "r" távolsága: $r \gg l$

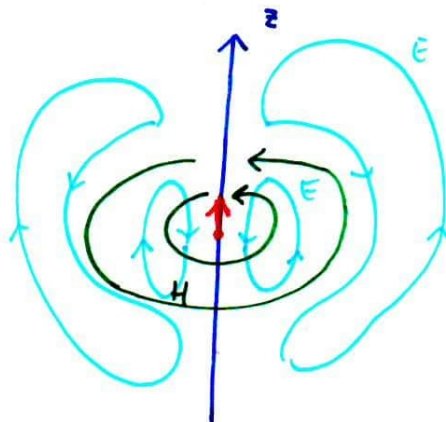


$$\left. \begin{array}{l} l \rightarrow 0 \\ \hat{r} \rightarrow \infty \end{array} \right\} e \cdot \hat{r} = \text{const.}$$

$$\vec{p} = \hat{e}_z \cdot l \cdot \hat{I} \quad \text{nyomatek (Hertz-dipólus)} \quad (\vec{p} = \hat{e}_z \cdot l \cdot \hat{I} \text{ komplex})$$

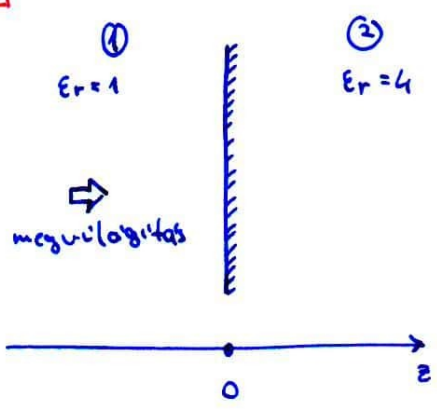
Hertz-dipólussal jellemezhető:

- egyenes antenna
- $a \ll l$: vonalszerű
- $r \gg l$: pontszerű
- $\lambda \gg l$: állandó (helyfüggetlen) áramú (időtől függ)



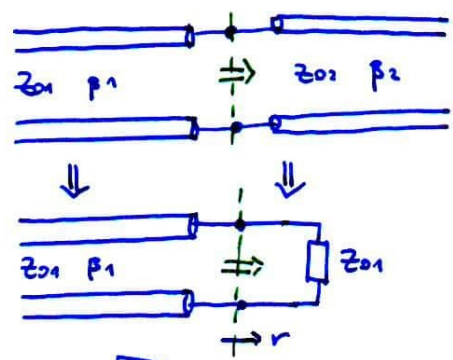
Gym.

1 $\vec{E}(z) = ?$ $U(z) = ?$



$f = 10 \text{ MHz}$
 $E = 120 \text{ V/m}$ amplitúdó
 $\vec{E} = E_x(z) \hat{e}_x$
 $\Rightarrow |E_x(0)| = 120 \frac{\text{V}}{\text{m}}$

Távvezetők helyettesítés: csak merőleges beesés esetén.



$$Z_{01} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \Omega \approx 377 \Omega$$

$$Z_{02} = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} = \frac{Z_{01}}{\sqrt{\epsilon_r}} = \frac{Z_{01}}{\sqrt{4}} = \frac{1}{2} Z_{01} = 60\pi \Omega$$

$$\Gamma_{12} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = -\frac{1}{3}$$

$$\beta_1 = \frac{2\pi \cdot f}{c} = 0,209 \frac{1}{\text{m}} : 1\text{m-en hány radióhullám van?}$$

$$\beta_2 = \frac{2\pi f}{v} = \frac{2\pi f}{\frac{c}{\sqrt{\epsilon_r}}} = 0,419 \frac{1}{\text{m}}$$

① ($z < 0$)

$$E_x(z) = E_{01}^+ e^{-j\beta_1 z} + E_{01}^- e^{j\beta_1 z}$$

$$E_x(0) = E_{01}^+ + E_{01}^- \hat{=} 120 \frac{V}{m}$$

$$\hat{=} E_1^+ + r_{12} E_1^+ = E_1^+ (1 + r_{12})$$

$$\Rightarrow \underline{E_1^+ = 180 \frac{V}{m}} \quad \text{e's} \quad \underline{E_1^- = -60 \frac{V}{m}}$$

$$\Rightarrow \underline{\underline{E_x(z) = (180 e^{-j0,209z} - 60 e^{j0,209z}) \frac{V}{m}}}$$

$$\underline{\underline{H = H_y \hat{e}_y}}$$

$$H_y(z) = H_1^+ e^{-j\beta_1 z} + H_1^- e^{j\beta_1 z}$$

$$H_1^+ = \frac{E_1^+}{Z_{01}} \quad H_1^- = -\frac{E_1^-}{Z_{01}}$$

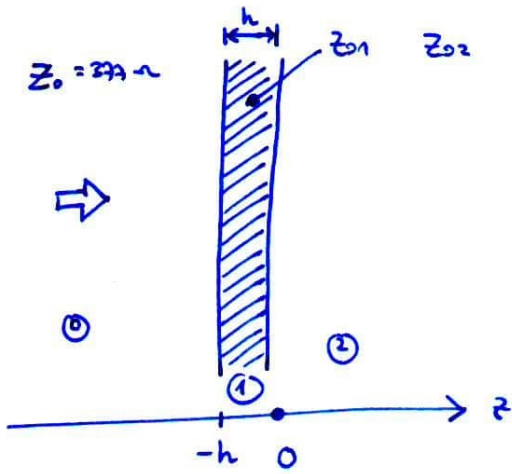
$$\underline{\underline{H_y(z) = (0,478 e^{-j0,209z} + 0,159 e^{j0,209z}) \frac{A}{m}}}$$

② ($z > 0$)

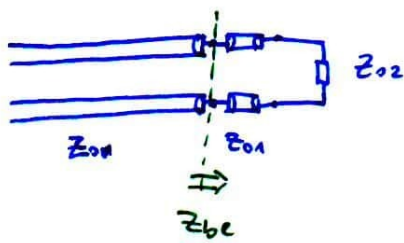
$$E_x(z) = E_2^+ e^{-j\beta_2 z} = 120 \frac{V}{m} e^{-j0,419z}$$

$$H_y(z) = \frac{E_2^+}{Z_{02}} e^{-j\beta_2 z} = 0,637 \frac{A}{m} \cdot e^{-j0,419z}$$

2 Reflexiómentes (illesztett) rálóg



$h = ?$ $Z_{01} = ?$, hogy $Z = -h$ -nél ne legyen reflexió.



$$Z_{be} = Z_0$$

$$\Downarrow$$

$$Z_{01} \frac{Z_{02} - Z_{01} j \tan \beta h}{Z_{01} - Z_{02} j \tan \beta h} = Z_0 \Rightarrow$$

$$\Rightarrow Z_{01} Z_{02} - j Z_{01}^2 \tan \beta h = Z_0 \cdot Z_{01} - j Z_0 Z_{02} \tan \beta h \quad / \div \tan \beta h$$

$$\frac{Z_{01} Z_{02}}{\tan \beta h} - j Z_{01}^2 = \frac{Z_0 Z_{01}}{\tan \beta h} - j Z_0 Z_{02}$$

Ha $\tan \beta h \rightarrow \infty \Rightarrow$ valós részek egyenlőek

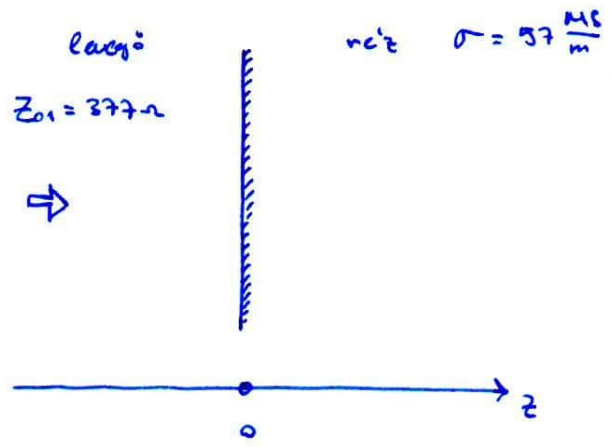
$$\Rightarrow \beta h = \frac{\pi}{2} + k\pi \quad \lambda_1 = \frac{2\pi}{\beta}$$

$$\underline{\underline{h = \frac{\lambda_1}{4} + k \cdot \frac{\lambda_1}{2}}} \quad (\lambda \text{ negyedek illesztés}) \quad (\text{szűk frekvencia tartományban működik})$$

$$Z_{01}^2 = Z_0 Z_{02} \rightarrow \underline{\underline{Z_{01} = \sqrt{Z_0 Z_{02}}}} \quad : \text{méslik kétféle mértékű kábelre}$$

\Rightarrow E_{01} meghatározható.

3. Vasteseesige liiteseel



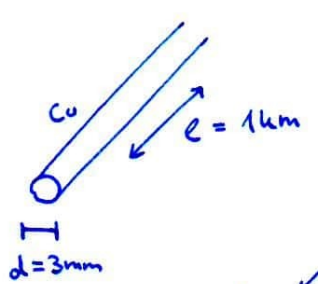
$f = 1 \text{ MHz}$
 $r_{12} = ? \quad E_e(z=0) = ?$
 $Z_{02} = \sqrt{\frac{j\omega\mu_0}{\sigma}} = 0,372 e^{j\frac{\pi}{4}} \text{ m}\Omega$

$r_{12} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = -1 + 1,974 \cdot 10^{-6} e^{j\frac{\pi}{4}} \approx -1$

PEC:
 perfect electric
 conductor

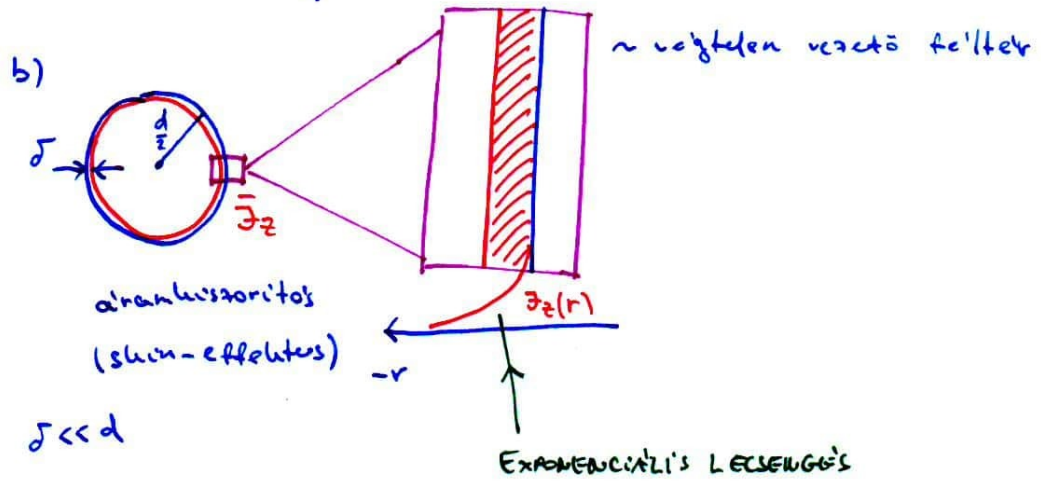
$E(0) = E_1^+ + E_1^- = E_1^+(1 + r_{12}) = 1,974 \cdot 10^{-6} e^{j\frac{\pi}{4}} \cdot E_1^+ \approx 0$

4. Vasteseesike elementaals

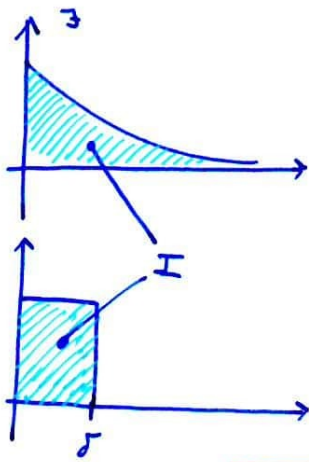


- a) $R_0 = ?$ ($f = 0$)
- b) $R = ?$ ($f = 30 \text{ MHz}$)

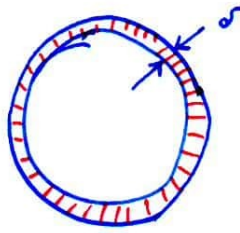
a) $R_0 = \frac{1}{\sigma_{Cu}} \cdot \frac{1000}{\frac{d^2}{4} \cdot \pi} = 2,48 \Omega$



← e



=>



$$\delta = \sqrt{\frac{2}{\mu \sigma \omega}} \quad , \quad \delta_{cu}^{mn} = \frac{66,7}{\sqrt{f^{Hz}}} = 0,38 \text{ mm} \ll d$$

$$R = \frac{1}{\sigma_{cu}} \frac{l}{2\pi\delta} = \underline{\underline{5,52 \Omega}} > R_0$$

↑
Lévelítés

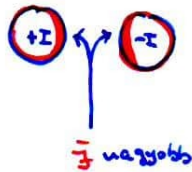
5

- $\delta \gg d, r \Rightarrow R_0$
- $\delta \ll d, r \Rightarrow R$
- $\delta \sim d, r \Rightarrow$ Nomenklátur módosít

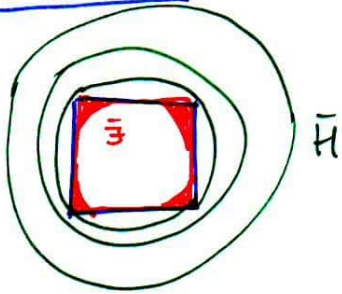
Négyzetű keresztmetszetű vezeték:



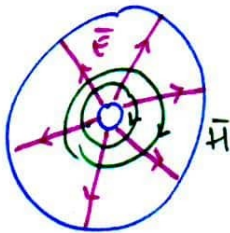
Lechner vezeték:



Magyaraitat



Koax:

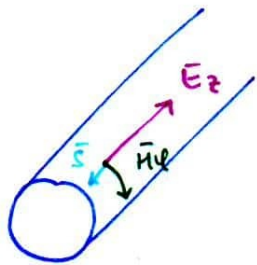


külső poláris $\rightarrow \infty$ visszaverés \rightarrow elterelés
(szimmetria miatt)

TEM: transverz. elektr. mágneses

$$\vec{S} = \vec{E} \times \vec{H} = \vec{S}_z \hat{e}_z$$

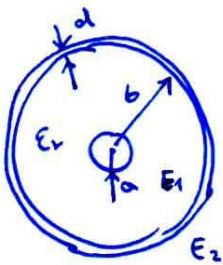
\rightarrow : töltéselosztást jelöl



$$\vec{S} = -S_r \hat{e}_r \quad (\vec{S} \text{ "r" irányú})$$

\vec{S} és \vec{H} merőleges a vezetőkre

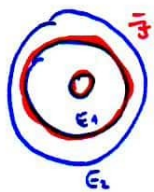
6 Koaxiális kábel anyagköltségi hatása



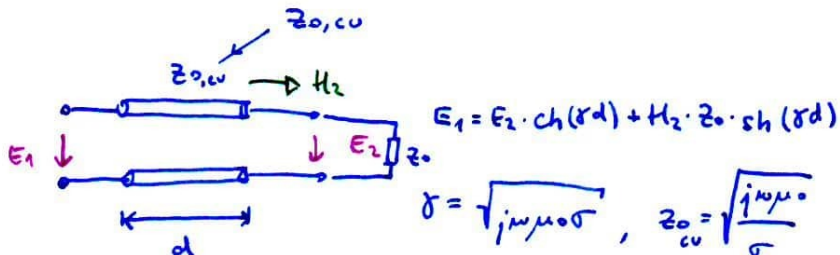
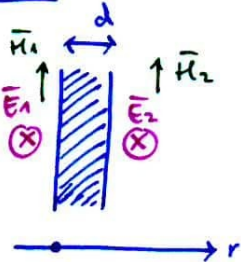
$a = 2,07 \text{ mm}$ $d = 0,6 \text{ mm}$ $\text{tg } \delta = 80 \cdot 10^{-4}$

$b = 4a = 8,28 \text{ mm}$ $\epsilon_r = 3,5$ $v_{cu} = 57 \frac{\text{M}}{\text{m}}$

$f_{\text{min}} = ?$, hogy $\frac{E_2}{E_1} < 10^{-3}$ $Z_0 = 377 \Omega$



hőtelítés:



$$E_1 = E_2 \cdot \text{ch}(\gamma d) + H_2 \cdot Z_0 \cdot \text{sh}(\gamma d)$$

$$\gamma = \sqrt{j\omega\mu_0\sigma}, \quad Z_{0,cu} = \sqrt{\frac{j\omega\mu_0}{\sigma}}$$

$$H_2 = \frac{E_2}{Z_0} \Rightarrow E_2 = \frac{E_1}{\text{ch}(\gamma d) + \frac{Z_{0,cu}}{Z_0} \cdot \text{sh}(\gamma d)} \quad (\Rightarrow)$$

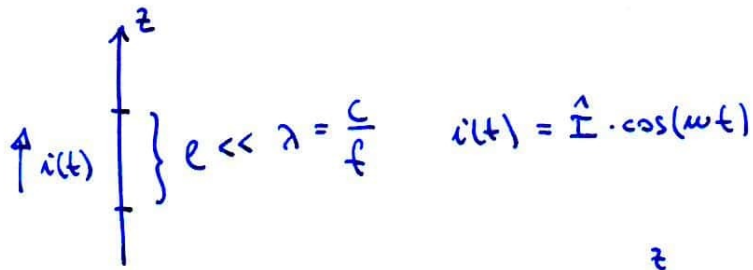
$= 10^{-3}$

iteratív megoldás

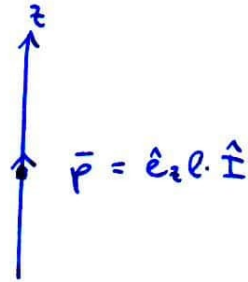
\Rightarrow erre a geometriára: $f = 7,15 \cdot 10^5 \text{ Hz}$

Tereh E.
15.12.02.

EM hullámok

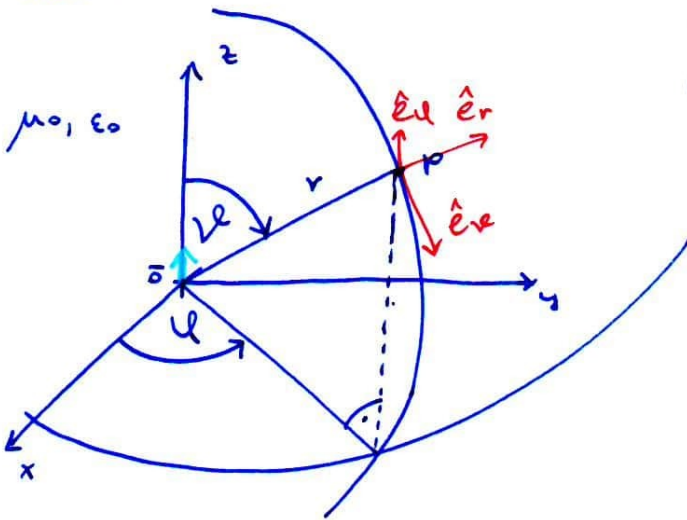


Hertz-dipólussal \Rightarrow



A Hertz-dipólus elektromágneses tere

• gömbkoordináta-rendszer



θ : elevációs szög
 ϕ : azimutális szög

$$E_r(r, \theta, t) = \frac{\hat{I} l}{4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{2}{r^2} \left[\cos(\omega t - \beta r) + \frac{1}{\beta r} \cdot \sin(\omega t - \beta r) \right] \cdot \cos(\theta)$$

$$E_\theta(r, \theta, t) = \frac{\hat{I} l}{4\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{\beta}{r} \left[-\left(1 - \frac{1}{\beta^2 r^2}\right) \sin(\omega t - \beta r) + \frac{1}{\beta r} \cdot \cos(\omega t - \beta r) \right] \cdot \sin(\theta)$$

$$E_\phi \equiv 0$$

ez tűnik el leglassabban

$$H_r = H_\varphi = 0$$

$$H_\varphi(r, \varphi, t) = \frac{\hat{I} \ell}{4\pi} \cdot \frac{\beta}{r} \left[-\sin(\omega t - \beta r) + \frac{1}{\beta r} \cdot \cos(\omega t - \beta r) \right] \cdot \sin(\varphi)$$

ez kicsi el. ~~$\frac{1}{\beta r}$~~ leghalább

Hullámter tulajdonságai:

- forgósszimmetrikus : $\frac{\partial}{\partial \varphi} = 0$

- $\left. \begin{matrix} \cos(\omega t - \beta r) \\ \sin(\omega t - \beta r) \end{matrix} \right\} \Rightarrow$ haladási hullámok "r" irányba (gömbhullámok)

hullámhossz : $\lambda = \frac{2\pi}{\beta}$

- Amplitúdó távolcsé függése:

$$E_r \sim \frac{1}{r^2} \quad \text{és} \sim \frac{1}{r^3}$$

$$E_\varphi \sim \frac{1}{r^2} \quad \text{és} \sim \frac{1}{r^3}$$

$$\left. \begin{matrix} \sim \frac{1}{r^3} : \text{ "sztatikus" ter } (E_r, E_\varphi) \\ \sim \frac{1}{r^2} : \text{ "indukciós" ter } (E_r, H_\varphi) \end{matrix} \right\} \text{ közelter}$$

$$\sim \frac{1}{r} : \text{ "sugarzó" ter } (E_\varphi, H_\varphi) \left. \right\} \text{ távolter}$$

A távolter:

$$\cos(\omega t - \beta r - \frac{\pi}{2})$$

$$E_\varphi(r, \varphi, t) = \cancel{\frac{\hat{I} \ell}{4\pi}} \cdot \frac{\hat{I}}{2} \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{\ell}{\lambda} \cdot \frac{1}{r} \cdot \sin(\varphi) \cdot \sin(\omega t - \beta r)$$

$$H_\varphi(r, \varphi, t) = -\frac{\hat{I}}{2} \cdot \frac{\ell}{\lambda} \cdot \frac{1}{r} \cdot \sin(\varphi) \cdot \sin(\omega t - \beta r)$$

ℓ : antenna hossza

$\frac{\ell}{\lambda}$: dimenzió nélküli

Komplex amplitúdókkal: $i(t) = \hat{I} \cdot \cos(\omega t) \rightarrow \hat{I}$

$$\tilde{E}_\varphi(r, \varphi) = j \frac{\hat{I}}{2} \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{\ell}{\lambda} \cdot \frac{\sin \varphi}{r} \cdot e^{-j\beta r}$$

$$\tilde{H}_\varphi(r, \varphi) = j \frac{\hat{I}}{2} \cdot \frac{\ell}{\lambda} \cdot \frac{\sin \varphi}{r} \cdot e^{-j\beta r}$$

$$|\tilde{E}_\varphi| \sim \frac{\sin \varphi}{r}$$

$$|\tilde{H}_\varphi| \sim \frac{\sin \varphi}{r}$$

I. $\vec{E} \perp \vec{H}$ ($\hat{e}_u \perp \hat{e}_\varphi$)
 II. Áronos fázisban vannak

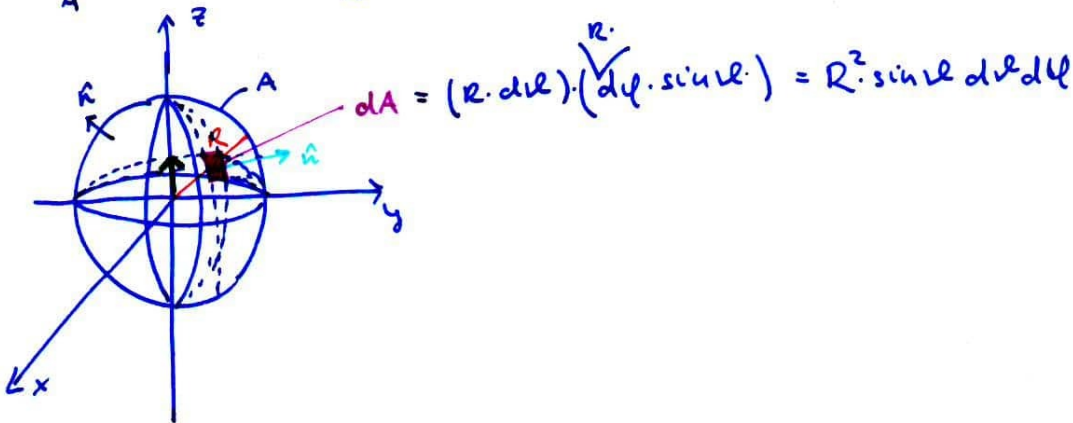
$$\left. \begin{array}{l} \text{I. } \vec{E} \perp \vec{H} \text{ (}\hat{e}_u \perp \hat{e}_\varphi\text{)} \\ \text{II. Áronos fázisban vannak} \end{array} \right\} \vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^*$$

$\vec{S} = \hat{e}_r S_r, S_r \in \mathbb{R} \Rightarrow$ nincs meddő teljesítmény áramlás

\Rightarrow Teljesítményáramlás a távvezetékben.

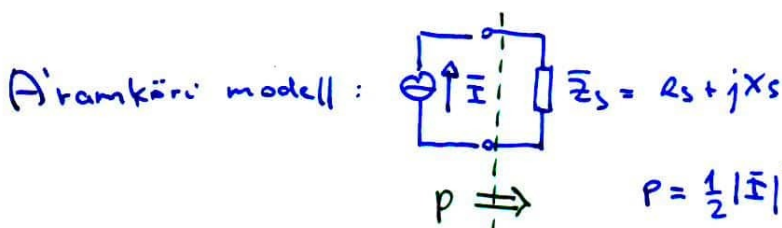
$$\vec{S} = \frac{1}{2} (\hat{e}_u \tilde{E}_u(r, \varphi)) \times (\hat{e}_\varphi \tilde{H}_\varphi(r, \varphi)) = \frac{1}{2} \hat{e}_r \frac{|\tilde{E}|^2}{4} \sqrt{\frac{\mu_0}{\epsilon_0}} \left(\frac{e}{\lambda}\right)^2 \left(\frac{\sin \varphi}{r}\right)^2 \cdot \underbrace{1}_{e^{jkr} \cdot (e^{-jkr})^*} \in \mathbb{R}$$

$$P = \oint_A \operatorname{Re}\{\vec{S}\} \cdot d\vec{A} = \oint_A \hat{e}_r S_r \cdot (d\vec{A} \cdot \hat{n}) \Leftrightarrow$$



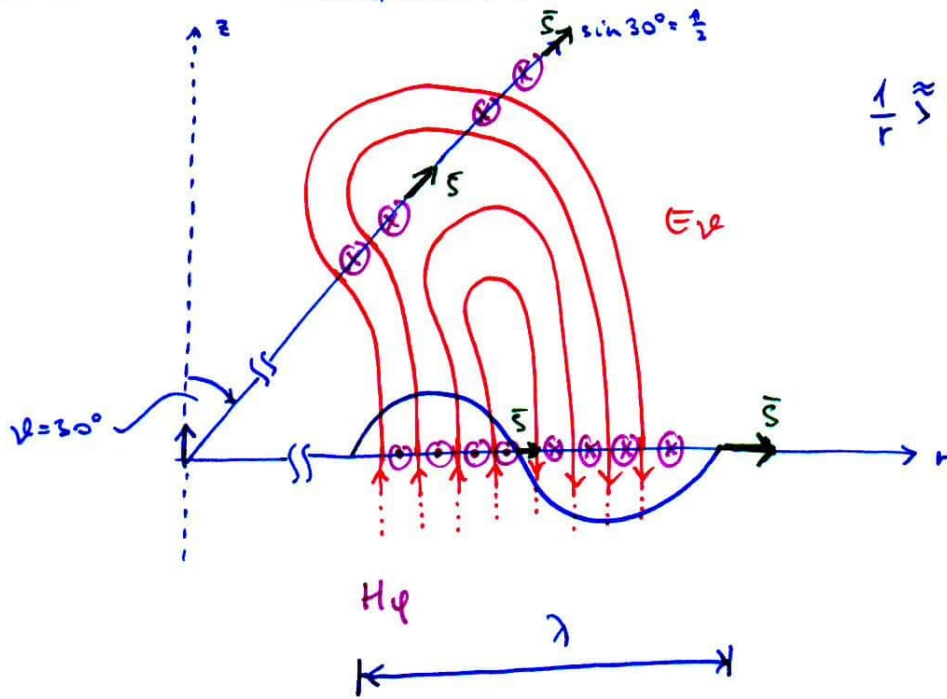
$$\Leftrightarrow \int_{\varphi=0}^{\pi} \int_{\varphi=0}^{2\pi} S_r(r, \varphi) \cdot R^2 \sin \varphi \, dl \, d\varphi = \dots = \frac{1}{2} |\tilde{E}|^2 \left(80\pi^2 \left(\frac{e}{\lambda}\right)^2 R \right)$$

R_S : sugárhosszi ellenállás



$P = \frac{1}{2} |\tilde{I}|^2 \cdot R_S \Rightarrow$ szelvénytároló teljesítmény

Várlatos abra a tévóterőről:



$$\frac{1}{r} \approx \frac{1}{r+\lambda} \quad (r \gg \lambda)$$

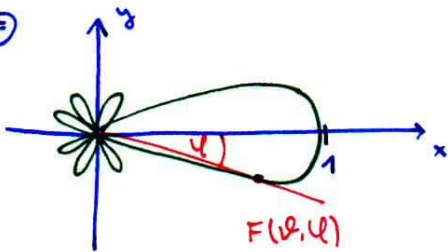
Antenna jellemzők

I. Sugárzási karakterisztika (amplitúdó irányszáma) tévóterben:

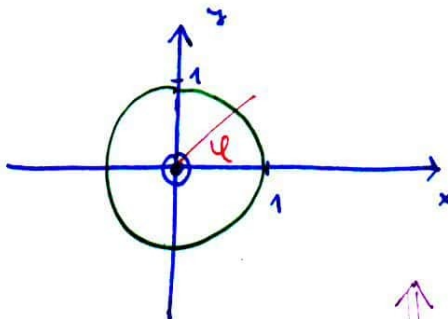
$$F(\psi, \varphi) \triangleq \frac{|E(r=R, \psi, \varphi)|}{|E(r=R, \psi, \varphi)|_{\max}}, \quad 0 \leq F \leq 1$$

R-től független

pl.: ①

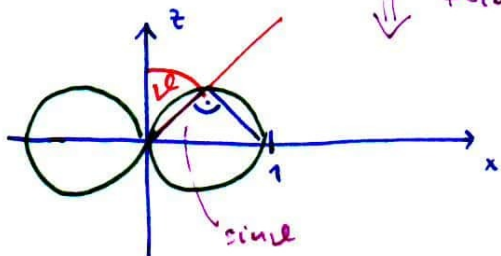


: irányított antenna



: irányítatlan antenna

↕ felső-alsó



: Törés

$$F(\vartheta, \varphi) \equiv F(\vartheta) = \frac{|K \cdot \frac{\sin \vartheta}{r}|_{r=R}}{|K \cdot \frac{\sin \vartheta}{r}|_{\max, r=R}} = \frac{|\sin \vartheta|}{1} = |\sin \vartheta|$$

II. Irányhatás:

$$D \equiv \frac{S_r(\vartheta, \varphi, r=R)|_{\max}}{S_r(\vartheta, \varphi, r=R)|_{\text{átl}} \rightarrow \vartheta, \varphi \text{ szerinti átlag}}$$

$$\equiv \frac{S_r(\vartheta, \varphi, r=R)|_{\max}}{\frac{P_{\text{avg}}}{4R^2\pi} \leftarrow \text{átlag}} : 1 < D < \infty$$

\leftarrow gömbfelületen sugarazik át

• Hertz-dipólusnál:

$$S_r(\vartheta, r) = \frac{1}{8} |I|^2 \sqrt{\frac{\mu_0}{\epsilon_0}} \left(\frac{e}{\lambda}\right)^2 \cdot \frac{\sin^2 \vartheta}{r^2}$$

$$S_r(\vartheta, r=R)|_{\max} \stackrel{\vartheta=90^\circ}{=} \frac{1}{8} |I|^2 \sqrt{\frac{\mu_0}{\epsilon_0}} \left(\frac{e}{\lambda}\right)^2 \cdot \frac{1}{R^2}$$

$$P_{\text{avg}} = \frac{1}{2} |I|^2 \cdot R_s = \frac{1}{2} |I|^2 \left(80\pi^2 \left(\frac{e}{\lambda}\right)^2 \cdot R \right)$$

$$D = 1,5$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = Z_0 = 377 \Omega \approx 120\pi \Omega$$

Antennák

• irányhatás:

$$D = \frac{S_r(r=R, \vartheta, \varphi)_{\max}}{S_r(r=R, \vartheta, \varphi)_{\text{átlag}}} = \frac{S_r(r=R, \vartheta, \varphi)_{\max}}{P_s / 4R^2\pi}$$

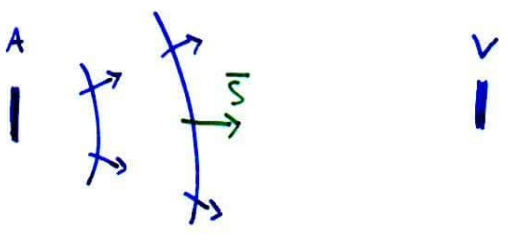
Hertz-dipólus: 1.5

↑
sugárzott teljesítmény

• Nyereség:

$$G = \frac{S_r(r=R, \vartheta, \varphi)}{\frac{P_{be}}{4R^2\pi}}, \quad P_{be} = P_s + P_{\text{veszt}} \Rightarrow G < D$$

Hatásos felület:



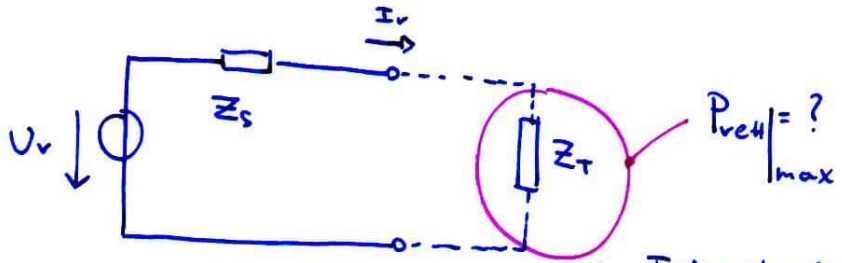
A vevő antenna helyén:

E_0 amplitúdó

$$\vec{S}_0 = \frac{1}{2} \vec{E}_0 \times \vec{H}_0^* \Rightarrow$$

$$\Rightarrow S_0 = \frac{1}{2} \frac{|E_0|^2}{Z_0}$$

A vevőből kivett hatásos teljesítmény:



Teljesítményillesztés:

$$Z_T \triangleq Z_s^*$$

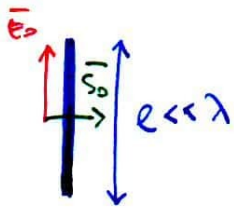
$$I_v = \frac{U_v}{2R_s} \Rightarrow P_{\text{veht}}^{\max} = \frac{1}{2} R_s \cdot |I_v|^2$$

$$\uparrow$$

$$\text{Re}\{Z_s + Z_s^*\}$$

$$\text{Def.: } A_h: P_{\text{veht}}^{\max} = S_0 \cdot A_h \quad \text{Hatásos felület}$$

Rövid dipólus hatósíks felülete:



$$U_v = \ell \cdot E_0$$

$$R_s = 80\pi^2 \left(\frac{\ell}{\lambda}\right)^2 \Omega$$

$$P_{\text{rüll}}^{\text{max}} = \frac{1}{2} R_s \frac{|U_v|^2}{4R_s^2} = \frac{|U_v|^2}{8R_s} = \frac{\ell^2 E_0^2}{8 \cdot 80\pi^2 \left(\frac{\ell}{\lambda}\right)^2}$$

$$S_0 = \frac{1}{2} \frac{|E_0|^2}{Z_0} \quad \underline{\underline{A_h}} = \frac{P_{\text{rüll}}^{\text{max}}}{S_0} = \frac{2Z_0 \lambda^2}{8 \cdot 80\pi^2} = \frac{240\pi \cdot \lambda^2}{80 \cdot 8 \cdot \pi^2} = \underline{\underline{\frac{3\lambda^2}{8\pi}}} \rightarrow$$

← antenna hosszától nem függ

Átviteli egyenlet



1. (A) az adó: P_A , (B) a vevő: P_B -t vesz.
2. (A) a vevő: P_A' , (B) az adó: P_B' -t ad.

$$1) P_B = \underbrace{\left(\frac{P_A}{4\pi d^2}\right)}_{S_{\text{max}} \# \#} D_A \cdot A_B$$

$$2) P_A' = \left(\frac{P_B'}{4\pi d^2}\right) D_B \cdot A_A$$

Reciprocitás: Ha $P_A = P_B' \Rightarrow P_B = P_A'$

$$\Rightarrow \frac{D_A \cdot A_B}{4\pi d^2} = \frac{D_B \cdot A_A}{4\pi d^2} \Rightarrow \boxed{\frac{A_A}{D_A} = \frac{A_B}{D_B} = \text{const.}}$$

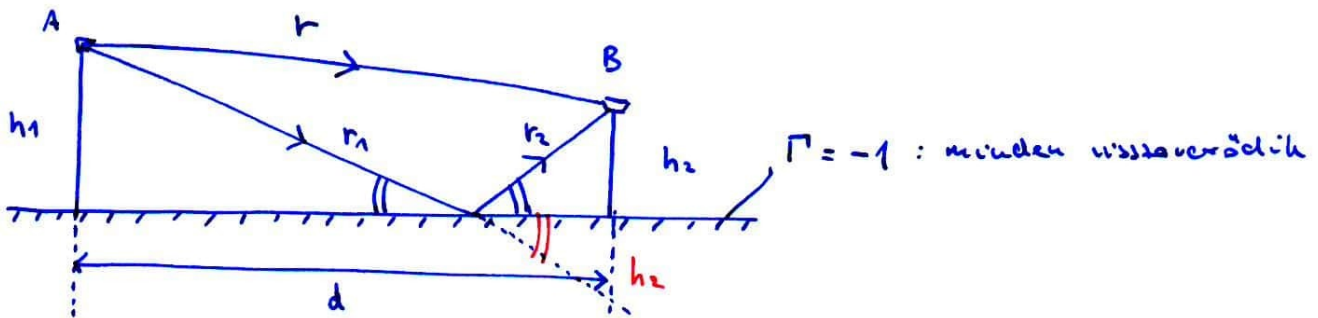
Rövid dipólus antenna:

$$A_h = \frac{3\lambda^2}{8\pi}, \quad D = 1.5 \Rightarrow \boxed{\frac{A_h}{D} = \frac{\lambda^2}{4\pi}}$$

← $P_B = P_A \cdot \frac{D_A D_B \lambda^2}{(4\pi d)^2}$ A'utitelés egyenlet

Kétutas földfelszíni terjedés

$d \gg h_1, h_2$



Útösszék kiértékelés: $\Delta = (r_1 + r_2) - r = \sqrt{d^2 + (h_1 + h_2)^2} - \sqrt{d^2 + (h_1 - h_2)^2}$

$d^2 + h_1^2 + 2h_1h_2 + h_2^2$ $(d^2 + h_1^2 + h_2^2) - 2h_1h_2$

$d \gg h_1, h_2$

$\sqrt{1+x} \approx 1 + \frac{x}{2}$, ha $|x| \ll 1$

$\Rightarrow \Delta = \frac{2h_1h_2}{d}$

fázis változás: $e^{-i\beta z}$

fázis kiértékelés: $\Delta \cdot \beta = \Delta \frac{2\pi}{\lambda}$

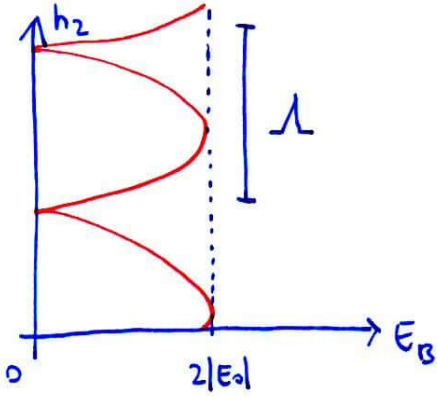
$E_B = E_0 + \Gamma \cdot E_0 \cdot e^{-i \frac{2\pi \Delta}{\lambda}} \rightarrow |E_B| = |E_0| \left| 1 - e^{-i \frac{2\pi \Delta}{\lambda}} \right| \quad \ominus \rightarrow$

az elsőnél nagyobb az amplitúdója, de $r_1 + r_2 \approx r \Rightarrow$
 \Rightarrow elhanyagolható

$$\ll \Leftrightarrow |E_0| \cdot |e^{-i\alpha \frac{\pi}{\lambda}}| \cdot |e^{i\alpha \frac{\pi}{\lambda}} - e^{-i\alpha \frac{\pi}{\lambda}}| =$$

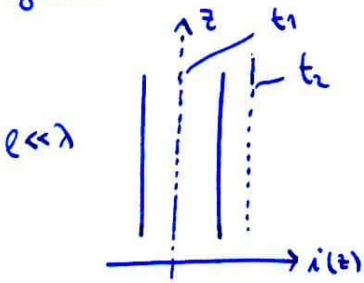
$$= |E_0| \cdot |2 \cdot \sin(\alpha \frac{\pi}{\lambda})| =$$

$$= |E_0| \cdot |2 \cdot \sin(\frac{h_1 h_2 \beta}{d})|$$

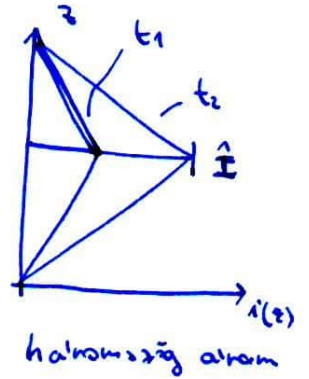
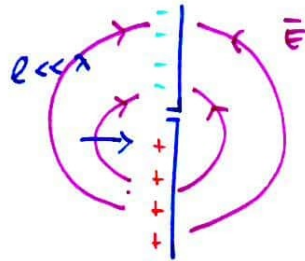
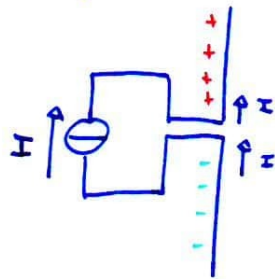


Valószínűs rövid dipólantenna

gond:



valóság:



$$i(z) = \hat{I} \cdot \cos(\omega t)$$

Tárolótekr \approx Tárolótekr
 (Δ áramú) (egyenletes áram elosztású)
 $\hat{I} = \frac{1}{2} \hat{I}_\Delta$

$$P_s = \frac{1}{2} R_s |\hat{I}|^2 = \frac{1}{2} 80 \pi^2 \left(\frac{l}{\lambda}\right) \cdot \left(\frac{\hat{I}_\Delta}{2}\right)^2 =$$

$$= \frac{1}{2} \underbrace{\left(20 \pi^2 \left(\frac{l}{\lambda}\right)^2\right)}_{R_{s\Delta}} \cdot |\hat{I}_\Delta|^2$$