

1. Elemi algebrai azonosságok

Műveletek hatványokkal: tegyük fel, hogy $a, b \in \mathbb{R}^+$, $\alpha, \beta \in \mathbb{N}^+$, ekkor

$$\begin{array}{l|l|l} a^\alpha \cdot b^\alpha = (a \cdot b)^\alpha & a^\alpha \cdot a^\beta = a^{\alpha+\beta} & (a^\alpha)^\beta = a^{\alpha \cdot \beta} \\ a^{-\alpha} = \frac{1}{a^\alpha} & \frac{a^\alpha}{a^\beta} = a^{\alpha-\beta} & a^0 = 1 \\ \sqrt[\alpha]{a} = a^{\frac{1}{\alpha}} & \sqrt[\alpha]{a^\beta} = a^{\frac{\beta}{\alpha}} = (\sqrt[\alpha]{a})^\beta & \sqrt[\alpha]{a \cdot b} = \sqrt[\alpha]{a} \cdot \sqrt[\alpha]{b} \end{array}$$

2. Műveletek logaritmussal

Tegyük fel, hogy $a \in \mathbb{R}^+ \setminus \{1\}$, $b, c, d \in \mathbb{R}^+$, $\alpha \in \mathbb{R}$ ekkor definíció szerint $\log_a b = c \Leftrightarrow a^c = b$, és

$$\begin{array}{l|l} \log_a(c \cdot d) = \log_a c + \log_a d & \log_a(b^\alpha) = \alpha \log_a b \\ \log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c & \log_a 1 = 0, \log_a a = 1 \end{array}$$

3. Trigonometrikus azonosságok

Tegyük fel, hogy $\alpha, \beta \in \mathbb{R}$, ekkor $\sin^2 \alpha + \cos^2 \alpha = 1$, továbbá

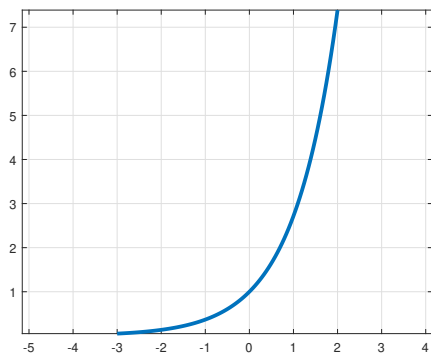
$$\begin{array}{l|l} \sin(-\alpha) = -\sin \alpha & \cos(-\alpha) = \cos(\alpha) \\ \sin(\alpha + \pi) = -\sin(\alpha) & \cos(\alpha + \pi) = -\cos \alpha \\ \sin\left(\alpha + \frac{\pi}{2}\right) = \cos \alpha & \cos\left(\alpha + \frac{\pi}{2}\right) = -\sin \alpha \\ \sin(2\alpha) = 2 \sin \alpha \cos \alpha & \cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha \\ \sin^2 \alpha = \frac{1 - \cos(2\alpha)}{2} & \cos^2 \alpha = \frac{1 + \cos(2\alpha)}{2} \\ \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha & \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha & \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \end{array}$$

4. Nevezetes szögek szögfüggvényei

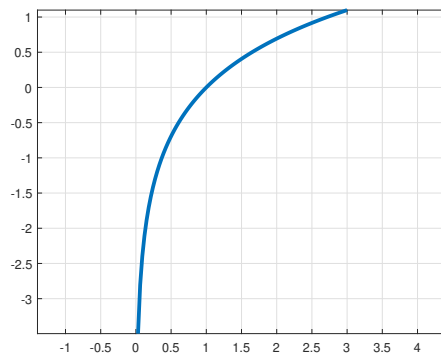
Tegyük fel, hogy $k \in \mathbb{Z}$, ekkor

$\sin \frac{\pi}{6} = \frac{1}{2}$	$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$	$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$
$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\cos \frac{\pi}{3} = \frac{1}{2}$	$\tan \frac{\pi}{3} = \sqrt{3}$
$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\tan \frac{\pi}{4} = 1$
$\sin \left(\frac{\pi}{2} + k\pi\right) = (-1)^k$	$\cos \left(\frac{\pi}{2} + k\pi\right) = 0$	$\tan \left(\frac{\pi}{2} + k\pi\right)$ nem ért.
$\sin(k\pi) = 0$	$\cos(k\pi) = (-1)^k$	$\tan(k\pi) = 0$

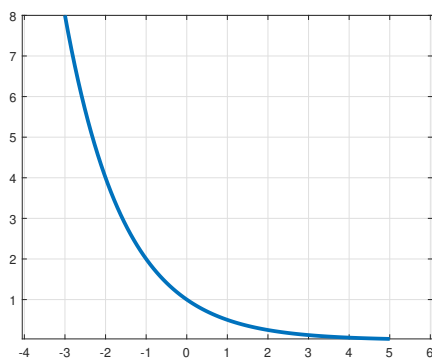
5. Elemi függvények



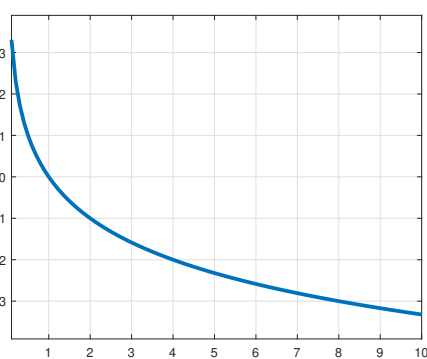
$f(x) = a^x, a > 1$
 $D_f = \mathbb{R}$
 $R_f = \mathbb{R}^+$
 $\lim_{x \rightarrow \infty} a^x = \infty$
 $\lim_{x \rightarrow -\infty} a^x = 0$
 szig. mon. nő, folytonos \mathbb{R} -en
 inverze : $\log_a x$
 $(a^x)' = \ln(a) a^x$



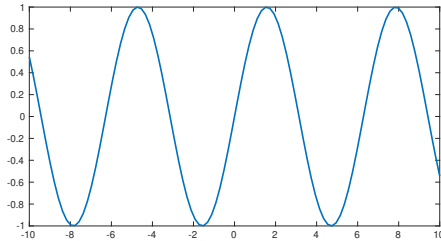
$f(x) = \log_a(x), a > 1$
 $D_f = \mathbb{R}^+$
 $R_f = \mathbb{R}$
 $\lim_{x \rightarrow \infty} \log_a(x) = \infty$
 $\lim_{x \rightarrow 0^+} \log_a(x) = -\infty$
 szig. mon. nő, folytonos \mathbb{R}^+ -en
 inverze : a^x
 $(\log_a x)' = \frac{1}{\ln(a) \cdot x}$



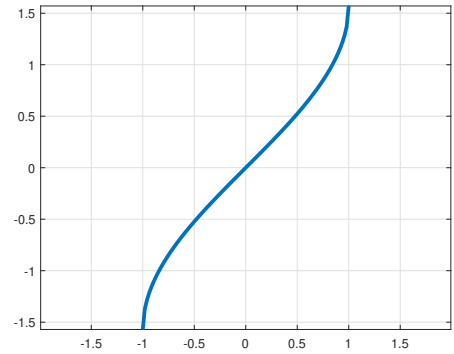
$f(x) = a^x, 0 < a < 1$
 $D_f = \mathbb{R}$
 $R_f = \mathbb{R}^+$
 $\lim_{x \rightarrow \infty} a^x = 0$
 $\lim_{x \rightarrow -\infty} a^x = \infty$
 szig. mon. csökk., folytonos \mathbb{R} -en
 inverze : $\log_a x$
 $(a^x)' = \ln(a) a^x$



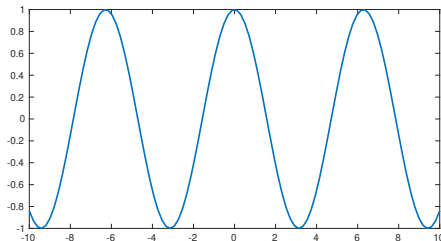
$f(x) = \log_a(x), 0 < a < 1$
 $D_f = \mathbb{R}^+$
 $R_f = \mathbb{R}$
 $\lim_{x \rightarrow \infty} \log_a(x) = -\infty$
 $\lim_{x \rightarrow 0^+} \log_a(x) = \infty$
 szig. mon. csökk., folytonos \mathbb{R}^+ -en
 inverze : a^x
 $(\log_a x)' = \frac{1}{\ln(a) \cdot x}$



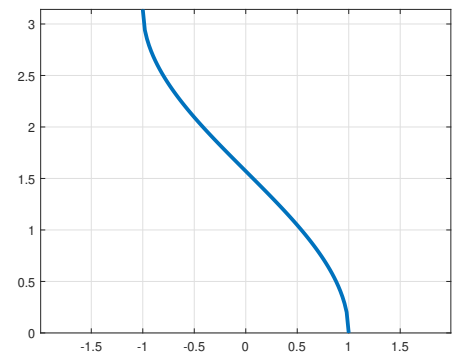
$f(x) = \sin(x)$,
 $D_f = \mathbb{R}$
 $R_f = [-1, 1]$
 folytonos \mathbb{R} -en, páratlan, 2π -periódikus
 inverze : $\arcsin(x)$
 $(\sin x)' = \cos(x)$



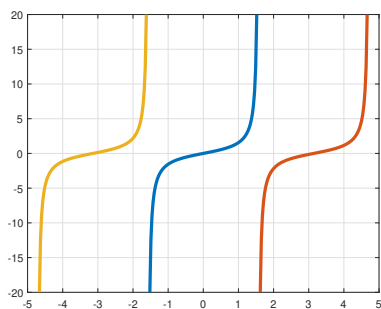
$f(x) = \arcsin(x)$
 $D_f = [-1, 1]$
 $R_f = [-\frac{\pi}{2}, \frac{\pi}{2}]$
 szig. mon. nő, folytonos $[-1, 1]$ -en, páratlan
 inverze : $\sin(x)$
 $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$



$f(x) = \cos(x)$,
 $D_f = \mathbb{R}$
 $R_f = [-1, 1]$
 folytonos \mathbb{R} -en, páros, 2π -periódikus
 inverze : $\arccos(x)$
 $(\cos x)' = -\sin(x)$

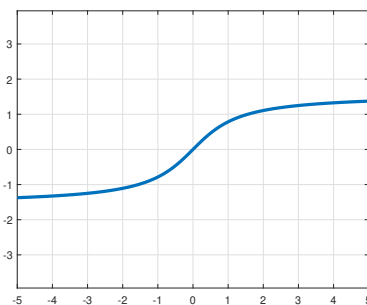


$f(x) = \arccos(x)$
 $D_f = [-1, 1]$
 $R_f = [0, \pi]$
 szig. mon. csökken, folytonos $[-1, 1]$ -en
 inverze : $\cos(x)$
 $(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$



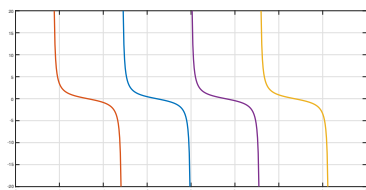
$f(x) = \tan(x)$,
 $D_f = \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$
 $R_f = \mathbb{R}$
 folytonos $\mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$ -en, páratlan,
 π -periódikus

inverze : $\arctan(x)$
 $(\tan x)' = 1 + \tan^2(x) = \frac{1}{\cos^2(x)}$



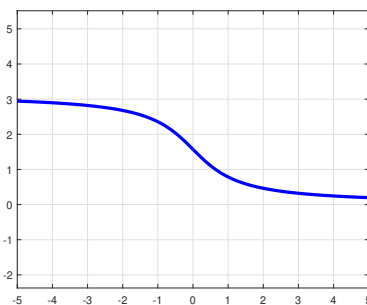
$f(x) = \arctan(x)$
 $D_f = \mathbb{R}$
 $R_f = (-\frac{\pi}{2}, \frac{\pi}{2})$
 szig. mon. nő, folytonos \mathbb{R} -en, páratlan

$\lim_{x \rightarrow \infty} \arctan x = \frac{\pi}{2}$, $\lim_{x \rightarrow -\infty} \arctan x = -\frac{\pi}{2}$
 inverze : $\tan(x)$
 $(\arctan x)' = \frac{1}{1+x^2}$



$f(x) = \cotan(x)$,
 $D_f = \mathbb{R} \setminus \{k\pi\}$
 $R_f = \mathbb{R}$
 folytonos $\mathbb{R} \setminus \{k\pi\}$ -en, páratlan, π -periódikus

inverze : $\operatorname{acotan}(x)$
 $(\cotan x)' = -1 - \cotan^2(x) = -\frac{1}{\sin^2(x)}$



$f(x) = \operatorname{acotan}(x)$
 $D_f = \mathbb{R}$
 $R_f = (0, \pi)$
 szig. mon. csökken, folytonos \mathbb{R} -en
 $\lim_{x \rightarrow \infty} \operatorname{acotan} x = 0$, $\lim_{x \rightarrow -\infty} \operatorname{acotan} x = \pi$
 inverze : $\cotan(x)$
 $(\operatorname{acotan} x)' = \frac{-1}{1+x^2}$