1, $\boxed{15}$

$$y' - \frac{2y}{x} = 2x + 3$$

(H) $y' - \frac{2y}{x} = 0$; $\quad \frac{1}{2}\int \frac{dy}{y} = \int \frac{1}{x}\,dx$ (2)

$$\frac{1}{2}\ln|y| = \ln|x| + C \quad (2)$$

$$y_{H,\text{ált}}(x) = A \cdot x^2 \quad (2) \; ; \; A \in \mathbb{R}$$

(I) $y_{I,p}(x) = A(x)\, x^2$, (2)

$$A' \cdot x^2 + \cancel{2xA} - \cancel{2xA} = 2x + 3$$

$$A'(x) = \frac{2}{x} + \frac{3}{x^2} \; (2) \; ; \quad A(x) = \int A'(x)\,dx = 2\ln|x| - \frac{3}{x} \quad (2)$$

$$y_{I,\text{ált}}(x) = y_{H,\text{ált}}(x) + y_{I,\text{part}}(x) \overset{(2)}{=} \underline{\underline{Ax^2 + 2x^2\ln|x| - 3x}} \quad (1)$$

— ∘ —

2, a, $\displaystyle f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!}\, x^k + \frac{f^{(n+1)}(\xi)}{(n+1)!}\, x^{n+1}$

ahol ξ 0 és x között van. (4)

b, $\displaystyle \cos x = \underbrace{1 - \frac{x^2}{2!}}_{T_3(x)} + \overbrace{\underbrace{\frac{\cos^{(4)}(\xi)}{4!}\, x^4}_{R(x)}}^{(3)}$ $\qquad T_3 = 1 - \frac{x^2}{2!}$ (4)

$|R(x)| \le \frac{0{,}1^4}{4!}$, mert $\cos^{(4)}(\xi) = \cos(\xi)$, és $|\cos(\xi)| \le 1$ (4)

— ∘ —

3, $\displaystyle f(x) = \frac{1}{1-2x} = \sum_{n=0}^{\infty} 2^n x^n$ (4), ha $|2x| < 1$, azaz $R = \frac{1}{2}$ (2)

$g(x) = \ln(1-2x)$; $\displaystyle \int_0^x f(t)\,dt = \int_0^x \frac{1}{1-2t}\,dt = \frac{-1}{2}\Big[\ln(1-2t)\Big]_0^x = \frac{-\ln(1-2x)}{2}$

$$g(x) = -2\int_0^x f(t)\,dt \; (2) = -2\int_0^x \Big(\sum_{n=0}^{\infty} 2^n t^n\Big)dt \; (2) = -2\sum_{n=0}^{\infty} 2^n \frac{x^{n+1}}{n+1} = -\sum_{n=0}^{\infty} \frac{2^{n+1} x^{n+1}}{n+1} \; (3)$$

4, a, $$\left.\frac{df}{de}\right|_{(x_0,y_0)} = \lim_{t \to 0+0} \frac{f(x_0+te_x,\, y_0+te_y) - f(x_0,y_0)}{t}$$ (3)

b, T.: Ha $\exists$ grad $f(x_0,y_0)$, akkor $\left.\frac{df}{d\underline{e}}\right|_{(x_0,y_0)} = \underline{e} \cdot \text{grad } f(x_0,y_0)$ (3)

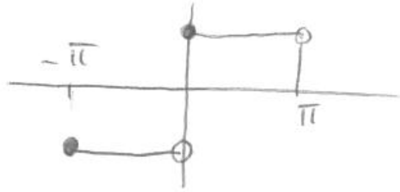
c, $f(x,y) = \sqrt{x^2+2y^2}$; $(x_0,y_0) = (1,2)$

$$\left.\begin{aligned} f'_x(x,y) &= \frac{x}{\sqrt{x^2+2y^2}} \ (2); \quad f'_x(x_0,y_0) = \frac{1}{3} \ (1) \\ f'_y(x,y) &= \frac{2y}{\sqrt{x^2+2y^2}} \ (2); \quad f'_y(x_0,y_0) = \frac{4}{3} \ (1) \end{aligned}\right\} \quad \exists \text{ grad } f(x_0,y_0) = \begin{bmatrix} 1/3 \\ 4/3 \end{bmatrix}$$

$$\left.\frac{df}{d\underline{e}}\right|_{(x_0,y_0)} = \text{grad } f(x_0,y_0) \cdot \underline{e} = \frac{1}{3}\cdot\frac{4}{5} + \frac{4}{3}\cdot\frac{3}{5} = \frac{4+12}{15} = \underline{\underline{\frac{16}{15}}} \ (2)$$

$$\underline{e} = \frac{1}{5}\begin{bmatrix} 4 \\ 3 \end{bmatrix} \ (1)$$

5,* a, $$f(x) = \begin{cases} +1, & \text{ha } x \in [2k\pi, (2k+1)\pi) \\ -1, & \text{ha } x \in [(2k+1)\pi, (2k+2)\pi) \end{cases}$$



Szimmetria miatt $a_k = 0$, (4)

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx \ (2) = \frac{2}{\pi}\int_0^{\pi}\sin(kx)\,dx = \frac{2}{\pi}\left[\frac{-\cos(kx)}{k}\right]_0^{\pi} \ (2) =$$

$$= \frac{2}{k\pi}\left(-(-1)^k+1\right) = \begin{cases} \frac{4}{k\pi}, & \text{ha } k \text{ páratlan,} \\ 0, & \text{ha } k \text{ páros} \end{cases} \ (2)$$

$$\Phi(x) = \frac{4}{\pi}\left(\sin x + \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5} + \dots\right)$$

b, Dirichlet-tétel alapján:

$$\Phi(x) = \begin{cases} +1, & \text{ha } x \in (2k\pi, (2k+1)\pi) \\ 0, & \text{ha } x = k\pi \\ -1, & \text{ha } x \in ((2k+1)\pi, (2k+2)\pi) \end{cases} \ (3)$$

6, *

$e^{2+3i} = e^2 \cos 3 + i\, e^2 \sin 3$ ③

$\sin(2+3i) = \sin 2 \cos(3i) + \cos 2 \sin(3i) = \sin 2 \operatorname{ch} 3 + i \cos 2 \operatorname{sh} 3$ ④

$\ln(2-2i) = \ln(2\sqrt{2}) - \frac{\pi}{4} i$ ③

$|2-2i| = \sqrt{2^2+(-2)^2} = \sqrt{8} = 2\sqrt{2}$

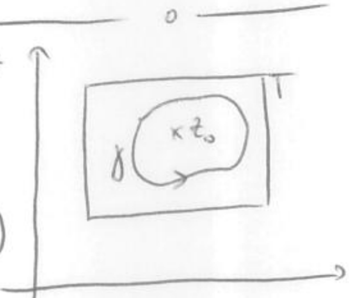
$\varphi = -\frac{\pi}{4}$

$(1+i)^i = e^{i \ln(1+i)} = e^{i \ln\sqrt{2} - \frac{\pi}{4}} = e^{-\frac{\pi}{4}} \cos(\ln\sqrt{2}) + i\, e^{-\frac{\pi}{4}} \sin(\ln\sqrt{2})$ ④

$\ln(1+i) = \ln\sqrt{2} + \frac{\pi}{4} i$

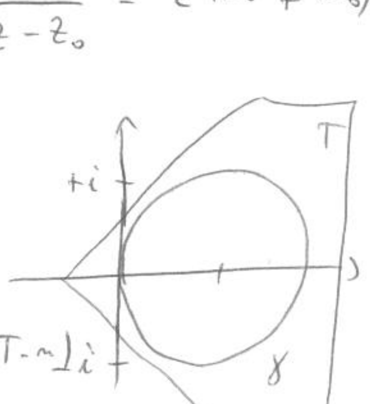
7, *

a, ④



T egysz. öf. tart.
f reg. T-n
$z_0 \in T$
γ zárt görbe, egyszer járja körbe z_0-t poz. irányban.

$$\oint_\gamma \frac{f(z)}{z-z_0} = 2\pi i\, f(z_0)$$



b, 1, $\displaystyle\oint_{|z-1|=1} \frac{z}{z^2+1}\, dz = 0$ ③ (Ch. tétel; $f = \frac{z}{z^2+1}$ reg. T-n)

$z^2+1 = (z+i)(z-i)$

2, $\displaystyle\oint_{|z-1|=2} \frac{z}{z^2+1}\, dz = \frac{1}{2} \oint_{|z-1|=2} \frac{1}{z+i}\, dz + \frac{1}{2} \oint_{|z-1|=2} \frac{1}{z-i}\, dz = \frac{1}{2}(2\pi i + 2\pi i) = \underline{\underline{2\pi i}}$ ② ②

$$\frac{z}{z^2+1} = \frac{A}{z+i} + \frac{B}{z-i} = \frac{1/2}{z+i} + \frac{+1/2}{z-i}$$ ②

8, $\sin(2x),\ x \Rightarrow \lambda_{1,2} = \pm 2i,\ \lambda_{3,4} = 0$ ③ ; $(\lambda+2i)(\lambda-2i)\lambda^2 = (\lambda^2+4)\lambda^2 = \lambda^4 + 4\lambda^2$ ②

$\underline{\underline{y^{(4)} + 4y'' = 0}}$ ② ; $y_{ált}(x) = A \sin(2x) + B\cos(2x) + Cx + D$ ③

9, a, : 4p. ; b, $\displaystyle\frac{a_{n+1}}{a_n} = \frac{(n+1)^{n+1}(2n)!}{n^n (2n+2)!} = \underbrace{\left(1+\frac{1}{n}\right)^n}_{\downarrow e} \underbrace{\frac{n+1}{(2n+1)(2n+2)}}_{\downarrow 0} \to 0$ 6p. $\Rightarrow$ konv.