A május 28.-i valószínűségszámítás vizsgasor megoldása és pontozása

1. $\mathbf{P}(A_p \cdot A_z \cdot A_t \cdot A_m) = 1 - \mathbf{P}\left(\overline{A_p \cdot A_z \cdot A_t \cdot A_m}\right) = 1 - \mathbf{P}\left(\overline{A}_p + \overline{A}_z + \overline{A}_t + \overline{A}_m\right) =$ (**5 pont**)
$= 1 - \mathbf{P}(\overline{A}_p) - \mathbf{P}(\overline{A}_z) - \mathbf{P}(\overline{A}_t) - \mathbf{P}(\overline{A}_m) + \mathbf{P}(\overline{A}_p \cdot \overline{A}_z) + \mathbf{P}(\overline{A}_p \cdot \overline{A}_t) +$
$\mathbf{P}(\overline{A}_p \cdot \overline{A}_m) + \mathbf{P}(\overline{A}_z \cdot \overline{A}_t) +$
$+\mathbf{P}(\overline{A}_z \cdot \overline{A}_m) + \mathbf{P}(\overline{A}_m \cdot \overline{A}_t) - \mathbf{P}(\overline{A}_p \cdot \overline{A}_m \cdot \overline{A}_z) - \mathbf{P}(\overline{A}_p \cdot \overline{A}_t \cdot \overline{A}_z) - \mathbf{P}(\overline{A}_p \cdot \overline{A}_m \cdot \overline{A}_z) -$
$\mathbf{P}(\overline{A}_t \cdot \overline{A}_m \cdot \overline{A}_z) +$
$+\mathbf{P}(\overline{A}_p \cdot \overline{A}_m \cdot \overline{A}_z \cdot \overline{A}_t) = 1 - 3 \cdot \mathbf{P}(\overline{A}_p) + 6 \cdot \mathbf{P}(\overline{A}_p \cdot \overline{A}_z) - 4 \cdot \mathbf{P}(\overline{A}_p \cdot \overline{A}_t \cdot \overline{A}_z) +$
$\mathbf{P}(\overline{A}_p \cdot \overline{A}_m \cdot \overline{A}_z \cdot \overline{A}_t)$. (**5 pont**)
1. csoport: $\mathbf{P}(\overline{A}_p) = \frac{\binom{24}{6}}{\binom{32}{6}}, \mathbf{P}(\overline{A}_p \cdot \overline{A}_z) = \frac{\binom{16}{6}}{\binom{32}{6}}, \mathbf{P}(\overline{A}_p \cdot \overline{A}_t \cdot \overline{A}_z) = \frac{\binom{8}{6}}{\binom{32}{6}},$
$\mathbf{P}(\overline{A}_p \cdot \overline{A}_m \cdot \overline{A}_z \cdot \overline{A}_t) = 0.$ (**10 pont**)
2.csoport: Hasonlóan. $\mathbf{P}(\overline{A}_p) = \frac{\binom{39}{10}}{\binom{52}{10}}, \mathbf{P}(\overline{A}_p \cdot \overline{A}_z) = \frac{\binom{26}{10}}{\binom{52}{10}}, \mathbf{P}(\overline{A}_p \cdot \overline{A}_t \cdot \overline{A}_z) =$
$\frac{\binom{13}{10}}{\binom{52}{10}},$
$\mathbf{P}(\overline{A}_p \cdot \overline{A}_m \cdot \overline{A}_z \cdot \overline{A}_t) = 0.$

2. 1. csoport: $X \in B\left(100, \frac{1}{100}\right)$, (**2 pont**) $k_{\max} = [(n+1)p] = \left[\frac{100}{101}\right] =$
1 (**5 pont**), $\mathbf{P}(X=1) = 100 \cdot \frac{1}{100} \cdot \left(\frac{99}{100}\right)^{99}$. (**3 pont**)
$\lambda = 1$ paraméterű Poisson eloszlással közelítve: $\mathbf{P}(X=1) = \frac{1}{e}$.
2. csoport: $X \in B(50,\ 0,03)$, (**2 pont**) $k_{\max} = [(n+1)p] = \left[\frac{153}{101}\right] =$
2 (**5 pont**), $\mathbf{P}(X=2) = \binom{50}{2} \cdot 0,03^2 \cdot (0,97)^{48}$. (**3 pont**)

3. 1. csoport: $\mathbf{E}X = \int_0^1 2xe^{-2x}dx + \int_1^2 \frac{2x^2}{3e^2}dx =$ (**5 pont**) $= \left[-xe^{-2x} - \frac{1}{2}e^{-2x}\right]_0^1 +$
$\frac{2}{3e^2}\left[\frac{x^3}{3}\right]_1^2 = \frac{1}{27 \cdot e^2} + \frac{1}{2}$. (**5 pont**)
2. csoport: $\mathbf{P}\left(\frac{1}{2} \leq X < \frac{3}{2}\right) = \int_{\frac{1}{2}}^1 2e^{-2x}dx + \int_1^{\frac{3}{2}} \frac{2x}{3e^2}dx =$ (**5 pont**) $= \left[-e^{-2x}\right]_{\frac{1}{2}}^1 +$
$\frac{2}{3e^2}\left[\frac{x^2}{2}\right]_1^{\frac{3}{2}} = \frac{1}{e} - \frac{7}{12e^2}$ (**5 pont**)

4. 1. csoport: $f_{X+Y}(v) = \int_{-\infty}^{\infty} f_x(u) f_Y(v-u) du = \int_{\max\{0,v-3\}}^{\min\{1,v-2\}} \frac{2(v-u)}{5} du$. (**10 pont**)
Ha $v \in [2,3] : f_{X+Y}(v) = \int_0^{v-2} \frac{2}{5}(v-u)\, du = \frac{v^2}{5} - \frac{4}{5}$ (**5 pont**)
Ha $v \in [3,4] : f_{X+Y}(v) = \int_{v-3}^1 \frac{2}{5}(v-u)\, du = \frac{2}{5}\left[vu - \frac{u^2}{2}\right]_{v-3}^1 = ...$ (**5 pont**)
2. csoport: $f_{X+Y}(v) = \int_{-\infty}^{\infty} f_x(u) f_Y(v-u) du = \int_{\max\{2,v-3\}}^{\min\{3,v-2\}} \frac{2(v-u)}{5} du$. (**10 pont**)

Ha $v \in [4,5]$: $f_{X+Y}(v) = \int\limits_{2}^{v-2} \frac{2}{5}(v-u)\,du = \frac{2}{5}\left[vu - \frac{u^2}{2}\right]_2^{v-2} = ...$ (**5 pont**)

Ha $v \in [5,6]$: $f_{X+Y}(v) = \int\limits_{v-3}^{3} \frac{2}{5}(v-u)\,du = \frac{2}{5}\left[vu - \frac{u^2}{2}\right]_{v-3}^{3} = ...$ (**5 pont**)

5. 1. csoport: $1 = A \cdot \int\limits_0^1 \int\limits_0^1 (x+y+xy)\,dxdy = A \int\limits_0^1 \left[\frac{x^2}{2} + xy + \frac{x^2}{2}y\right]_0^1 =$

   $= A \int\limits_0^1 \frac{1}{2} + \frac{3}{2}y dy = A\left[\frac{y}{2} + \frac{3y^2}{4}\right]_0^1 = A\frac{5}{4} \Longrightarrow A = 0,8$ (**5 pont**)

   $f_X(x) = \int\limits_0^1 0,8\,(x+y+yx)\,dy = 0,8\left(\frac{3}{2}x + \frac{1}{2}\right)$ (**5 pont**)

   $f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{2x+2y+2xy}{3x+1}$ (**3 pont**)

   $\mathbf{E}(Y \mid X = x) = \int\limits_0^1 y \cdot f_{Y|X}(y \mid x)\,dy = \int\limits_0^1 y \cdot \frac{2x+2y+2xy}{3x+1} dy =$

   $= \frac{1}{3x+1}\left[xy^2 + \frac{2y^3}{3} + \frac{2x}{3}y^3\right]_0^1 = \frac{5x+2}{9x+3}$ (**5 pont**)

   $\mathbf{E}(Y \mid X) = \frac{5X+2}{9X+3}$ (**2 pont**)

   2. csoport: $1 = A \cdot \int\limits_0^1 \int\limits_0^1 (x+2y+xy)\,dxdy = A \int\limits_0^1 \left[\frac{x^2}{2} + 2xy + \frac{x^2}{2}y\right]_0^1 =$

   $= A \int\limits_0^1 \frac{1}{2} + \frac{5}{2}y dy = A\left[\frac{y}{2} + \frac{5y^2}{4}\right]_0^1 = A \cdot \frac{7}{4} \Longrightarrow A = \frac{4}{7}$ (**5 pont**)

   $f_Y(y) = \int\limits_0^1 A\,(x+2y+yx)\,dx = A\left(\frac{5}{2}y + \frac{1}{2}\right)$ (**5 pont**)

   $f_{X|Y}(x \mid y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{x+2y+xy}{\frac{5}{2}y+\frac{1}{2}}$ (**3 pont**)

   $\mathbf{E}(X \mid Y = y) = \int\limits_0^1 x \cdot f_{X|Y}(x \mid y)\,dx = \int\limits_0^1 x \cdot \frac{x+2y+xy}{\frac{5}{2}y+\frac{1}{2}} dx =$

   $= \frac{1}{\frac{5}{2}y+\frac{1}{2}}\left[\frac{x^3}{3} + yx^2 + \frac{x^3}{3}y\right]_0^1 = \frac{\frac{4}{3}y+\frac{1}{3}}{\frac{5}{2}y+\frac{1}{2}}$ (**5 pont**)

   $\mathbf{E}(X \mid Y) = \frac{8Y+2}{15Y+3}$ (**2 pont**)

6. 1. csoport: A minták függetlenek, normális eloszlásúak és azonos szórásúak.(**5 pont**)
   $H_0 : m_1 = m_2$, azaz egyenlő a két minta várható értéke. (**5 pont**)
   A próbastatisztika:$\frac{\frac{\overline{X_n} - \overline{Y_m}}{\sqrt{\frac{1}{n}+\frac{1}{m}}}}{\sqrt{\frac{(n-1)s^{*2}_{n,X} + (m-1)s^{*2}_{m,Y}}{n+m-2}}}$. (**10 pont**)

   2. csoport: A minták függetlenek, normális eloszlásúak.(**5 pont**) H_0 : $\sigma_1 = \sigma_2$, azaz egyenlő a két minta szórása.(**5 pont**) A próbastatisztika: $\frac{s^{*2}_{n,X}}{s^{*2}_{m,Y}}$. (**10 pont**)