

1. Feladat

$$X + Y \in N(0, \sqrt{2}), Z = (X + Y)^2$$

$$Fz(t) = P(Z < t) = P((X + Y)^2 < t) = P(-\sqrt{t} < X + Y < \sqrt{t}) = 2\Phi\left(\frac{\sqrt{t}}{\sqrt{2}}\right) - 1; \quad t > 0$$

$$fz(t) = F'z(t) = 2 \cdot \varphi\left(\frac{\sqrt{t}}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{\pi \cdot t}} \cdot e^{-\frac{t}{4}}; \quad t > 0$$

2. Feladat

$$P(\overline{ABC} + \overline{A}BC + \overline{AB}\overline{C} + \overline{A}B\overline{C} + A\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C}) = 1 - P(ABC) = 1 - \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{5} = \frac{29}{30}$$

3. Feladat

$$p = \iint_{x < y} (x + y) dx dy = \int_0^1 \int_0^y (x + y) dx dy = \int_0^1 \left[ \frac{x^2}{2} + yx \right]_0^y dy = \int_0^1 \frac{3}{2} y^2 dy = \left[ \frac{y^3}{2} \right]_0^1 = \frac{1}{2}$$

4. Feladat

$$P(X = 0) = \frac{1}{3} = e^{-\lambda} \rightarrow \lambda = \ln 3$$

$$\begin{aligned} P(X > 3) &= 1 - P(X = 0) - P(X = 1) - P(X = 2) = 1 - \frac{1}{3} - \frac{1}{3} \cdot \ln 3 - \frac{1}{3} \cdot \frac{(\ln 3)^2}{2} \\ &= \frac{2}{3} - \frac{1}{3} \cdot \left( \ln 3 + \frac{(\ln 3)^2}{2} \right) = \frac{2}{3} - \frac{1}{3} \cdot \left( 1,099 + \frac{1,207}{2} \right) = \frac{2}{3} - \frac{1}{3} \cdot \ln 3 \cdot \left( 1 + \frac{\ln 3}{2} \right) \end{aligned}$$

5. Feladat

$$X, Y \in B\left(3, \frac{1}{2}\right)$$

$$\sigma^2 X = \sigma X \sigma Y = \frac{3}{4}$$

$$EX = EY = \frac{3}{2}$$

$$X = 3 - Y \rightarrow R(X, Y) = -1 = \frac{E(XY) - EXEY}{\sigma X \sigma Y}$$

$$-1 = \frac{E(XY) - \frac{9}{4}}{\frac{3}{4}} \rightarrow E(XY) = \frac{9}{4} - \frac{3}{4} = \frac{3}{2}$$

$$EZ = E(X(3 - X)) = 3EX - E(X^2)$$