

# "Association Rules"

Market Baskets

Frequent Itemsets

A-priori Algorithm

# The Market-Basket Model

- ◆ A large set of *items*, e.g., things sold in a supermarket.
- ◆ A large set of *baskets*, each of which is a small set of the items, e.g., the things one customer buys on one day.

# Support

- ◆ Simplest question: find sets of items that appear “frequently” in the baskets.
- ◆ *Support* for itemset  $I$  = the number of baskets containing all items in  $I$ .
- ◆ Given a support threshold  $s$ , sets of items that appear in  $\geq s$  baskets are called *frequent itemsets*.

# Example

◆ Items = {milk, coke, pepsi, beer, juice}.

◆ Support = 3 baskets.

B1 = {m, c, b}

B2 = {m, p, j}

B3 = {m, b}

B4 = {c, j}

B5 = {m, p, b}

B6 = {m, c, b, j}

B7 = {c, b, j}

B8 = {b, c}

◆ Frequent itemsets: {m}, {c}, {b}, {j},  
{m, b}, {c, b}, {j, c}.

# Applications --- (1)

- ◆ Real market baskets: chain stores keep terabytes of information about what customers buy together.
  - ◆ Tells how typical customers navigate stores, lets them position tempting items.
  - ◆ Suggests tie-in “tricks,” e.g., run sale on diapers and raise the price of beer.
- ◆ High support needed, or no \$\$'s .

# Applications --- (2)

- ◆ “Baskets” = documents; “items” = words in those documents.
  - ◆ Lets us find words that appear together unusually frequently, i.e., linked concepts.
- ◆ “Baskets” = sentences, “items” = documents containing those sentences.
  - ◆ Items that appear together too often could represent plagiarism.

# Applications --- (3)

- ◆ “Baskets” = Web pages; “items” = linked pages.
  - ◆ Pairs of pages with many common references may be about the same topic.
- ◆ “Baskets” = Web pages  $p$  ; “items” = pages that link to  $p$  .
  - ◆ Pages with many of the same links may be mirrors or about the same topic.

# Important Point

- ◆ “Market Baskets” is an abstraction that models any many-many relationship between two concepts: “items” and “baskets.”
  - ◆ Items need not be “contained” in baskets.
- ◆ The only difference is that we count co-occurrences of items related to a basket, not vice-versa.



# Scale of Problem

- ◆ WalMart sells 100,000 items and can store billions of baskets.
- ◆ The Web has over 100,000,000 words and billions of pages.

# Association Rules

- ◆ If-then rules about the contents of baskets.
- ◆  $\{i_1, i_2, \dots, i_k\} \rightarrow j$  means: "if a basket contains all of  $i_1, \dots, i_k$  then it is likely to contain  $j$ ."
- ◆ *Confidence* of this association rule is the probability of  $j$  given  $i_1, \dots, i_k$ .

# Example

+ B1 = {m, c, b}

B2 = {m, p, j}

– B3 = {m, b}

B4 = {c, j}

– B5 = {m, p, b}

+ B6 = {m, c, b, j}

B7 = {c, b, j}

B8 = {b, c}

◆ An association rule: {m, b} → c.

◆ Confidence =  $2/4 = 50\%$ .

# Interest

- ◆ The *interest* of an association rule is the absolute value of the amount by which the confidence differs from what you would expect, were items selected independently of one another.

# Example

$$B1 = \{m, c, b\}$$

$$B2 = \{m, p, j\}$$

$$B3 = \{m, b\}$$

$$B4 = \{c, j\}$$

$$B5 = \{m, p, b\}$$

$$B6 = \{m, c, b, j\}$$

$$B7 = \{c, b, j\}$$

$$B8 = \{b, c\}$$

- ◆ For association rule  $\{m, b\} \rightarrow c$ , item  $c$  appears in 5/8 of the baskets.
- ◆ Interest =  $| 2/4 - 5/8 | = 1/8$  --- not very interesting.

# Relationships Among Measures

- ◆ Rules with high support and confidence may be useful even if they are not “interesting.”
  - ◆ We don’t care if buying bread causes people to buy milk, or whether simply a lot of people buy both bread and milk.
- ◆ But high interest suggests a cause that might be worth investigating.

# Finding Association Rules

- ◆ A typical question: “find all association rules with support  $\geq s$  and confidence  $\geq c$ .”
  - ◆ Note: “support” of an association rule is the support of the set of items it mentions.
- ◆ Hard part: finding the high-support (*frequent*) itemsets.
  - ◆ Checking the confidence of association rules involving those sets is relatively easy.

# Computation Model

- ◆ Typically, data is kept in a “flat file” rather than a database system.
  - ◆ Stored on disk.
  - ◆ Stored basket-by-basket.
  - ◆ Expand baskets into pairs, triples, etc. as you read baskets.



## Computation Model --- (2)

- ◆ The true cost of mining disk-resident data is usually the **number of disk I/O's**.
- ◆ In practice, association-rule algorithms read the data in passes --- all baskets read in turn.
- ◆ Thus, we measure the cost by the number of passes an algorithm takes.

# Main-Memory Bottleneck

- ◆ In many algorithms to find frequent itemsets we need to worry about how main memory is used.
  - ◆ As we read baskets, we need to count something, e.g., occurrences of pairs.
  - ◆ The number of different things we can count is limited by main memory.
  - ◆ Swapping counts in/out is a disaster.

# Finding Frequent Pairs

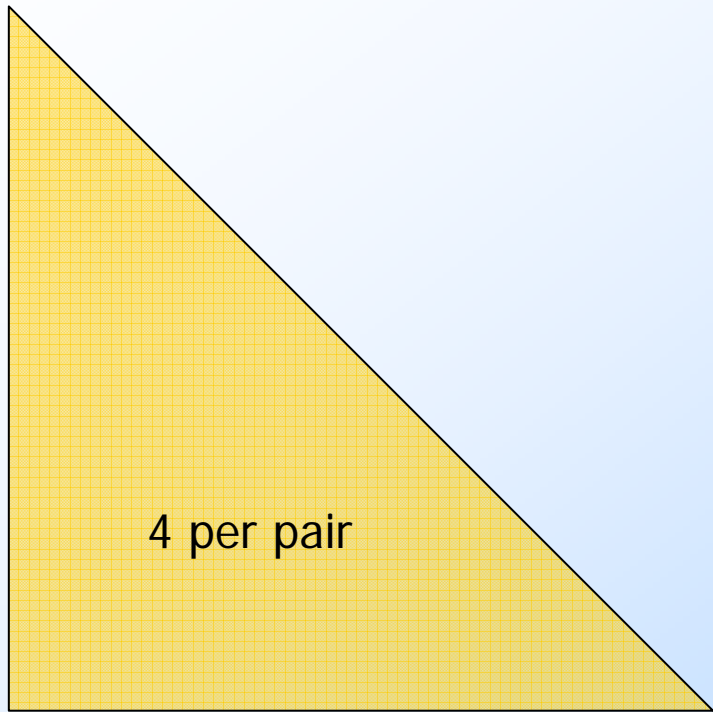
- ◆ The hardest problem often turns out to be finding the frequent pairs.
- ◆ We'll concentrate on how to do that, then discuss extensions to finding frequent triples, etc.

# Naïve Algorithm

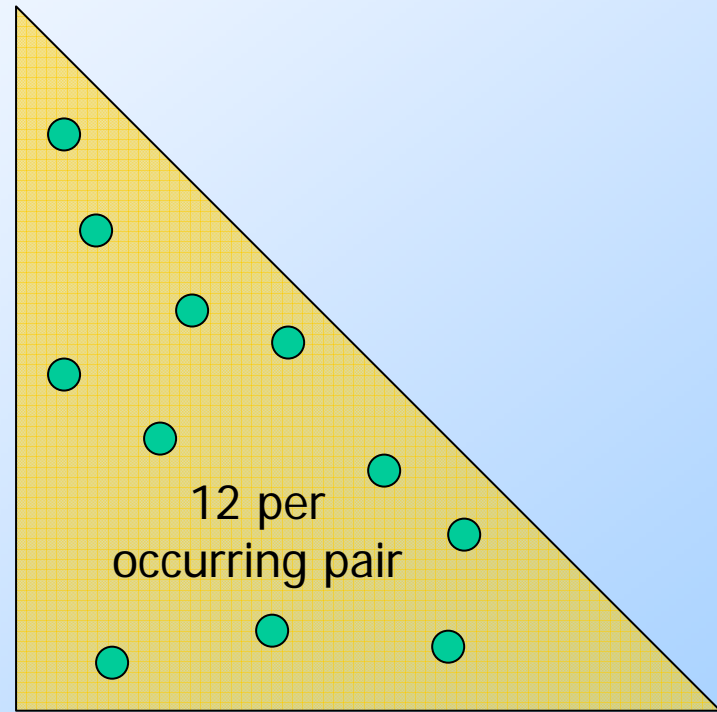
- ◆ A simple way to find frequent pairs is:
  - ◆ Read file once, counting in main memory the occurrences of each pair.
    - Expand each basket of  $n$  items into its  $n(n-1)/2$  pairs.
- ◆ Fails if #items-squared exceeds main memory.

# Details of Main-Memory Counting

- ◆ There are two basic approaches:
  1. Count all item pairs, using a triangular matrix.
  2. Keep a table of triples  $[i, j, c]$  = the count of the pair of items  $\{i, j\}$  is  $c$ .
- ◆ (1) requires only (say) 4 bytes/pair;  
(2) requires 12 bytes, but only for those pairs with  $>0$  counts.



Method (1)



Method (2)

# Details of Approach (1)

- ◆ Number items  $1, 2, \dots$
- ◆ Keep pairs in the order  $\{1, 2\}, \{1, 3\}, \dots, \{1, n\}, \{2, 3\}, \{2, 4\}, \dots, \{2, n\}, \{3, 4\}, \dots, \{3, n\}, \dots, \{n-1, n\}$ .
- ◆ Find pair  $\{i, j\}$  at the position  $(i-1)(n-i/2) + j - i$ .
- ◆ Total number of pairs  $n(n-1)/2$ ; total bytes about  $2n^2$ .

## Details of Approach (2)

- ◆ You need a hash table, with  $i$  and  $j$  as the key, to locate  $(i, j, c)$  triples efficiently.
  - ◆ Typically, the cost of the hash structure can be neglected.
- ◆ Total bytes used is about  $12p$ , where  $p$  is the number of pairs that actually occur.
  - ◆ Beats triangular matrix if at most  $1/3$  of possible pairs actually occur.



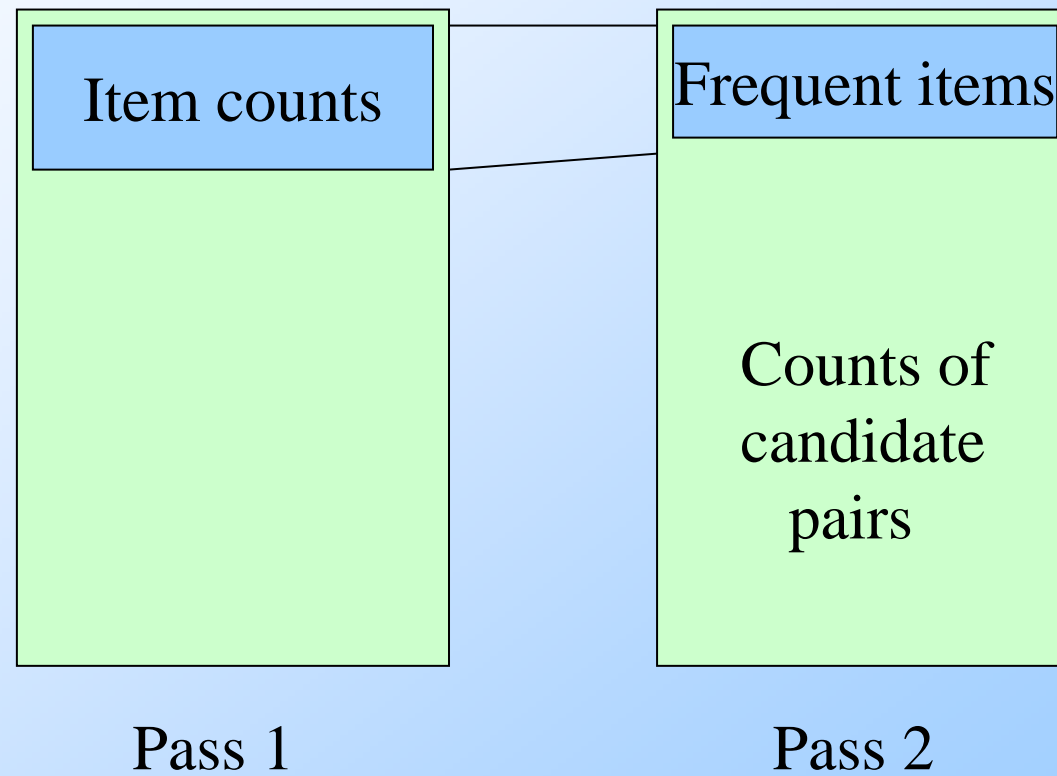
# A-Priori Algorithm --- (1)

- ◆ A two-pass approach called *a-priori* limits the need for main memory.
- ◆ Key idea: *monotonicity*: if a set of items appears at least  $s$  times, so does every subset.
  - ◆ **Contrapositive for pairs**: if item  $i$  does not appear in  $s$  baskets, then no pair including  $i$  can appear in  $s$  baskets.

## A-Priori Algorithm --- (2)

- ◆ **Pass 1:** Read baskets and count in main memory the occurrences of each item.
  - ◆ Requires only memory proportional to #items.
- ◆ **Pass 2:** Read baskets again and count in main memory only those pairs both of which were found in Pass 1 to be frequent.
  - ◆ Requires memory proportional to square of frequent items only.

# Picture of A-Priori

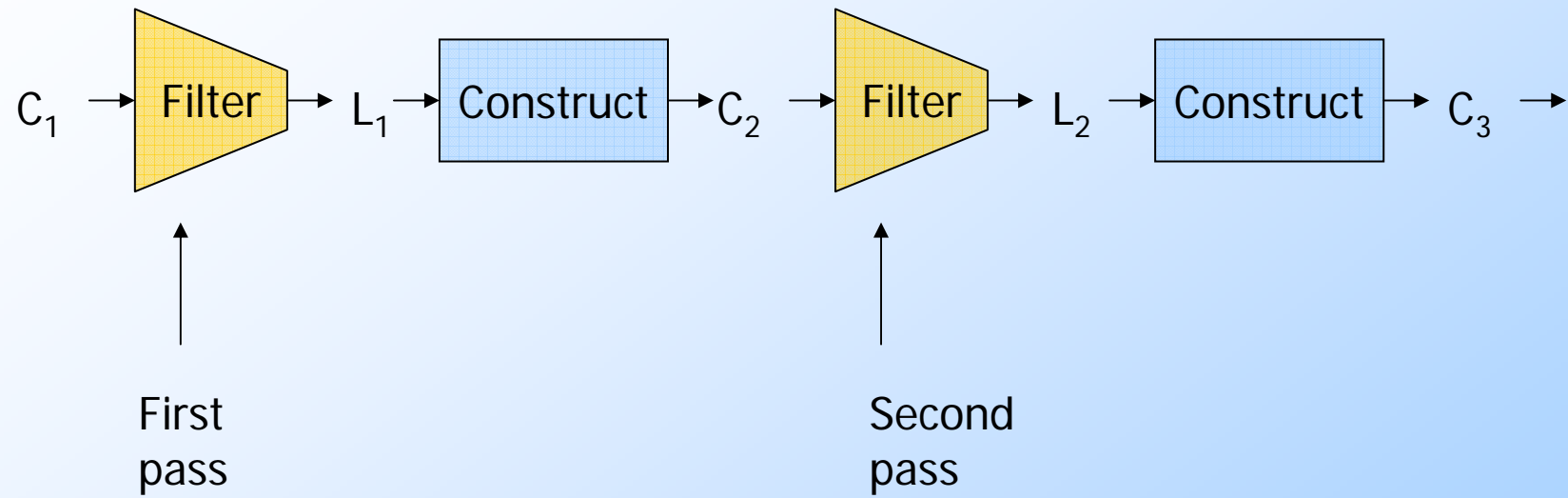


# Detail for A-Priori

- ◆ You can use the triangular matrix method with  $n$  = number of frequent items.
  - ◆ Saves space compared with storing triples.
- ◆ **Trick:** number frequent items 1,2,... and keep a table relating new numbers to original item numbers.

# Frequent Triples, Etc.

- ◆ For each  $k$ , we construct two sets of  $k$ -tuples:
  - ◆  $C_k$  = **candidate**  $k$ -tuples = those that might be frequent sets (support  $\geq s$ ) based on information from the pass for  $k-1$ .
  - ◆  $L_k$  = the set of truly frequent  $k$ -tuples.



# A-Priori for All Frequent Itemsets

- ◆ One pass for each  $k$ .
- ◆ Needs room in main memory to count each candidate  $k$ -tuple.
- ◆ For typical market-basket data and reasonable support (e.g., 1%),  $k = 2$  requires the most memory.

## Frequent Itemsets --- (2)

- ◆  $C_1$  = all items
- ◆  $L_1$  = those counted on first pass to be frequent.
- ◆  $C_2$  = pairs, both chosen from  $L_1$ .
- ◆ In general,  $C_k$  =  $k$ -tuples each  $k-1$  of which is in  $L_{k-1}$ .
- ◆  $L_k$  = those candidates with support  $\geq s$ .