

Nevezetes eloszlások

Diszkrét

Név	Eloszlás	Várható érték	Szórás
Bernoulli (indikátor változó) $X \sim I(p)$	$P(X=1) = p$ $P(X=0) = 1-p$	p	$\sqrt{p \cdot (1-p)}$
Binomiális (visszatevéses mintavétel) $X \sim B_n(p)$	$P(X=k) = \binom{n}{k} \cdot p^k (1-p)^{n-k}$, $k = 0, 1, 2, \dots, n$	np	$\sqrt{np \cdot (1-p)}$
Geometriai (k-adikra következnek be A) $X \sim G(p)$	$P(X=k) = p \cdot (1-p)^{k-1}$, $k = 1, 2, \dots$	$\frac{1}{p}$	$\frac{\sqrt{1-p}}{p}$
Negatív binomiális vagy Pascal (k-adikra következnek be m-edszer A) $X \sim Nb_m(p)$	$P(X=k) = \binom{k-1}{m-1} \cdot p^m (1-p)^{k-m}$, $k = m, m+1, \dots$	$\frac{m}{p}$	$\sqrt{m} \cdot \frac{\sqrt{1-p}}{p}$
Hípergeometriai (visszatevés nélküli mintavétel) $X \sim Hg(N, M, n)$ $M < N, n \leq \min\{M, N-M\}$	$P(X=k) = \frac{\binom{M}{k} \binom{N-M}{n-k}}{\binom{N}{n}}$, $k = 0, 1, 2, \dots, n$	$n \frac{M}{N}$	$\sqrt{n \frac{M}{N} \left(1 - \frac{M}{N}\right) \left(1 - \frac{M-1}{N-1}\right)}$
Poisson $X \sim P(\lambda)$	$P(X=k) = \frac{\lambda^k}{k!} \cdot e^{-\lambda}$, $k = 0, 1, 2, \dots$	λ	$\sqrt{\lambda}$
$\{1, 2, \dots, n\}$ -en Egyenletes $X \sim E(1, 2, \dots, n)$	$P(X=k) = \frac{1}{n}$, $k = 1, 2, \dots, n$	$\frac{n+1}{2}$	$\frac{\sqrt{n^2-1}}{2\sqrt{3}}$

Abszolút folytonos

Név	Sűrűségfüggvény	Eloszlásfüggvény	Várható érték	Szórás
[a,b]-n Egyenletes $X \sim U[a,b]$	$f(x) = \frac{1}{b-a}$, $a \leq x \leq b$	$F(x) = \frac{x-a}{b-a}$, $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{b-a}{2\sqrt{3}}$
Exponenciális $X \sim Exp(\lambda)$	$f(x) = \lambda e^{-\lambda x}$, $x > 0$	$F(x) = 1 - e^{-\lambda x}$, $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Cauchy $X \sim C(\mathcal{G}, \sigma)$, $\sigma > 0$	$f(x) = \frac{1}{\pi \sigma} \frac{1}{1 + \left(\frac{x-\mathcal{G}}{\sigma}\right)^2}$	$F(x) = \frac{1}{2} + \frac{1}{\pi} \arctg \frac{x-\mathcal{G}}{\sigma}$	nem létezik	nem létezik
Gamma $\Gamma_\alpha(\lambda)$, $\alpha, \lambda > 0$	$f(x) = \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$, $x > 0$	$F(x) = \frac{\lambda^\alpha}{(n-1)!} \int_0^x t^{n-1} e^{-\lambda t} dt$, $x > 0$	$\frac{\alpha}{\lambda}$	$\frac{\sqrt{\alpha}}{\lambda}$
Gauss - Normális $X \sim N(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{2\pi\sigma}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$F(x) = \int_{-\infty}^x f(t) dt$	μ	σ
Standard normális $X \sim N(0, 1)$	$\varphi(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$	$\Phi(x) = \text{lásd a táblázatot!}$	0	1
Pareto $X \sim P(\alpha, \beta)$ $\alpha > 0, \beta > 0$	$f(x) = \beta \alpha^\beta / x^{\beta+1}$ ha $x \geq \alpha$	$F(x) = \int_\alpha^x f(t) dt$	$\frac{\alpha\beta}{\beta-1}$ $\beta > 1$	$\sqrt{\frac{\alpha^2\beta}{(\beta-1)(\beta-2)}}$ $\beta > 2$
Beta $X \sim B(a, b)$ $a > 0, b > 0$	$f(x) = \frac{1}{B(a,b)} \cdot x^{a-1} (1-x)^{b-1}$, $0 \leq x \leq 1$	$F(x) = \int_0^x f(t) dt$	$\frac{a}{a+b}$	$\sqrt{\frac{ab}{(a+b)^2(a+b+1)}}$
$\chi^2(n)$ $\equiv \Gamma_{\frac{n}{2}}\left(\frac{1}{2}\right)$ $X = \sum_{i=1}^n X_i^2$ $X_i, \dots, X_n \sim N(0, 1)$ fae.	$f(x) = x^{\frac{n}{2}-1} \cdot \frac{e^{-\frac{x}{2}}}{2^{\frac{n}{2}} \Gamma\left(\frac{n}{2}\right)}$	$F(x) = \int_0^x f(t) dt$, $x > 0$	n	$\sqrt{2n}$
Student t $X = \frac{Y}{\sqrt{Z/n}}$ $\sim t(n)$ $Y \sim N(0, 1)$ $Z \sim \chi^2(n)$ fgtn!	$f(x) = \frac{1}{\sqrt{\pi \cdot n}} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}$		0, ha $n \geq 2$	$\sqrt{\frac{n}{n-2}}$, ha $n \geq 3$
Fisher F $X = \frac{Y/n}{Z/m}$ $\sim F(n, m)$ $Y \sim \chi^2(n)$ $Z \sim \chi^2(m)$ fgtn!	$f(x) = \frac{n \Gamma\left(\frac{n+m}{2}\right) \left(\frac{n}{m} x\right)^{\frac{n}{2}-1}}{m \Gamma\left(\frac{n}{2}\right) \Gamma\left(\frac{m}{2}\right) \left(1 + \frac{n}{m} x\right)^{\frac{n+m}{2}}$, $x > 0$		$\frac{m}{m-2}$ ha $m > 2$	$\sqrt{\frac{2m^2(m+n-2)}{n(m-2)^2(m-4)}}$ $m > 4$

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx, \alpha > 0; \quad \Gamma(n) = (n-1)!, \quad B(a,b) = \frac{\Gamma(a) \cdot \Gamma(b)}{\Gamma(a+b)}$$

HIPOTÉZISVIZSGÁLAT

$$s_n^2 = \frac{1}{n} \sum_{i=1}^n x_i^2 - \bar{x}^2, \quad s_n^{*2} = \frac{n}{n-1} s_n^2, \quad r = \frac{\overline{xy} - \bar{x}\bar{y}}{s_x s_y}$$

$$y = ax + b \text{ lineáris regresszió: } \hat{a} = r \frac{s_y}{s_x} = \frac{\overline{xy} - \bar{x}\bar{y}}{s_x^2}, \quad \hat{b} = \bar{y} - \hat{a} \cdot \bar{x}$$

u-próba:

$$1. \text{ Kétoldali, egymintás: } u = \frac{\bar{x} - \mu}{\sigma} \sqrt{n}, \quad u_{\epsilon/2} = \Phi^{-1}(1 - \epsilon/2).$$

$$\text{konfidenciaintervallum } \mu\text{-re: } [\bar{x} - u_{\epsilon/2} \frac{\sigma}{\sqrt{n}}, \bar{x} + u_{\epsilon/2} \frac{\sigma}{\sqrt{n}}].$$

$$2. \text{ Egyoldali, egymintás: } u = \frac{\bar{x} - \mu}{\sigma} \sqrt{n}, \quad u_{\epsilon} = \Phi^{-1}(1 - \epsilon).$$

$$3. \text{ Kétoldali, kétmintás: } u = \frac{\bar{x} - \bar{y}}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}, \quad u_{\epsilon/2} = \Phi^{-1}(1 - \epsilon/2).$$

$$4. \text{ Egyoldali, kétmintás: } u = \frac{\bar{x} - \bar{y}}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}, \quad u_{\epsilon} = \Phi^{-1}(1 - \epsilon).$$

t-próba:

$$1. \text{ Kétoldali, egymintás: } t = \frac{\bar{x} - \mu}{s_n^*} \sqrt{n}, \quad t_{\epsilon/2} = \text{a } t_{n-1}\text{-eloszlás } 1 - \epsilon/2\text{-kvantilise.}$$

$$\text{konfidenciaintervallum } \mu\text{-re: } [\bar{x} - t_{\epsilon/2} \frac{s_n^*}{\sqrt{n}}, \bar{x} + t_{\epsilon/2} \frac{s_n^*}{\sqrt{n}}].$$

$$2. \text{ Egyoldali, egymintás: } t = \frac{\bar{x} - \mu}{s_n^*} \sqrt{n}, \quad t_{\epsilon} = \text{a } t_{n-1}\text{-eloszlás } 1 - \epsilon\text{-kvantilise.}$$

$$3. \text{ Kétoldali, kétmintás: } t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{(n_1-1)\sigma_1^{*2} + (n_2-1)\sigma_2^{*2}}{n_1+n_2-2}}} \sqrt{\frac{n_1 n_2}{n_1+n_2}}, \quad t_{\epsilon/2} = \text{a}$$

$t_{n_1+n_2-2}$ -eloszlás $1 - \epsilon/2$ -kvantilise.

$$4. \text{ Egyoldali, kétmintás: } t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{(n_1-1)\sigma_1^{*2} + (n_2-1)\sigma_2^{*2}}{n_1+n_2-2}}} \sqrt{\frac{n_1 n_2}{n_1+n_2}}, \quad t_{\epsilon} = \text{a}$$

$t_{n_1+n_2-2}$ -eloszlás $1 - \epsilon$ -kvantilise.

χ^2 -próba:

$$1. \text{ Illeszkedésvizsgálat: } \chi^2 = \sum_{i=1}^r \frac{(v_i - np_i)^2}{np_i} \text{ összehasonlítva a } \chi_{r-1}\text{-eloszlás}$$

$(1 - \epsilon)$ -kvantilisével.

$$2. \text{ Homogenitásvizsgálat: } \chi^2 = nm \sum_{i=1}^r \frac{\left(\frac{v_i}{n} - \frac{\mu_i}{m}\right)^2}{\frac{v_i}{n} + \frac{\mu_i}{m}} \text{ összehasonlítva a } \chi_{r-1}\text{-eloszlás}$$

$(1 - \epsilon)$ -kvantilisével.

$$3. \text{ Függetlenségvizsgálat: } \chi^2 = n \sum_{i=1}^r \sum_{j=1}^s \frac{\left(v_{ij} - \frac{v_i \cdot v_j}{n}\right)^2}{\frac{v_i \cdot v_j}{n}} \text{ összehasonlítva a } \chi_{(r-1)(s-1)}\text{-}$$

eloszlás $(1 - \epsilon)$ -kvantilisével.

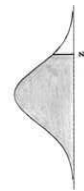
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$$\text{Kritérium: } t' = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_x^{*2}}{n_1} + \frac{s_y^{*2}}{n_2}}}, \quad c = \frac{s_x^{*2}/n_1}{s_x^{*2}/n_1 + s_y^{*2}/n_2}, \quad \frac{1-c}{f} = \frac{1-c}{n_1-1} + \frac{(1-c)^2}{n_2-1}$$

$$\mathcal{X}_{\epsilon} = \{(x, y) : |t'(x, y)| \geq t_{\epsilon/2}(f)\} \quad 2\text{-oldali}$$

$$\mathcal{X}_{\epsilon} = \{(x, y) : t'(x, y) \geq t_{\epsilon}(f)\} \quad 1\text{-oldali}$$

Standard normális eloszlás táblázat



Annak a valószínűsége, hogy a valószínűségi változó
-∞ és x közé esik

x		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0		0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1		0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2		0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3		0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4		0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5		0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6		0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7		0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8		0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9		0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0		0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1		0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2		0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3		0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4		0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5		0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6		0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7		0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8		0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9		0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0		0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1		0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2		0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3		0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4		0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5		0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6		0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7		0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8		0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9		0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0		0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

PERCENTAGE POINTS OF THE T DISTRIBUTION

Tail Probabilities															
One Tail	Two Tails	0.10	0.05	0.025	0.01	0.005	0.001	0.0005					0.001	0.0005	
		0.20	0.10	0.05	0.02	0.01	0.002	0.001					0.002	0.001	
D	1	3.078	6.314	12.71	31.82	63.66	318.3	637					318.3	637	1
E	2	1.886	2.920	4.303	6.965	9.925	22.330	31.6					22.330	31.6	2
G	3	1.638	2.353	3.182	4.541	5.841	10.210	12.92					10.210	12.92	3
R	4	1.533	2.132	2.776	3.747	4.604	7.173	8.610					7.173	8.610	4
E	5	1.476	2.015	2.571	3.365	4.032	5.893	6.869					5.893	6.869	5
E	6	1.440	1.943	2.447	3.143	3.707	5.208	5.959					5.208	5.959	6
S	7	1.415	1.895	2.365	2.998	3.499	4.785	5.408					4.785	5.408	7
	8	1.397	1.860	2.306	2.896	3.355	4.501	5.041					4.501	5.041	8
O	9	1.383	1.833	2.262	2.821	3.250	4.297	4.781					4.297	4.781	9
F	10	1.372	1.812	2.228	2.764	3.169	4.144	4.537					4.144	4.537	10
F	11	1.363	1.796	2.201	2.718	3.106	4.025	4.387					4.025	4.387	11
F	12	1.356	1.782	2.179	2.681	3.055	3.930	4.318					3.930	4.318	12
R	13	1.350	1.771	2.160	2.650	3.012	3.852	4.221					3.852	4.221	13
E	14	1.345	1.761	2.145	2.624	2.977	3.787	4.140					3.787	4.140	14
E	15	1.341	1.753	2.131	2.602	2.947	3.733	4.073					3.733	4.073	15
D	16	1.337	1.746	2.110	2.567	2.921	3.686	4.015					3.686	4.015	16
M	17	1.333	1.740	2.110	2.567	2.921	3.646	3.965					3.646	3.965	17
	18	1.330	1.734	2.101	2.552	2.878	3.610	3.922					3.610	3.922	18
	19	1.328	1.729	2.093	2.539	2.861	3.579	3.883					3.579	3.883	19
	20	1.325	1.725	2.086	2.528	2.845	3.552	3.850					3.552	3.850	20
	21	1.323	1.721	2.080	2.518	2.831	3.527	3.819					3.527	3.819	21
	22	1.321	1.717	2.074	2.508	2.819	3.505	3.792					3.505	3.792	22
	23	1.319	1.714	2.069	2.500	2.807	3.485	3.768					3.485	3.768	23
	24	1.318	1.711	2.064	2.492	2.797	3.467	3.745					3.467	3.745	24
	25	1.316	1.708	2.060	2.485	2.787	3.450	3.725					3.450	3.725	25
	26	1.315	1.706	2.056	2.479	2.779	3.435	3.707					3.435	3.707	26
	27	1.314	1.703	2.052	2.473	2.771	3.421	3.690					3.421	3.690	27
	28	1.313	1.701	2.048	2.467	2.763	3.408	3.674					3.408	3.674	28
	29	1.311	1.699	2.045	2.462	2.756	3.396	3.659					3.396	3.659	29
	30	1.310	1.697	2.042	2.457	2.750	3.385	3.646					3.385	3.646	30
	32	1.309	1.694	2.037	2.449	2.738	3.365	3.622					3.365	3.622	32
	34	1.307	1.691	2.032	2.441	2.728	3.348	3.601					3.348	3.601	34
	36	1.306	1.688	2.028	2.434	2.719	3.333	3.582					3.333	3.582	36
	38	1.304	1.686	2.024	2.429	2.712	3.319	3.566					3.319	3.566	38
	40	1.303	1.684	2.021	2.423	2.704	3.307	3.551					3.307	3.551	40
	42	1.302	1.682	2.018	2.418	2.698	3.296	3.538					3.296	3.538	42
	44	1.301	1.680	2.015	2.414	2.692	3.286	3.526					3.286	3.526	44
	46	1.300	1.679	2.013	2.410	2.687	3.277	3.515					3.277	3.515	46
	48	1.299	1.677	2.011	2.407	2.682	3.269	3.505					3.269	3.505	48
	50	1.299	1.676	2.009	2.403	2.678	3.261	3.496					3.261	3.496	50
	55	1.297	1.673	2.004	2.396	2.668	3.245	3.476					3.245	3.476	55
	60	1.296	1.671	2.000	2.390	2.660	3.232	3.460					3.232	3.460	60
	65	1.295	1.669	1.997	2.385	2.654	3.220	3.447					3.220	3.447	65
	70	1.294	1.667	1.994	2.381	2.648	3.211	3.435					3.211	3.435	70
	80	1.292	1.664	1.990	2.374	2.639	3.195	3.416					3.195	3.416	80
	100	1.290	1.660	1.984	2.364	2.626	3.174	3.390					3.174	3.390	100
	150	1.287	1.655	1.976	2.351	2.609	3.145	3.357					3.145	3.357	150
	200	1.286	1.653	1.972	2.345	2.601	3.131	3.340					3.131	3.340	200

khi-négyszet eloszlás farok percentilisei					
df	0.1	0.05	0.025	0.01	0.005
1	2.706	3.841	5.024	6.635	7.879
2	4.605	5.991	7.378	9.210	10.597
3	6.251	7.815	9.348	11.345	12.838
4	7.779	9.488	11.143	13.277	14.860
5	9.236	11.070	12.833	15.086	16.750
6	10.645	12.592	14.449	16.812	18.548
7	12.017	14.067	16.013	18.475	20.278
8	13.362	15.507	17.535	20.090	21.955
9	14.684	16.919	19.023	21.666	23.589
10	15.987	18.307	20.483	23.209	25.188
11	17.275	19.675	21.920	24.725	26.757
12	18.549	21.026	23.337	26.217	28.300
13	19.812	22.362	24.736	27.688	29.819
14	21.064	23.685	26.119	29.141	31.319
15	22.307	24.996	27.488	30.578	32.801
16	23.542	26.296	28.845	32.000	34.267
17	24.769	27.587	30.191	33.409	35.718
18	25.989	28.869	31.526	34.805	37.156
19	27.204	30.144	32.852	36.191	38.582
20	28.412	31.410	34.170	37.566	39.997
21	29.615	32.671	35.479	38.932	41.401
22	30.813	33.924	36.781	40.289	42.796
23	32.007	35.172	38.076	41.638	44.181
24	33.196	36.415	39.364	42.980	45.559
25	34.382	37.652	40.646	44.314	46.928
26	35.563	38.885	41.923	45.642	48.290
27	36.741	40.113	43.195	46.963	49.645
28	37.916	41.337	44.461	48.278	50.993
29	39.087	42.557	45.722	49.588	52.336
30	40.256	43.773	46.979	50.892	53.672
40	51.805	55.758	59.342	63.691	66.766
50	63.167	67.505	71.420	76.154	79.490
60	74.397	79.082	83.298	88.379	91.952
70	85.527	90.531	95.023	100.425	104.215
80	96.578	101.879	106.629	112.329	116.321
90	107.565	113.145	118.136	124.116	128.299
100	118.498	124.342	129.561	135.807	140.169