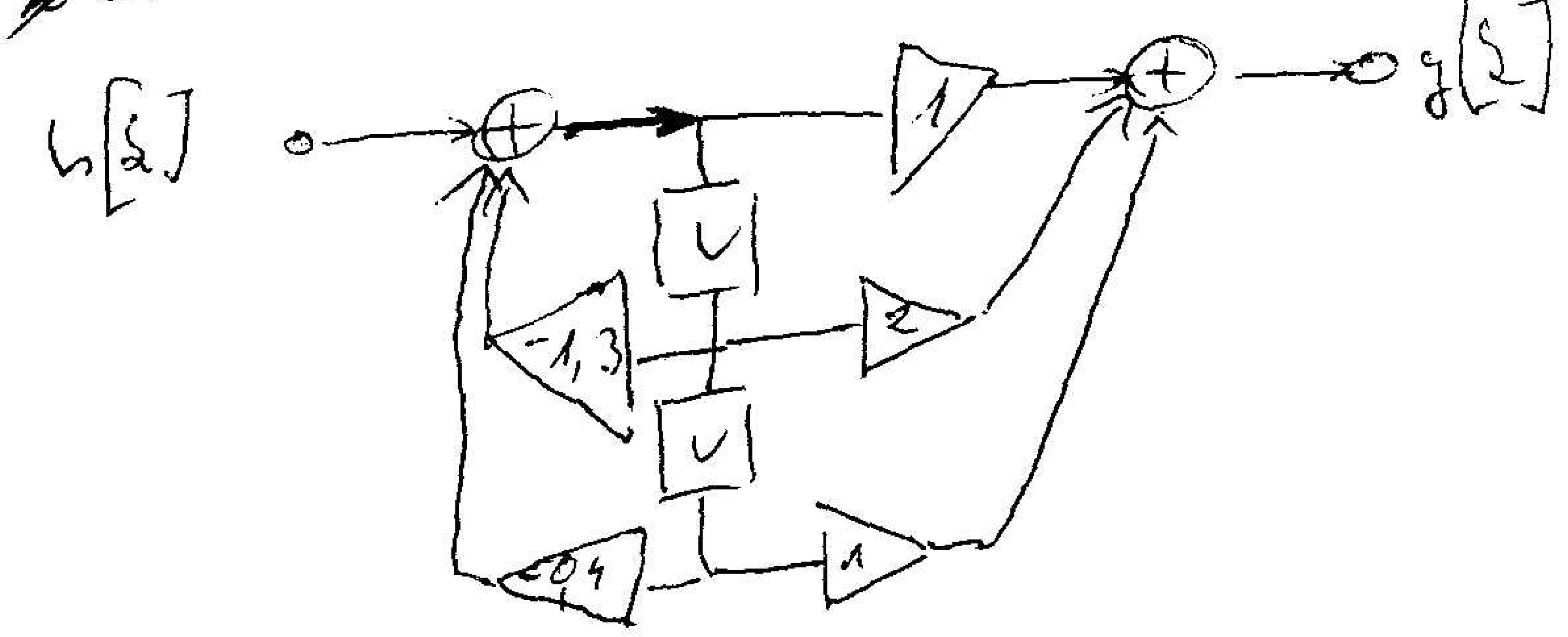
Jelek Rendszerek 2012. 11. 30. ZH 2012.

2 NP + 10 KP

A csoport

1 NP

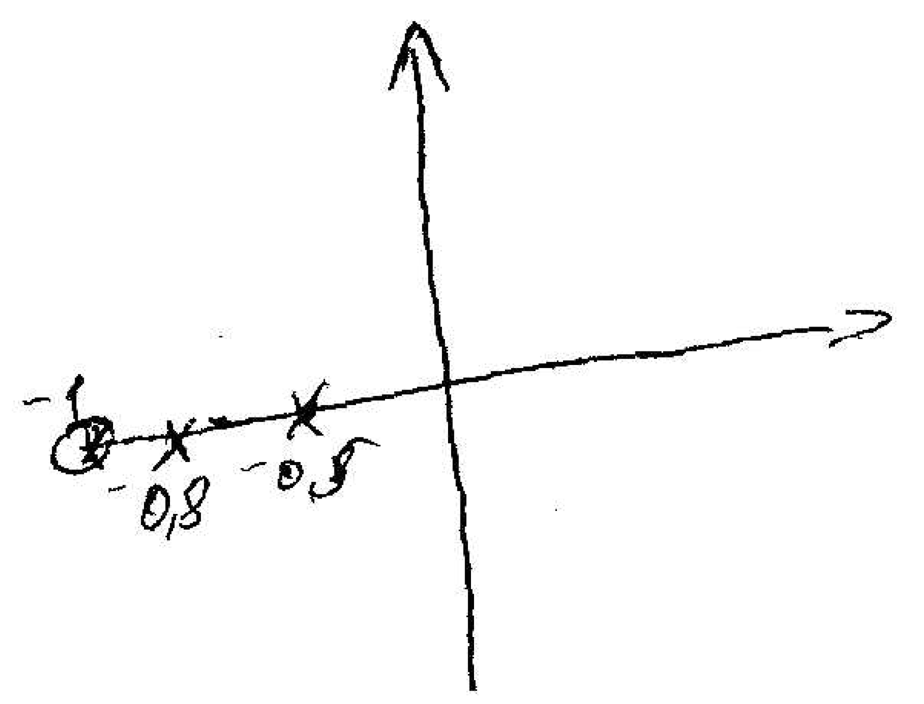
kanonikus alak



5 p) a) $H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 + 1{,}3z^{-1} + 0{,}4z^{-2}} =$

$= \frac{z^2 + 2z + 1}{z^2 + 1{,}3z + 0{,}4}$

Ábrázolva



5 p) b) P→Z kép.

p. nev. null. h.: $z^2 + 1{,}3z + 0{,}4 = 0$

$\frac{-1{,}3 \pm 0{,}3}{2}$ → $-0{,}5$, $-0{,}8$

Z: nevező null helyei

$z^2 + 2z + 1 = (z+1)^2 = 0$

$z_1 = z_2 = -1$

c) mivel minden pólus abszolút értéke < 1 =>

=> Aszimptotikusan stabil => GV stabil.

↓

mert a fokszám megegyezik

d) $u[k] = 117 \cdot \varepsilon[k]$ ← $Z\{a^k\} = \frac{z}{z-a}$

↓ Z transzformált

$U(z) = 117 \cdot \frac{z}{z-1}$

$H(z) = \frac{z^2 + 2z + 1}{(z+0{,}5)(z+0{,}8)} \Longrightarrow Y(z) = H(z) \cdot U(z) = \frac{z^2 + 2z + 1}{(z+0{,}5)(z+0{,}8)} \cdot 117 \cdot \frac{z}{z-1} =$

$= 117 \cdot \frac{z^3 + 2z^2 + z}{(z-1)(z+0{,}5)(z+0{,}8)}$

probléma: áltört → a fokszám megegyezik → csináljunk belőle valódi törtet sz<N

$Y(z) = \frac{z^3 + 2z^2 + z}{z^3 + 0{,}3z^2 - 0{,}9z - 0{,}4} =$

$= \frac{z^3 + 0{,}3z^2 - 0{,}9z - 0{,}4 + 1{,}7z^2 + 1{,}9z + 0{,}4}{(\quad)} =$

$$= 1 + \frac{1{,}7z^2 + 1{,}9z + 0{,}4}{(z-1)(z+0{,}5)(z+0{,}8)} = 1 + \frac{A}{z-1} + \frac{B}{z+0{,}5} + \frac{C}{z+0{,}8}$$

$$A = \left.\frac{1{,}7z^2 + 1{,}9z + 0{,}4}{(z+0{,}5)(z+0{,}8)}\right|_{\substack{z-1=0\\ z=1}} = \frac{1{,}7+1{,}9+0{,}4}{1{,}5 \cdot 1{,}8} = \frac{4}{2{,}7} = \frac{40}{27}$$

$$B = \left.\frac{1{,}7z^2 + 1{,}9z + 0{,}4}{(z-1)(z+0{,}8)}\right|_{\substack{z+0{,}5=0\\ z=-0{,}5}} = \dots$$

$$C = \dots$$

$$H(z) = 1 + \left(\frac{Az}{z-1} + \frac{Bz}{z+0{,}5} + \frac{Cz}{z+0{,}8}\right) \cdot z^{-1}$$

probléma: nem z transzformált

mo.: bővítünk z-vel

kisleltetve van! Ha ezt csináljuk akkor mindenhol csináljuk.

$$y[k] = \delta[k] + \varepsilon[k-1] \quad A \cdot \underbrace{1^{(k-1)}}_{1} + B \cdot (0{,}5)^{(k-1)} + C \cdot (-0{,}8)^{(k-1)}$$

$$Z\{a^k\} = \frac{z}{z-a}$$

2) $h(t) = \delta(t) + \varepsilon(t) \cdot 4e^{-t}$

a) $H(j\omega) = ?$

feviteli karakterisztika $(j\omega)$ a kérdés és nem a kapcsolat (s)

$$\mathcal{L}\{h(t)\} = H(s) = 1 + 4 \cdot \frac{1}{s+1} = \frac{s+1+4}{s+1} = \boxed{\frac{s+5}{s+1}}$$

Ez még nem válasz

$$\mathcal{L}\{e^{-\lambda t}\} = \frac{1}{s+\lambda}$$

$$H(j\omega) = \frac{j\omega+5}{j\omega+1}$$

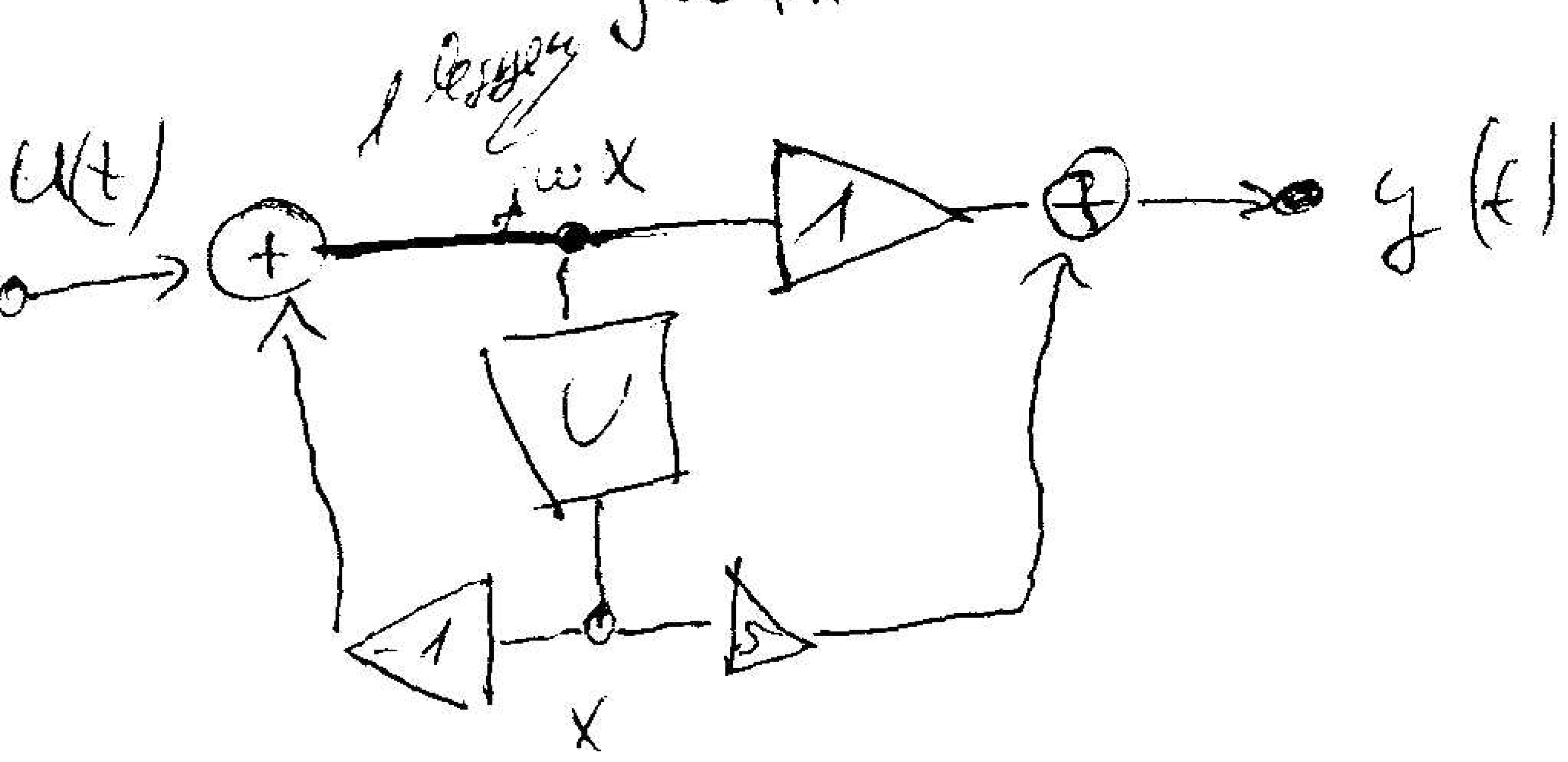
ez már igen

Fehér Sanzi 2012.11.30

2 NP

b) kanonikus alak beli realizáció

$$H(j\omega) = \frac{j\omega + 5}{j\omega + 1} \leftarrow \frac{\text{polinom}}{\text{polinom}} \text{ alak}$$



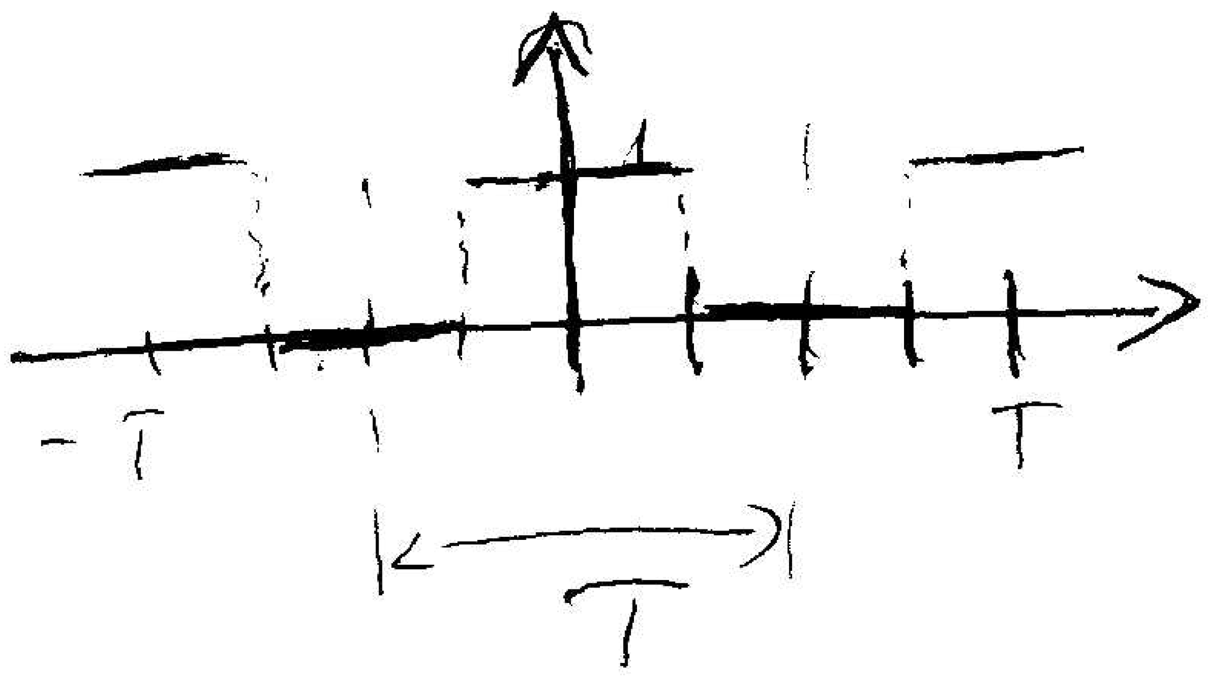
c)

$$u(t) = \begin{cases} 1 & 0 < t < \frac{T}{4} \\ 0 & \frac{T}{4} < t < \frac{3T}{4} \\ 1 & \frac{3T}{4} < t < T \end{cases}$$

$\omega_0 = \frac{2\pi}{T}$, $T = 1$

Négyszögjel lesz!

$\omega_0 = \frac{2\pi}{T} = 2\pi$ rad/s



$$U_p^C = ? = \frac{1}{T} \int_{\langle T \rangle} u(t) e^{-jp\omega_0 t} dt$$

p=0

$$U_0 = \frac{1}{T} \int_{-\frac{T}{4}}^{T/4} 1 \, dt = \frac{1}{T} \cdot [t]_{-\frac{T}{4}}^{\frac{T}{4}} = \frac{1}{T}\left(\frac{T}{4} - \left(\frac{-T}{4}\right)\right) = \frac{1}{2} \rightarrow \text{ez a DC átlag}$$

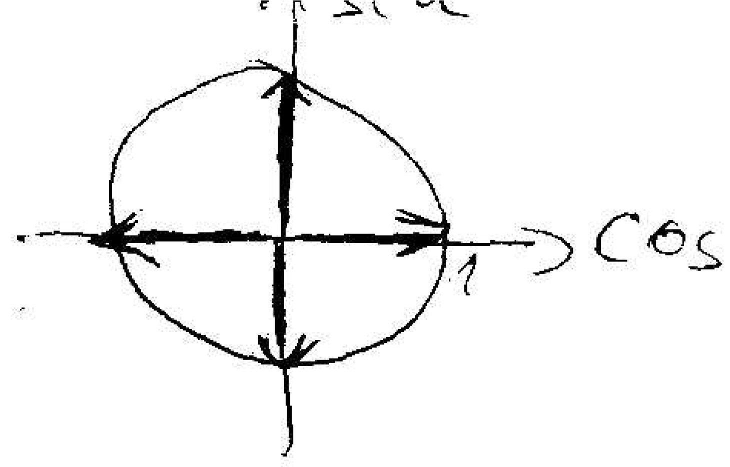
$p > 0$
$p \in \mathbb{Z}$

$$U_p^C \frac{1}{T} \int_{-T/4}^{T/4} 1 \cdot e^{-jp\omega_0 t} dt = \frac{1}{T} \left[\frac{e^{-jp\omega_0 t}}{-jp\omega_0}\right]_{-\frac{T}{4}}^{\frac{T}{4}} = \frac{1}{T} \cdot \frac{1}{-jp\omega_0} \cdot \left(e^{-jp\omega\frac{T}{4}} - e^{jp\omega_0 T/4}\right) =$$

$$= \frac{1}{T} \frac{2 \cdot 1}{p\omega_0} \cdot \left(\frac{e^{jp\omega_0 T/4} - e^{-jp\omega_0 T/4}}{2j}\right) = \frac{2}{Tp\omega_0} \cdot \sin\left(p\omega_0 \frac{T}{4}\right) \Bigg|_{\substack{\omega_0 = 2\pi \\ T=1}} = \frac{2}{1 \cdot p \cdot 2\pi} \cdot \sin\left(p \frac{2\pi \cdot 1}{4}\right)$$

$$= \frac{1}{p\pi} \cdot \sin\left(p\frac{\pi}{2}\right) \Rightarrow$$

$$\Rightarrow U_1^C = \frac{1}{\pi} \cdot \sin\left(\frac{\pi}{2}\right) = 1$$



$$U_2^C = \frac{1}{2\pi} \cdot \sin 2\frac{\pi}{2} = 0$$

$$U_3^C = \frac{1}{3\pi} \cdot \sin\left(\frac{3\pi}{2}\right) = \frac{-1}{3\pi}$$

<u>Fourier közelítés</u>

$$u(t) = u_0 + \sum_{p=1}^{\infty} 2 \cdot U_p^C \cdot \cos(p\omega_0 t + \rho_p)$$

$$\rho_p = \mathrm{arc}(U_p^C)$$

$$u(t) = \frac{1}{2} + \sum_{p=1}^{\infty} \frac{2}{p\pi} \cdot \sin\left(p\frac{\pi}{2}\right) \cdot \cos(p \cdot 2\pi t) \; \cancel{\cos(p \cdot 2\pi \cdot t +} =$$

$$= \frac{1}{2} + \underbrace{\frac{2}{\pi}\cos 2\pi t}_{1} + \underbrace{\frac{-2}{3\pi} \cdot \cos(6\pi t)}_{3} + \underbrace{\frac{2}{5\pi}\cos(10\pi t)}_{5} + \underbrace{\dots}_{7}$$

$$p = 0 \Rightarrow \omega = 0 \Rightarrow H(j\omega) = \left.\frac{j\omega + 5}{j\omega + 1}\right|_{\omega = 0} = \frac{5}{1} = 5 \rightarrow$$ 5szörös erősítés

$$p = 1 \Rightarrow \omega = 2\pi \Rightarrow H(j2\pi) = \frac{j2\pi + 5}{j2\pi + 1} = \frac{\sqrt{(2\pi)^2 + 5^2}}{\sqrt{(2\pi)^2 + 1}} \cdot e^{\left(j\,\mathrm{arctg}\frac{2\pi}{5} - \mathrm{arctg}\frac{2\pi}{1}\right)} = A \cdot e^{j\alpha}$$

nem kell tovább számolni, de lehet

$$p = 3 \Rightarrow \omega = 6\pi \Rightarrow H(j6\pi) = \frac{j6\pi + 5}{j6\pi + 1} = B \cdot e^{j\beta}$$

válaszjel:

$$y(t) = \frac{1}{2} \cdot 5 + \underbrace{\frac{2}{\pi} A \cdot \cos 2\pi t\alpha}_{1.\ \text{tag}} + \underbrace{\frac{-2}{3\pi} \cdot B \cdot \cos(6\pi t + \beta)}_{3.\ \text{tag}} + \dots$$

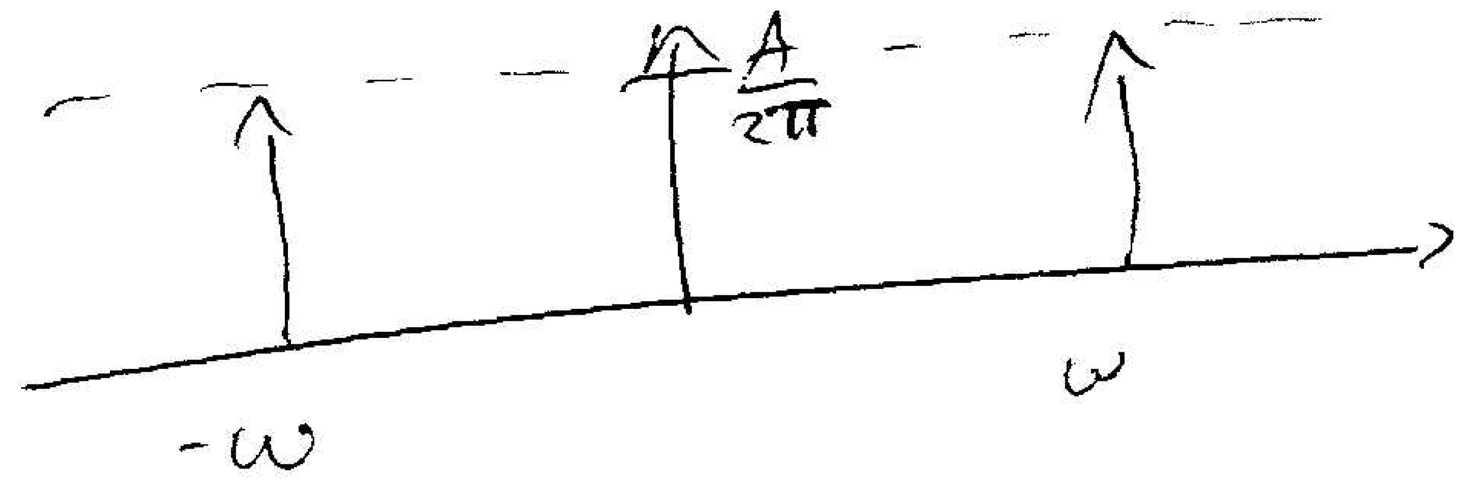
felelő konzi 2012.11.30

1kP)

$f(t) = \frac{2}{2\pi} \cdot A \cdot \cos(\omega t)$ jel spektruma és ábrázold!

$\downarrow$

$f(t) = \frac{\cancel{2}}{2\pi} \cdot A \, \frac{e^{j\omega t} + e^{-j\omega t}}{\cancel{2}}$



2kP) $F(j\omega)$ ~~spe~~; hat. meg a jel energiáját
spektrum

Parseval-tétel

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \underbrace{\frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega}$$ ez a válasz

miatt

3kP) Milyen sávkorlátú jelet lehet 196 kHz frekvenciájú jellel mintavételezni?

Shannon tétel: $B_{max} \leq \frac{f_s}{2}$

$B_{max} \leq \frac{196\,kHz}{2}$

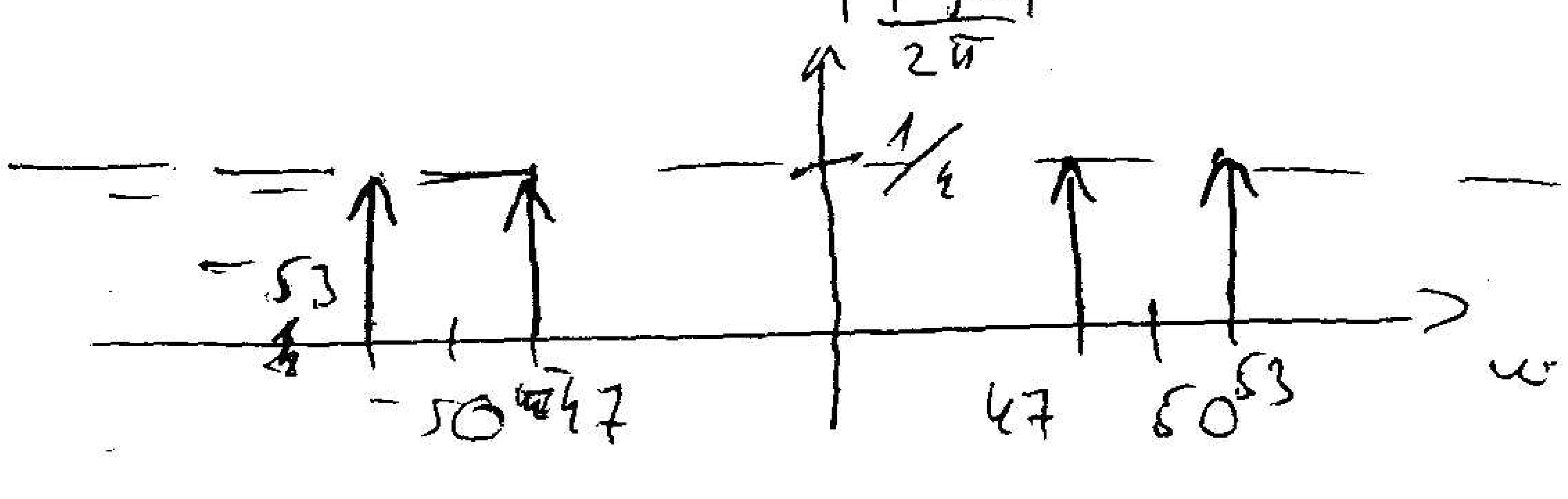
4kP)

$f(t) = \cos(3t)\cos(50t)$ spektruma ábra! körforgó vektor spektrumát kell rajzolni

$\cos\alpha \cdot \cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$

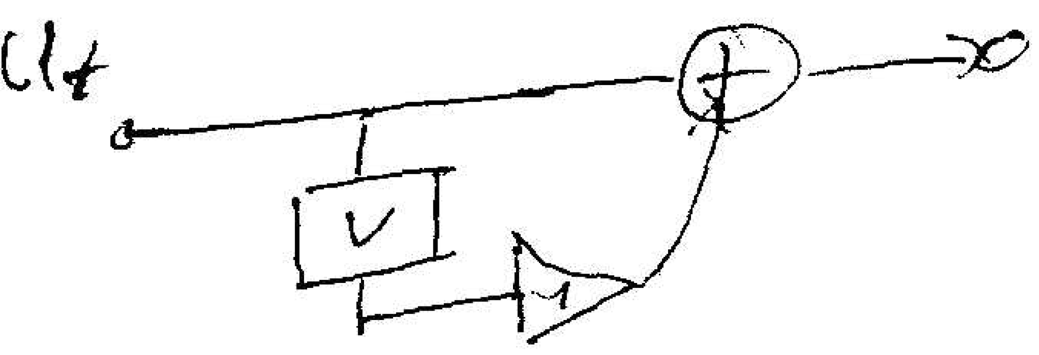
Euler alak így jön ki a $\frac{1}{4}$

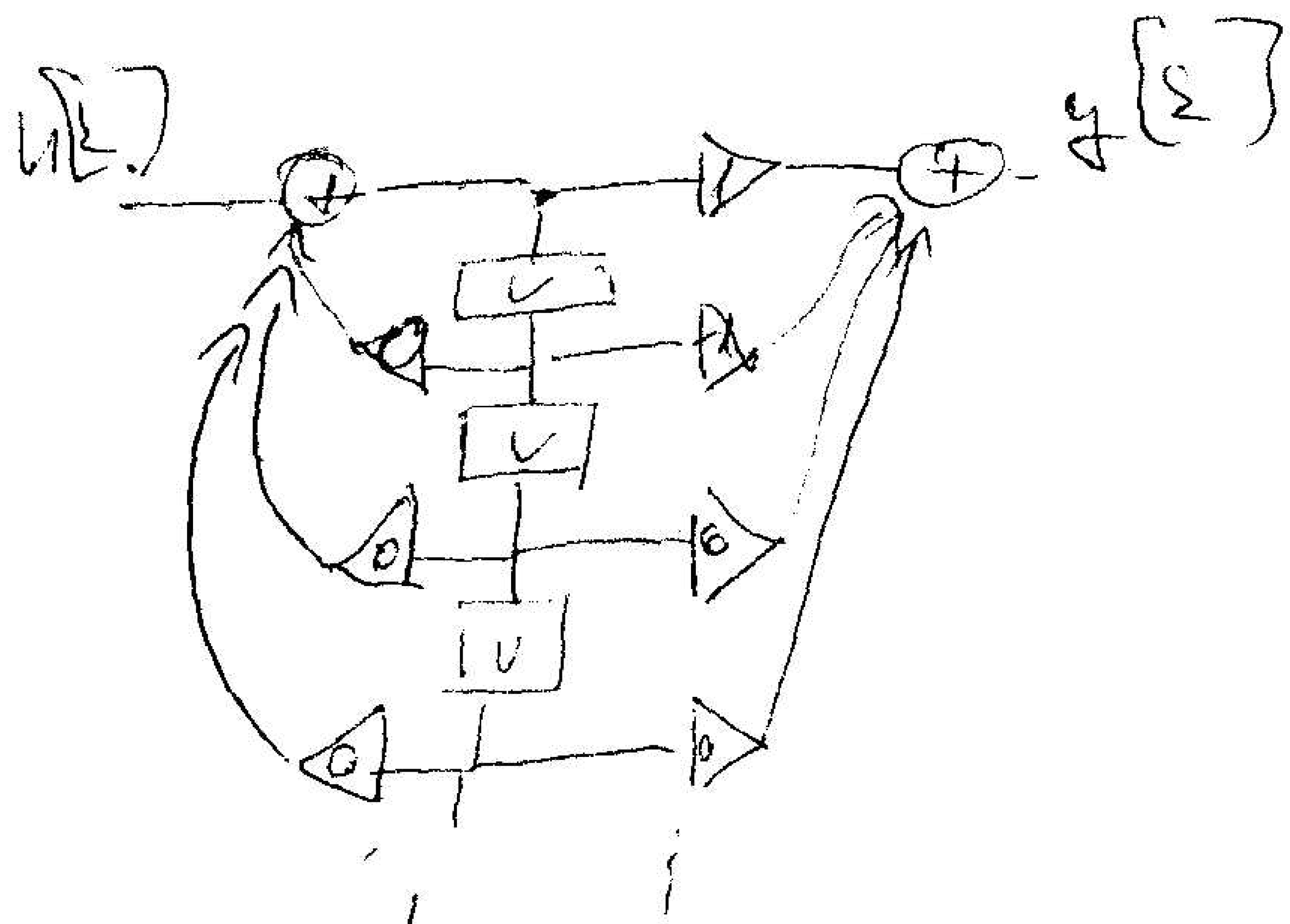
mert $\frac{1}{2} \cdot \frac{e^{\cdots} + e^{\cdots}}{2} = \frac{1}{4} \cdot e^{\cdots}$



5 KP)
Adja meg a $H(z) = \frac{1 - z^{-1}}{1}$ kanonikus alakjával!

DI: $\searrow$ 0. fokú a nevező $\rightarrow$ nincs visszacsatolás





6 KP)
$$F\{x(t-t_0)\} = X(j\omega) \cdot e^{-\omega t_0}$$
$$F\{x(t)\} = X(j\omega)$$

7. KP) Hat. meg. á. fv. fgv. normál alakját.

$$\left.\begin{array}{l} u[k] = \delta[k] \\ y[k] = \varepsilon[k] \cdot 0{,}5^k \end{array}\right\} H(z) = \frac{Y(z)}{U(z)} = \frac{\frac{z}{z-0{,}5}}{1}$$

8 KP)
PS fgv $F\{\ \}$ tisztán valós képz.
ptlen.

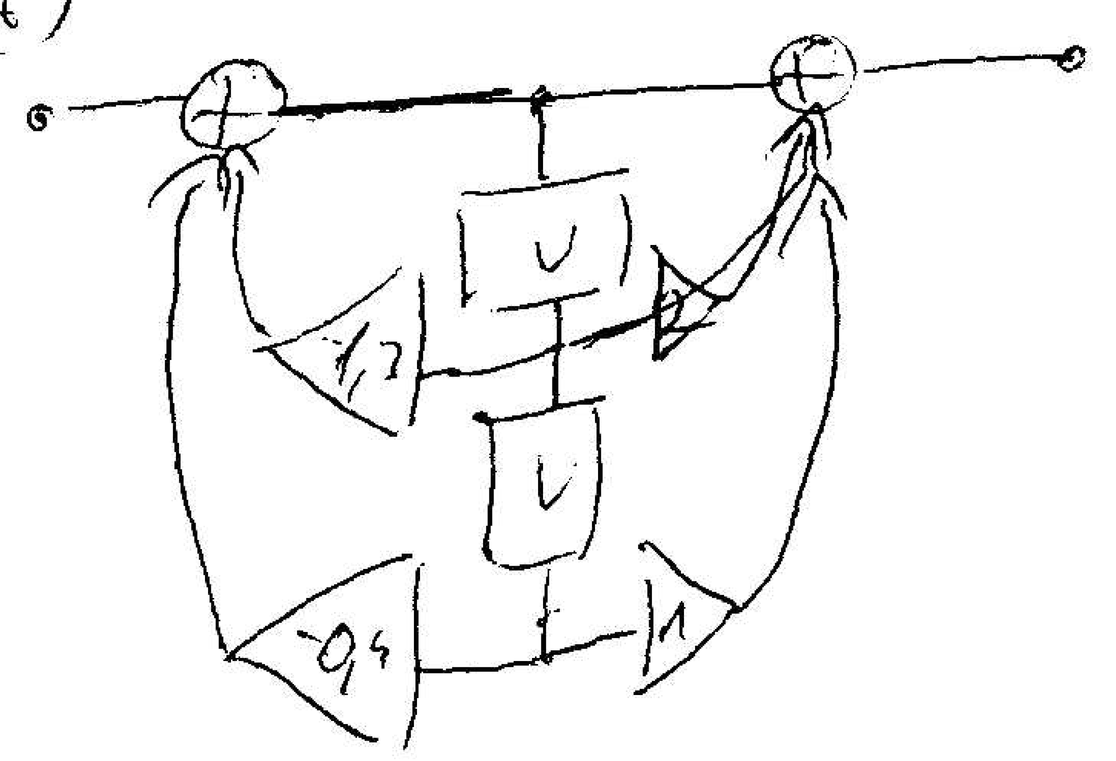
9 KP)
$$H(s) = \frac{1}{s+3} \rightarrow H(j\omega) = \frac{1}{j\omega+3} \Bigg| = \frac{1}{j4+3} = \frac{1}{5} e^{-j\,\mathrm{arctg}\frac{4}{3}}$$
$$u(t) = \cos(4t) \rightarrow \omega = 4$$
$$y(t) = ? = \frac{1}{5} \cdot \cos\left(4t - \mathrm{arctg}\frac{4}{3}\right)$$

10 KP)
Kezdeti érték tétel:
$$h(+0) = \lim_{t \to +0} h(t) = \lim_{s \to \infty} s H(s) \text{ ha } H(s) = \mathcal{L}\{h(t)\}$$
$$\ldots \lim h(t) = \lim s\, H(s) \text{ ha } - '' -$$

Jelek Zanzi 2012. 11. 30.

3 1NP)

$u(t)$ $y(t)$



a) $H(s) = ? = \dfrac{s^2 + 2s + 1}{1 \cdot s^2 + 1{,}3s + 0{,}4}$

b) $z_1 = z_2 = -1$

$p_1 = -0{,}5$

$p_2 = -0{,}8$

egyenletmegoldás

c) $\forall\ \mathrm{Re}\{p_i\} < 0 \Rightarrow$ asz stabil $\rightarrow$ GV stabil

Re: valós része

d) $u(t) = 2 \cdot \varepsilon(t)$ $\qquad \mathcal{L}\{e^{-\alpha t}\} = \dfrac{1}{s+\alpha}$

$U(s) = \dfrac{2}{s} \rightarrow \alpha = 0$

$Y(s) = H(s)\,U(s) = 2 \cdot \dfrac{s^2 + 2s + 1}{s(s+0{,}5)(s+0{,}8)} = 2\left(\dfrac{A}{s} + \dfrac{B}{s+0{,}5} + \dfrac{C}{s+0{,}8}\right)$

letakarásos módszer

$A\Big|_{s=0} = \dfrac{s^2 + 2s + 1}{(s+0{,}5)(s+0{,}8)} = 2{,}5$

$B\Big|_{s=-0{,}5} = \ldots$

$C\Big|_{s=-0{,}8} = \ldots$

Válaszjel:

$\underline{\boxed{y(t)}} = \varepsilon(t)\left(2A + 2B \cdot e^{-0{,}5t} + 2C \cdot e^{-0{,}8t}\right)$

B: 2NP)

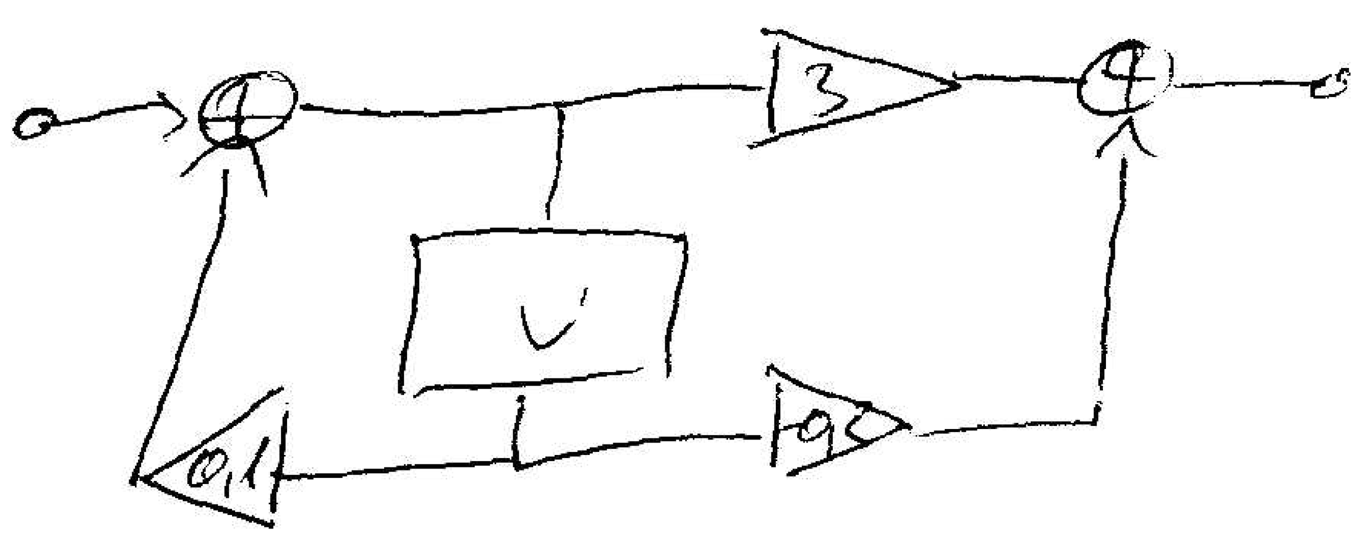
$h[k] = 2\,\delta[k] + \varepsilon[k] \cdot 0{,}1^k$ $\qquad \mathcal{Z}\{h[k]\} = 2 + \dfrac{z}{z-0{,}1} = \dfrac{z - 0{,}1 + z}{z - 0{,}1} = \dfrac{3z - 0{,}2}{z - 0{,}1}$

a) Normál alak

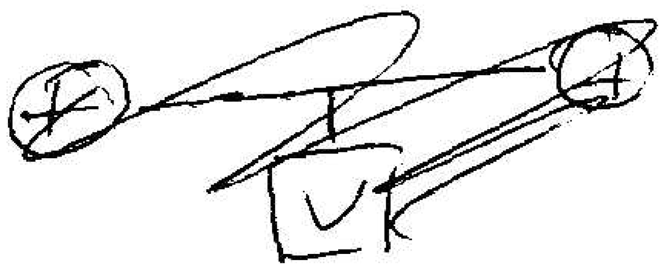
$H(e^{j\vartheta}) = ?$ $\qquad H(e^{j\vartheta}) = \dfrac{3 \cdot e^{j\vartheta} - 0{,}2}{e^{j\vartheta} - 0{,}1}$

kanonikus realizáció

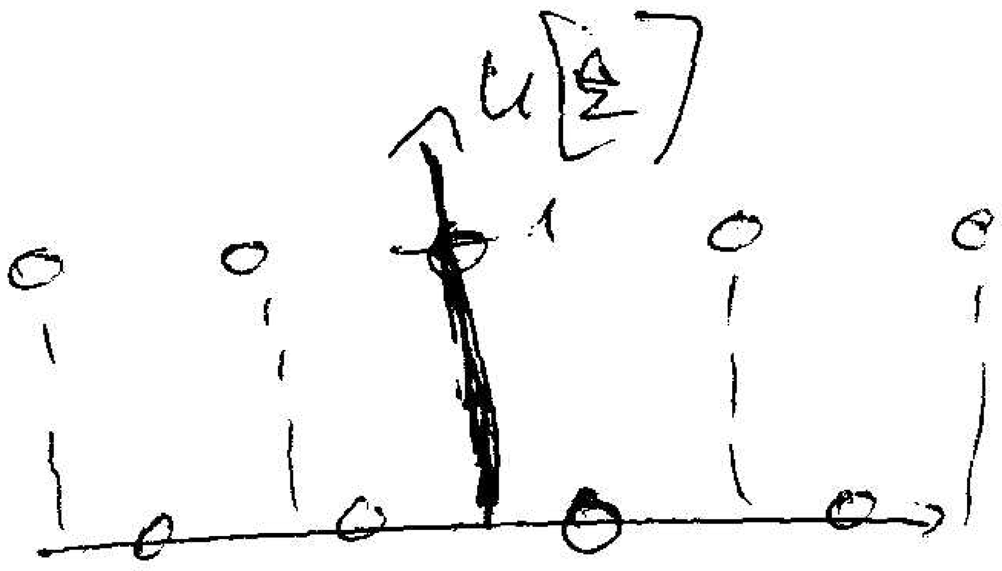
$$H(z) = \frac{3 - 0{,}2z^{-1}}{1 - 0{,}1z^{-1}}$$



miért elsőfokú ezért 1 db késleltető van.



c)



$u[0] = 1$

$u[1] = 0$

$L = 2$

$$U_0 = \frac{1}{L} \sum_{k=0}^{L-1} u[k] = \frac{1}{2}(1+0) = \frac{1}{2}$$

$$U_p^C = \frac{1}{L} \sum_{k=0}^{L-1} u[k] \cdot e^{-jp\vartheta k} = \frac{1}{2}\, 1 + 0 \cdot e^{-jp\vartheta} = \frac{1}{2}$$

$$\vartheta = \frac{2\pi}{L} = \frac{2\pi}{2} = \pi$$

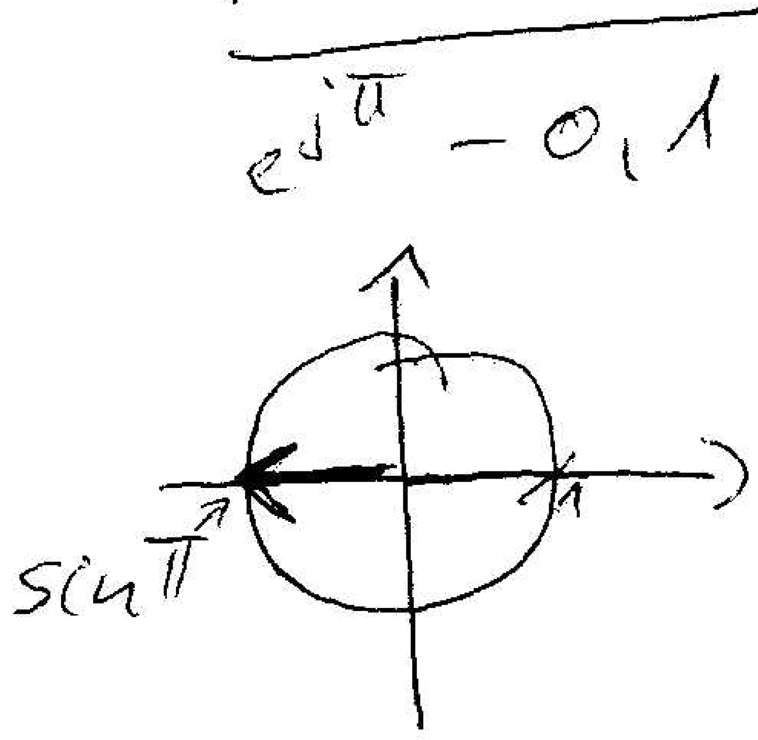
$$u[k] = \frac{1}{2} + \frac{1}{2} \cdot \cos \pi k$$

d)

$$H(e^{j\vartheta}) = \frac{3e^{j\vartheta} - 0{,}2}{e^{j\vartheta} - 0{,}1}$$

$$H(e^{j0}) = \frac{3 - 0{,}2}{1 - 0{,}1} = \frac{2{,}8}{0{,}9}$$

$$H(e^{j\pi}) = \frac{3 \cdot e^{j\pi} - 0{,}2}{e^{j\pi} - 0{,}1} = \frac{-3 - 0{,}2}{-1 - 0{,}1} = \frac{3{,}2}{1{,}1}$$

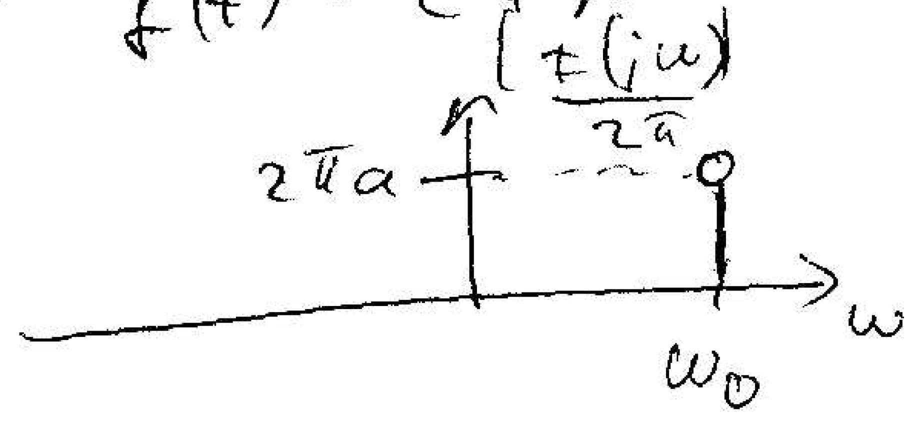


$$y(t) = \frac{1}{2} \cdot \frac{2{,}8}{0{,}9} + \frac{1}{2} \cdot \frac{3{,}2}{1{,}1} \cdot \cos(\pi k + 0^\circ)$$

valós

0° valós eltolódás

B: 1 KP) $f(t) = 2\pi A\, e^{j\omega_0 t}$ spektrum ábrája



Jelek konzi 2012.11.30.

2 $f(t) \rightarrow E$

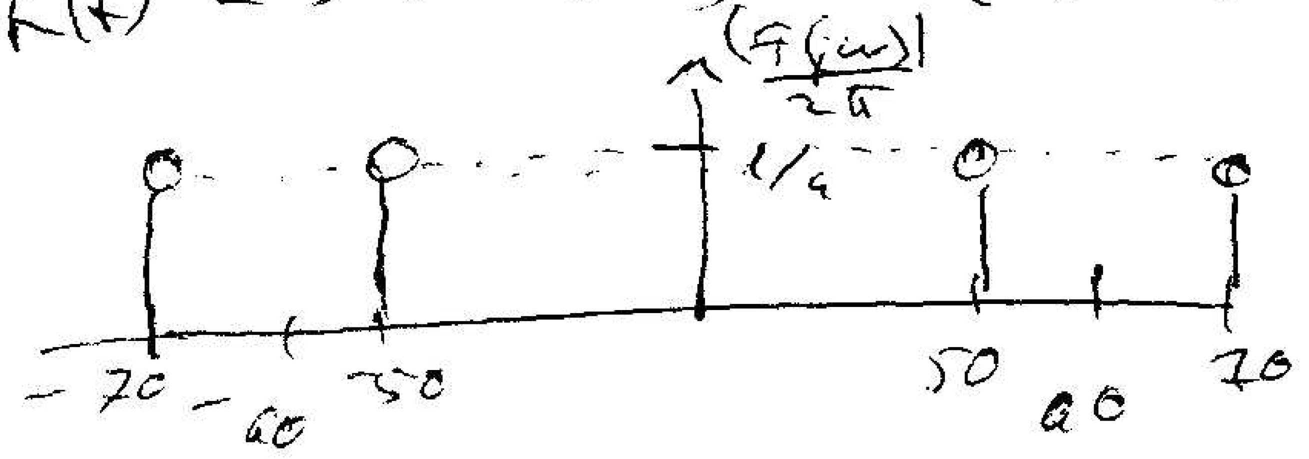
parseval

3/ $B_{max} = 3000\ Hz$

$F_{s\,min} = 2 \cdot B_{max} = 6\,kHz$

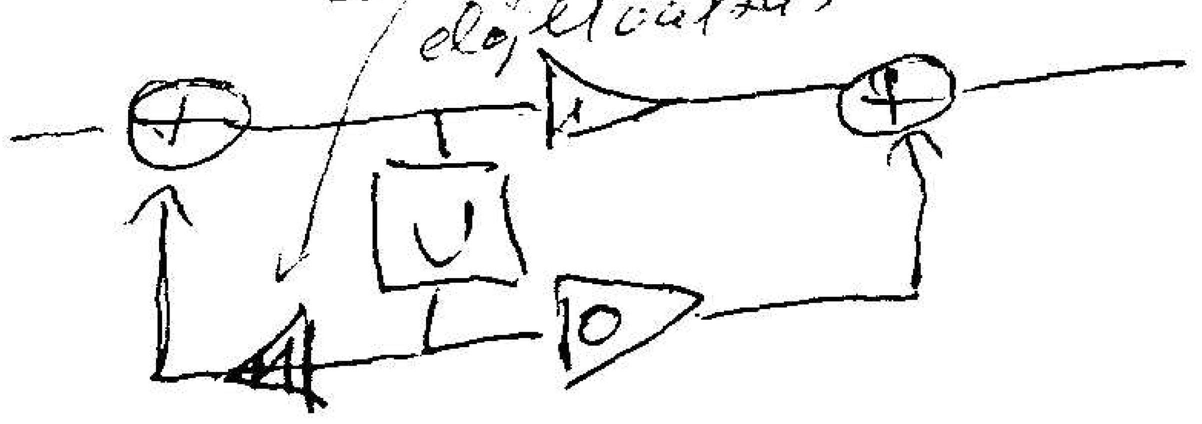
4.

$f(t) = \sin(10t)\,\sin(60t)$



5/ $H(z) = \dfrac{1}{1-z^{-1}}$ kanonikus

előjelváltás



6./ $\mathcal{L}\{\varepsilon(t-t_0)\cdot y(t-t_0)\} = Y(s)\cdot e^{-st_0}$

$\mathcal{L}\{y(t)\} = Y(s)$

7. $u[k] = 5\,\varepsilon[k]$ $\qquad Z\{a^k\} = \dfrac{z}{z-a}$ $\qquad$ a lehet 0, 1, ...

$y[k] = \varepsilon[k]\cdot 0{,}6^k$

$U(z) = 5\cdot\dfrac{z}{z-1}$

$Y(z) = \dfrac{z}{z-0{,}6}$ $\qquad H(z) = \dfrac{Y(z)}{U(z)} = \dfrac{\cancel{z}}{z-0{,}6}\cdot\dfrac{1}{5}\cdot\dfrac{z-1}{\cancel{z}} = \dfrac{1}{5}\cdot\dfrac{z-1}{z-0{,}6}$

8. PTLN...

9) $H(s) = \dfrac{1}{s+6} \rightarrow H(j\omega) = \dfrac{1}{\ldots}$ $\qquad u(t) = 3\sin(8t) \Rightarrow \omega = 8$ $\Rightarrow H(j8) = \dfrac{1}{6+j8} = \dfrac{1}{10}\cdot e^{-j\,\mathrm{arctg}\frac{8}{6}}$

9