# Rendszerelmélet

## 1. gyakorlat

2017.09.04.

1.

$$x(t)\begin{cases} 0, & t<0 \\ e^{-0,4t}, & t\geq 0 \end{cases}$$

a) $u[k] = x(kT)$

b) $v[k] = \dfrac{3x(kT)+x(kT-T)}{4} \qquad k\in\mathbb{Z}$

$T=0,5$

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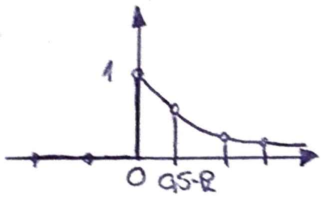
a) $u[k] = x(kT) = \begin{cases} 0, & k<0 \\ e^{-0,4\cdot k\cdot 0,5} & k\geq 0 \end{cases} = \begin{cases} 0, & k<0 \\ e^{-0,2k}, & k\geq 0 \end{cases} =$




$= \varepsilon[k]\cdot\left(e^{-0,2}\right)^k = \underline{\underline{\varepsilon[k]\, 0,819^k}}$

b) $v[k] = \dfrac{3x(kT)+x((k-1)T)}{4} = \dfrac{3}{4}\varepsilon[k]\cdot 0,819^k + \dfrac{1}{4}\cdot\varepsilon[k-1]\cdot 0,819^{k-1}$

$\dfrac{3}{4}\varepsilon[k]\cdot 0,819^k = \dfrac{3}{4}\delta[k]\cdot 0,819^0 + \dfrac{3}{4}\varepsilon[k-1]\, 0,819^{k}$

$\Rightarrow v[k] = \dfrac{3}{4}\delta[k] + \dfrac{3}{4}\varepsilon[k-1]\cdot 0,819\cdot 0,819^{k-1} + \dfrac{1}{4}\varepsilon[k-1]\cdot 0,819^{k-1} =$

$= 0,75\delta[k] + \varepsilon[k-1]\left(\dfrac{3}{4}\cdot 0,819 + \dfrac{1}{4}\right)\cdot 0,819^{k-1} =$

$= 0,75\delta[k] + \varepsilon[k-1]\, 0,864\cdot 0,819^{k-1}$

c) $y[k] = \frac{1}{T}\int\limits_{k\cdot T-T}^{kT} x(t)\,dt = \frac{1}{T}\int\limits_{kT-T}^{kT} e^{-0,4t}\,dt = \frac{1}{T}\left[\frac{e^{-0,4t}}{-0,4}\right]_{(k-1)T}^{kT} =$

(T=0,5)

$= 2\cdot\left(-2,5\left(e^{-0,4kT} - e^{-0,4(k-1)T}\right)\right) = 2\left(-2,5\left(e^{-0,2k} - e^{-0,2k+0,2}\right)\right) =$

$$= \left.\begin{cases} 2\left(-2,5\left(1-e^{0,2}\right)e^{-0,2k}\right) = 2\cdot 0,55\cdot 0,819^k & \text{ha } k\geq 0 \\ 0 & \text{ha } k<0 \end{cases}\right\} =$$

$= \underline{\underline{\varepsilon[k]\cdot 1,1\cdot 0,819^k}}$

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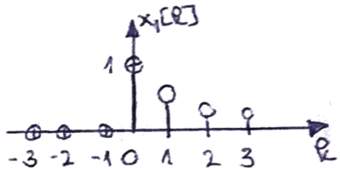
$x_1[k] = \begin{cases} 0, & k<0 \\ 0,5^k, & k\geq 0 \end{cases}$




$x_3[k] = \begin{cases} 0, & k<2 \\ 0,5^k, & k\geq 2 \end{cases} = \varepsilon[k-2]\cdot 0,5^k$

nem tisztán eltolt

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$y_1[k+1] = 3y_1[k]$ és $y_1[0] = 2$

| $k$ | $y_1[k]$ | $y_1[k+1]$ |
|---|---|---|
| 0 | 2 | 6 |
| 1 | 6 | 18 |
| 2 | 18 | 54 |
| 3 | ... | ... |
| 4 | ... | ... |

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$x_1[k] = \varepsilon[k]2^k$

$x_2[k] = \varepsilon[k]\cdot 0,5^{-k} = \varepsilon[k]\cdot(0,5^{-1})^k = \varepsilon[k]\cdot 2^k$

$x_3[k] = \underbrace{\left(1-\varepsilon[-k-1]\right)}_{\varepsilon[k]}2^k = \varepsilon[k]2^k$

$x_4[k] = \delta[k] + 2\delta[k-1] + 4\varepsilon[k-2]2^{k-2} = \varepsilon[k]2^k$

$$u^{(e)}(t) = \frac{1}{2}\left(u(t) + u(-t)\right)$$

$$u^{(o)}(t) = \frac{1}{2}\left(u(t) - u(-t)\right)$$

$$u(t) = A\cos(\omega t) + B(\sin(\omega t))$$

$$u^{(e)}(t) = A\cos(\omega t)$$

$$u^{(o)}(t) = B\sin(\omega t)$$

$$v[k] = A + B\cdot k \;\begin{array}{l} \nearrow\; v^{(e)}[k] = A \\ \searrow\; v^{(o)}[k] = B\cdot k \end{array}$$

$$e^{j\alpha} = \cos\alpha + j\sin\alpha$$

$$\vartheta = \frac{2\pi}{L}$$

$$\vartheta \cdot \underset{\text{periódusszám}}{L} = 2\pi \overset{\text{körszám}}{M}$$

$$\vartheta = 2\pi \cdot \frac{M}{L}$$

$$x_1[k] = \cos[0{,}17\pi k + \underbrace{0{,}2\pi}_{\varphi}]$$

$$\vartheta = 0{,}17\cancel{\pi} = 2\cancel{\pi}\frac{M}{L}$$

$$\frac{0{,}17}{2} = \frac{M}{L} = \frac{17}{200} \Rightarrow \text{periodikus}$$

$M = 17$

$L = 200$

# Rendszerelmélet

## 2. gyakorlat

2017.09.11.

1. DI:

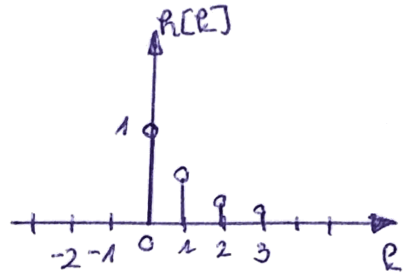
$h[0]=1$

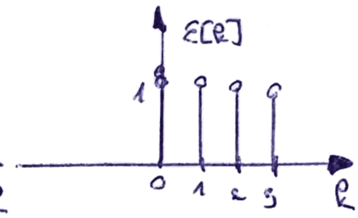
$h[1]=0,5$

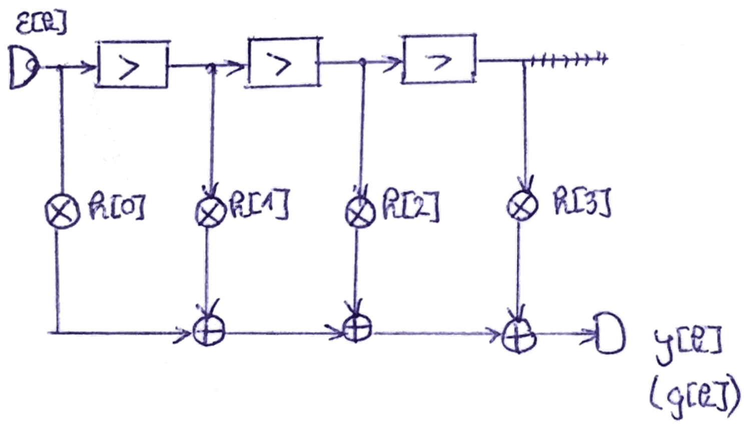
$h[2]=0,25$

$u[k]=\varepsilon[k]\ (\cancel{=g[k]})$ miatt

$y[k]=g[k]$




$g[0]=\varepsilon[0]\cdot h[0]=1$

$g[1]=\varepsilon[1]\cdot h[0]+\varepsilon[0]\cdot h[1]=1,5$

$g[2]=\varepsilon[2]\cdot h[0]+\varepsilon[1]\cdot h[1]+\varepsilon[0]\cdot h[2]=1,75$

$g[3]=\varepsilon[3]\cdot h[0]+\varepsilon[2]\cdot h[1]+\varepsilon[1]\cdot h[2]+\varepsilon[0]\cdot h[3]=1,75$

DI: $f[k]*g[k]=\sum_{i=-\infty}^{\infty} f[i]g[k-i]=\sum_{i=-\infty}^{\infty} f[k-i]g[i]=y[k]$

$y[k]=h[k]*u[k]=\sum_{i=0}^{\infty} h[i]\cdot u[k-i]=\sum_{i=-\infty}^{k} h[k-i]\cdot u[i]$

2. DI:

$$h[k] = -1{,}5 \cdot 0{,}8^k \cdot \varepsilon[k] + 2{,}5\,\delta[k]$$

a) $u[k] = \varepsilon[k] \cdot 0{,}6^k$

b) $u[k] = 3$

c) $u[k] = \varepsilon[k-L]\,0{,}6^{k-L}$

d) $u[k] = \varepsilon[k-L]\,0{,}6^k$

a) $y[k] = \sum_{i=-\infty}^{\infty} h[k-i] \cdot u[i] = \sum_{i=-\infty}^{\infty} \left(-1{,}5 \cdot 0{,}8^{k-i} \cdot \varepsilon[k-i] + 2{,}5\,\delta[k-i]\right)\left(\varepsilon[i] \cdot 0{,}6^i\right)$

$$y[k] = \underbrace{\sum_{i=-\infty}^{\infty} -1{,}5 \cdot 0{,}8^{k-i} \cdot \varepsilon[k-i] \cdot \varepsilon[i] \cdot 0{,}6^i}_{A} + \underbrace{\sum_{i=-\infty}^{\infty} 2{,}5\,\delta[k-i]\,\varepsilon[i] \cdot 0{,}6^i}_{B}$$

$$\boxed{\sum_{i=0}^{k} q^k = \frac{1-q^{k+1}}{1-q}}$$

B: Ra $i = k$: $2{,}5 \cdot 0{,}6^k \cdot \varepsilon[k]$

A: $\sum_{i=0}^{k} -1{,}5 \cdot 0{,}8^{k-i} \cdot 0{,}6^i = 0{,}8^k \sum_{i=1}^{0} -1{,}5 \left(\frac{0{,}6}{0{,}8}\right)^i =$

$$= -1{,}5 \cdot 0{,}8^k \, \frac{1-\left(\frac{0{,}6}{0{,}8}\right)^{k+1}}{1-\frac{0{,}6}{0{,}8}} \cdot \varepsilon[k] = \left(-6 \cdot 0{,}8^k \cdot \left(1 - \frac{3}{4} \cdot \left(\frac{3}{4}\right)^k\right)\right)\varepsilon[k] =$$

$$= \left(-6 \cdot 0{,}8^k + \frac{18}{4} \cdot 0{,}8^k \cdot \left(\frac{3}{4}\right)^k\right)\varepsilon[k] = \left(-6 \cdot 0{,}8^k + 4{,}5 \cdot 0{,}6^k\right)\varepsilon[k]$$

$$y[k] = \varepsilon[k] \cdot \left(-6 \cdot 0{,}8^k + 4{,}5 \cdot 0{,}6^k\right) + 2{,}5\,\varepsilon[k] \cdot 0{,}6^k = \left(-6 \cdot 0{,}8^k + 7 \cdot 0{,}6^k\right)\varepsilon[k]$$

b) $y[k] = \sum_{i=-\infty}^{\infty} h[i]\, u[k-i] = \sum_{i=-\infty}^{\infty} \left(-1{,}5 \cdot 0{,}8^{i} \cdot \varepsilon[i] + 2{,}5\, \delta[i]\right) \cdot 3 =$

$$= \underbrace{\sum_{i=-\infty}^{\infty} -1{,}5 \cdot 0{,}8^{i} \cdot \varepsilon[i] \cdot 3}_{A} + \underbrace{\sum_{i=-\infty}^{\infty} 2{,}5 \cdot \delta[i] \cdot 3}_{B}$$

B : ka $i=0$ : $2{,}5 \cdot 3 = 7{,}5$

$\boxed{\frac{1}{1-q}}$

$A = \sum_{i=0}^{\infty} -1{,}5 \cdot 3 \cdot 0{,}8^{i} = -4{,}5 \cdot \sum_{i=0}^{\infty} 0{,}8^{i}$

$$y[k] = 7{,}5 - 4{,}5 \cdot \frac{1}{1-0{,}8} = 7{,}5 - 22{,}5 = -15$$

c) $u_c[k] = u_a[k-L] \rightarrow y_c[k] = y_a[k-L] = \varepsilon[k-L]\left(-6 \cdot 0{,}8^{k-L} + 7 \cdot 0{,}6^{k-L}\right.$

d) $u_d[k] = u_c[k] \cdot 0{,}6^{L} \rightarrow y_d[k] = y_c\underset{k-L}{[k]} \cdot 0{,}6^{L} = \varepsilon\underset{k-L}{[k]}\left(-6 \cdot 0{,}8^{k-L} + 7 \cdot 0{,}6^{k-L}\right) \cdot 0{,}6^{L}$

$= P(2 \text{ v. } 3 \text{ 1-es} | \text{0-t}) \cdot P(\text{0-t küld}) + \ldots = (\underset{\substack{\uparrow \\ 3 \text{ 1-es}}}{0{,}01^3} + \binom{3}{2} \cdot \underset{\substack{\uparrow \\ 2 \text{ 1-es}}}{0{,}01^2} \cdot 0{,}99) \cdot \frac{1}{2} +$

$(\underset{\substack{\uparrow \\ 3 \text{ 0-ás}}}{0{,}01^3} + \binom{3}{2} \cdot \underset{\substack{\uparrow \\ 2 \text{ 0-ás}}}{0{,}01^2} \cdot 0{,}99) \cdot \frac{1}{2}$

115. kocka, majd egy érmét annyiszor, amennyit a kocka mutat

a) P(egyszer sem dobunk fejet) = $A_i$ = i-t dobunk a kockával

$i = 1, 2, \ldots, 6$

$$P(\text{nincs fej}) = \sum_{i=1}^{6} P(\text{nincs fej} | A_i) \cdot P(A_i) =$$

$$= \frac{1}{2} \cdot \frac{1}{6} + (\cancel{1-}\frac{1}{4}) \cdot \frac{1}{6} + (\cancel{1-}\frac{1}{8}) \cdot \frac{1}{6} + (\cancel{1-}\frac{1}{16}) \cdot \frac{1}{6} + (\cancel{1-}\frac{1}{32}) \cdot \frac{1}{6} + (\cancel{1-}\frac{1}{64}) \cdot \frac{1}{6} =$$

$$= \frac{1}{6} \cdot \sum_{i=1}^{6} (\cancel{1-}\frac{1}{2^i}) = \cancel{\frac{1}{6}(6-(1-\frac{1}{2^6}))} = \frac{1}{6}(1 - \frac{1}{2^6})$$

Bayes-tétel

$A_1, A_2, \ldots, A_n, \ldots$ teljes eseményrendszer

$B$

$$P(A_i | B) = \frac{P(A_i B)}{P(B)} = \frac{P(B | A_i) \cdot P(A_i)}{\sum_{j=1}^{\infty} P(B | A_j) \cdot P(A_j)}$$

b) Ha egyszer sem dobtunk fejet, mi a valószínűsége, hogy 6-ost dobtunk?

$$P(A_6 | \text{nincs fej}) = \frac{(1 - \frac{1}{2^6}) \cdot \frac{1}{6}}{\frac{1}{6} \cdot \sum_{i=1}^{6} (1 - \frac{1}{2^i})}$$

48. vizsgázók: 75% A szakos
15% B szakos
10% C szakos

ötöst kap: 0,4 – A szakos
0,7 – B szakos
0,6 – C szakos

ha ötöse vizsgázott, milyen valószínűséggel A, B illetve C szakos?

A, B, C szak: teljes eseményrendszer
O: ötöst kap
$P(A|O), P(B|O), P(C|O)$

$$P(A|O) = \frac{0{,}4 \cdot 0{,}75}{0{,}4 \cdot 0{,}75 + 0{,}7 \cdot 0{,}15 + 0{,}6 \cdot 0{,}1}$$

$$P(C|O) = \frac{0{,}6 \cdot 0{,}1}{0{,}4 \cdot 0{,}75 + 0{,}7 \cdot 0{,}15 + 0{,}6 \cdot 0{,}1}$$

$$P(B|O) = \frac{0{,}7 \cdot 0{,}1}{0{,}4 \cdot 0{,}75 + 0{,}7 \cdot 0{,}15 + 0{,}6 \cdot 0{,}1}$$

# Rendszerelmélet

## 3. gyakorlat

① $u[k] = 1000\,\delta[k]$

a) $k$ — $y[k]$

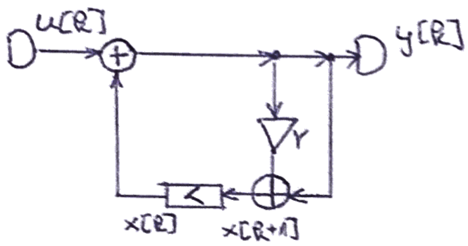
0 — $1000\,\delta[k]\ (= 1000\,u[0]) = 1000;$

1 — $u[0] + u[0]\cdot \underbrace{0{,}1}_{r} = u[0](1+r) = 1100;$

2 — $u[0] + u[0]\cdot r + (u[0] + u[0]r)r = 1210;$

3 — $u[0](1+r) + (u[0](1+r))r + (u[0](1+r) + (u[0](1+r))r)r = 1331;$

b)




c) $x[k+1] = (u[k] + x[k])r + u[k] + x[k] = \underbrace{(1+r)}_{\underline{\underline{A}}}x[k] + \underbrace{(1+r)}_{\underline{B}}u[k]$

$y[k] = x[k] + u[k]$

"1·" $\underline{C}^T$ "1·" $D$

d) $\underline{\underline{A}} = 1+r$

$\underline{B} = 1+r$

$\underline{C}^T = 1$

$D = 1$

e) $x[1] = (1+r)\underset{0}{x[0]} + (1+r)u[0] = (1+r)u[0]$

$y[0] = u[0]$

$x[2] = (1+r)x[1] = (1+r)^2 u[0]$

$y[1] = x[1] = (1+r)u[0]$

$x[3] = (1+r)x[2] = (1+r)^3 u[0]$

$y[2] = x[2] = (1+r)^2 u[0]$

$y[k] = (1+r)^k u[0]$

Rendszeregyenlet:

$y[k] = (1+r)y[k-1] + u[k]$

2.

$\alpha_i, \beta_i$

$1-\alpha_i-\beta_i$

$0 \leq \alpha_i, \beta_i \leq 1$

$0 \leq \alpha_i + \beta_i \leq 1$

$x_1[k+1] = \beta_1 x_1[k] + u[k]$

$x_2[k+1] = \alpha_1 x_1[k] + \beta_2 x_2[k]$

$x_3[k+1] = \alpha_2 x_2[k] + \beta_3 x_3[k]$

$y[k] = \alpha_3 \cdot x_3[k]$

$$\underline{x}[k+1] = \begin{bmatrix} \beta_1 & 0 & 0 \\ \alpha_1 & \beta_2 & 0 \\ 0 & \alpha_2 & \beta_3 \end{bmatrix} \underline{x}[k] + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u[k]$$

$y[k] = [0 \; 0 \; \alpha_3] \underline{x}[k] + 0 \cdot u[k]$

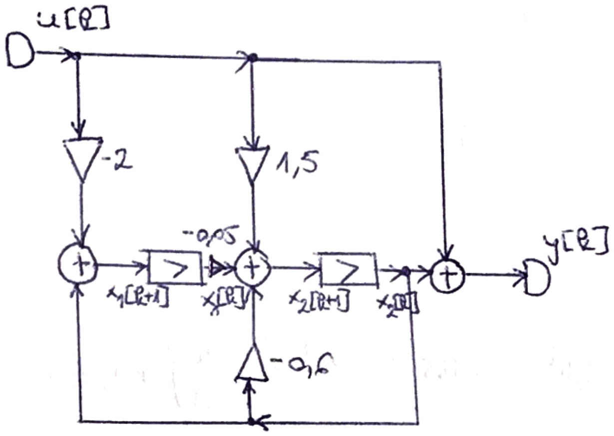
$\alpha_1 = 0{,}6 \quad \beta_1 = 0{,}2$

$\alpha_2 = 0{,}8 \quad \beta_2 = 0{,}15$

$\alpha_3 = 0{,}9 \quad \beta_3 = 0{,}08$

| $k$ | $u[k]$ | $x_1[k]$ | $x_2[k]$ | $x_3[k]$ | $y[k]$ |
|---|---|---|---|---|---|
| 2012 | 500 | 0 | 0 | 0 | 0 |
| 2013 | 500 | 500 | 0 | 0 | 0 |
| 2014 | 500 | 600 | 300 | 0 | 0 |
| 2015 | 500 | 620 | 405 | 240 | 216 |
| 2016 | 500 | | | | |
| 2017 | 500 | | | | |
| 2018 | 500 | | | | |
| ⋮ | ⋮ | | | | |
| $\infty$ | 500 | 625 | 441 | 384 | 345 |

3.




a) $x_1[k+1] = x_2[k] - 2u[k]$

$x_2[k+1] = -0{,}05x_1[k] - 0{,}6x_2[k] + 1{,}5u[k]$

$y[k] = x_2[k] + u[k]$

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 1 \\ -0{,}05 & -0{,}6 \end{bmatrix} \quad \underline{B} = \begin{bmatrix} -2 \\ 1{,}5 \end{bmatrix} \quad \underline{C}^T = [0 \ 1] \quad D = 1$$

$$h[k] = D \cdot \delta[k] + \varepsilon[k-1](\underline{C}^T \cdot \underline{\underline{A}}^{k-1} \underline{B})$$

$$\underline{\underline{A}}^k = \sum_{i=1}^{N} \lambda_i^k \cdot \underline{\underline{L}}_i$$

$$\underline{\underline{L}}_i = \prod_{\substack{p=1 \\ p \neq i}}^{N} \frac{\underline{\underline{A}} - \lambda_p \underline{\underline{I}}}{\lambda_i - \lambda_p} \qquad \sum_{i=1}^{N} \underline{\underline{L}}_i = \underline{\underline{I}} \text{ (egységmátrix)}$$

b)

1. sajátértékek
2. $\underline{\underline{L}}_i$
3. $\underline{C}^T \cdot \underline{\underline{A}}^{k-1} \cdot \underline{B}$
4. $y[k]$

1. $$0 = \det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = \begin{vmatrix} -\lambda & 1 \\ -0{,}05 & -0{,}6-\lambda \end{vmatrix} = \lambda^2 + 0{,}6\lambda + 0{,}05$$

$\lambda_1 = -0{,}1$

$\lambda_2 = -0{,}5$

2. $$\underline{\underline{L}}_1 = \frac{\underline{\underline{A}} - \lambda_2 \underline{\underline{I}}}{\lambda_1 - \lambda_2} = \frac{\begin{bmatrix} 0 & 1 \\ -0{,}05 & -0{,}6 \end{bmatrix} - \begin{bmatrix} -0{,}5 & 0 \\ 0 & -0{,}5 \end{bmatrix}}{(-0{,}1) - (-0{,}5)} = \frac{1}{0{,}4} \cdot \begin{bmatrix} 0{,}5 & 1 \\ -0{,}05 & -0{,}1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1{,}25 & 2{,}5 \\ -0{,}125 & -0{,}25 \end{bmatrix}$$

$$\underline{\underline{L}}_2 = \underline{\underline{I}} - \underline{\underline{L}}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1{,}25 & 2{,}5 \\ -0{,}125 & -0{,}25 \end{bmatrix} = \begin{bmatrix} -0{,}25 & -2{,}5 \\ 0{,}125 & 1{,}25 \end{bmatrix}$$

3. $$\underline{C}^T \cdot \underline{\underline{A}}^{k-1} \cdot \underline{B} = \underline{C}^T \cdot \{\lambda_1^{k-1} \cdot \underline{\underline{L}}_1 + \lambda_2^{k-1} \cdot \underline{\underline{L}}_2\} \underline{B} = [0 \ 1]\{(-0{,}1)^{k-1} \underline{\underline{L}}_1 + (-0{,}5)^{k-1} \underline{\underline{L}}_2\} \cdot \begin{bmatrix} -2 \\ 1{,}5 \end{bmatrix} =$$

$$= \{(-0{,}1)^{k-1}[-0{,}125 \ \ -0{,}25] + (-0{,}5)^{k-1}[0{,}125 \ \ 1{,}25]\} \begin{bmatrix} -2 \\ 1{,}5 \end{bmatrix} =$$

$$= (-0{,}1)^{k-1}(-0{,}125) + (-0{,}5)^{k-1}(1{,}625)$$

$$h[k] = 1 \cdot \delta[k] + \varepsilon[k-1]\left((-0{,}1)^{k-1} \cdot (-0{,}125) + 1{,}625(-0{,}5)^{k-1}\right)$$

c) $$y[k] = \underbrace{\underline{c}^T \cdot \underline{\underline{A}}^k \cdot \underline{x}[0]}_{=0} + \underbrace{\sum_{i=0}^{k-1} \underline{c}^T \cdot \underline{\underline{A}}^{k-1-i} \cdot B \cdot u[i]}_{=*} + D u[k]$$

$$u[k] = \varepsilon[k] \cdot 0{,}4^k$$

$$* = \sum_{i=0}^{k-1}\left(-0{,}125(-0{,}1)^{k-1-i} + 1{,}625(-0{,}5)^{k-1-i}\right)0{,}4^i = -0{,}125 \cdot (-0{,}1)^{k-1}\sum_{i=0}^{k-1}\left(\frac{0{,}4}{-0{,}1}\right)^i + 1{,}625 \cdot (-0{,}5)^{k-1}\sum_{i=0}^{k-1}\left(\frac{0{,}4}{-0{,}5}\right.$$

$$= -0{,}125 \cdot (-0{,}1)^{k-1} \cdot \frac{1-\left(\frac{0{,}4}{-0{,}1}\right)^k}{1-\frac{0{,}4}{-0{,}1}} + 1{,}625 \cdot (-0{,}5)^{k-1} \cdot \frac{1-\left(\frac{0{,}4}{-0{,}5}\right)^k}{1-\frac{0{,}4}{-0{,}5}} =$$

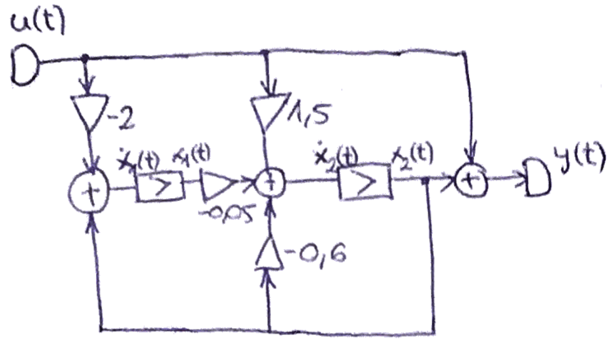
$$= 0{,}25 \cdot (-0{,}1)^k - 0{,}25 \cdot 0{,}4^k - 1{,}805(-0{,}5)^k + 1{,}805 \cdot 0{,}4^k =$$

$$= \left(0{,}25 \cdot (-0{,}1)^k - 1{,}805 \cdot (-0{,}5)^k + 2{,}555 \cdot 0{,}4^k\right)\varepsilon[k] = y[k]$$

$+0 \cdot u[k]$

1.

FI




a)
$$\dot{x}_1(t) = x_2(t) - 2u(t)$$
$$\dot{x}_2(t) = -0{,}05x_1(t) - 0{,}6x_2(t) + 1{,}5u(t)$$
$$y(t) = x_2(t) + u(t)$$

b)
$$\underline{\underline{A}} = \begin{bmatrix} \emptyset & 1 \\ -0{,}05 & -0{,}6 \end{bmatrix} \quad \underline{B} = \begin{bmatrix} -2 \\ 1{,}5 \end{bmatrix} \quad \underline{C}^T = [0 \ 1] \quad D = 1$$

c)
$$h(t) = D \cdot \delta(t) + \varepsilon(t)\left(\underline{C}^T e^{\underline{\underline{A}}t} \underline{B}\right)$$
$$\lambda_1 = -0{,}1$$
$$\lambda_2 = -0{,}5$$
$$\underline{\underline{L}}_1 = \frac{\underline{\underline{A}} - \lambda_2 \underline{\underline{E}}}{\lambda_1 - \lambda_2} = \begin{bmatrix} 1{,}25 & 2{,}5 \\ -0{,}125 & -0{,}25 \end{bmatrix} \quad \underline{\underline{L}}_2 = \underline{\underline{E}} - \underline{\underline{L}}_1 = \begin{bmatrix} -0{,}25 & -2{,}5 \\ 0{,}125 & 1{,}25 \end{bmatrix}$$
$$\underline{C}^T \cdot e^{\underline{\underline{A}}t} \underline{B} = [0 \ 1]\left(e^{-0{,}1t} \cdot \begin{bmatrix} 1{,}25 & 2{,}5 \\ -0{,}125 & -0{,}25 \end{bmatrix} + e^{-0{,}5t} \cdot \begin{bmatrix} -0{,}25 & -2{,}5 \\ 0{,}125 & 1{,}25 \end{bmatrix}\right)\begin{bmatrix} -2 \\ 1{,}5 \end{bmatrix} =$$
$$= e^{-0{,}1t} \cdot [-0{,}125 \ \ -0{,}25]\begin{bmatrix} -2 \\ 1{,}5 \end{bmatrix} + e^{-0{,}5t} \cdot [0{,}125 \ \ 1{,}25]\begin{bmatrix} -2 \\ 1{,}5 \end{bmatrix} =$$
$$= -0{,}125 \cdot e^{-0{,}1t} + 1{,}625 e^{-0{,}5t}$$
$$\Longrightarrow h(t) = \delta(t) + \varepsilon(t)\left(-0{,}125 e^{-0{,}1t} + 1{,}625 e^{-0{,}5t}\right)$$

d) $u(t) = 2 \cdot \varepsilon(t)$
$$y(t) = \cancel{\underline{C}^T \cdot e^{\underline{\underline{A}}t} \underline{x}(-0)} + \int_{-0}^{t} e^{\underline{\underline{A}}(t-\tau)} \cdot \underline{B} \cdot u(\tau) d\tau + D u(\tau)$$
$\underline{C}^T \cdot$
$$y(t) = \int_{-0}^{t} \left(-0{,}125 e^{-0{,}1(t-\tau)} + 1{,}625 e^{-0{,}5(t-\tau)}\right) 2 \cdot \varepsilon(t) d\tau + 1 \cdot 2\varepsilon(t) =$$
$$= \int_{-0}^{t} -0{,}25 e^{-0{,}1(t-\tau)} + 3{,}25 e^{-0{,}5(t-\tau)} d\tau + 2\varepsilon(t) =$$
$$= -0{,}25 e^{-0{,}1t} \int_{-0}^{t} e^{+0{,}1\tau} d\tau + 3{,}25 e^{-0{,}5t} \int_{-0}^{t} e^{+0{,}5\tau} d\tau + 2\varepsilon(t) = -0{,}25 e^{-0{,}1t} \left[\frac{e^{+0{,}1\tau}}{+0{,}1}\right]_{-0}^{t} + 3{,}25 e^{-0{,}5t} \left[\frac{e^{+0{,}5\tau}}{+0{,}5}\right]_{-0}^{t}$$
$+2\varepsilon(t)$

$$=\left(0{,}25e^{-0{,}1t}\frac{e^{0{,}1t}-1}{0{,}1}+3{,}25e^{-0{,}5t}\frac{e^{0{,}5t}-1}{0{,}5}\right)\varepsilon(t)+2\varepsilon(t)=$$

$$=\left(-2{,}5+2{,}5e^{-0{,}1t}+6{,}5-6{,}5\cdot e^{-0{,}5t}\right)\varepsilon(t)+2\varepsilon(t)=\underline{\underline{\varepsilon(t)\left(6+2{,}5e^{-0{,}1t}-6{,}5e^{-0{,}5t}\right)}}$$

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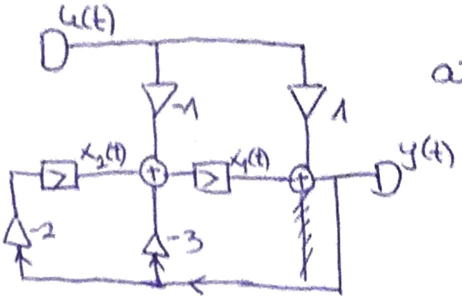
2.




a) $\dot{x}_1(t)=-3y(t)+x_2(t)-u(t)=-3x_1(t)+x_2(t)-4u(t)$

$\dot{x}_2(t)=-2y(t)=-2x_1(t)-2u(t)$

$y(t)=x_1(t)+u(t)$

$$\underline{\underline{A}}=\begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}\quad \underline{B}=\begin{bmatrix}-4\\ -2\end{bmatrix}\quad \underline{c}^T=[1\ 0]\quad D=1$$

b) $\lambda_1=-1\quad \lambda_2=-2\quad \underline{\underline{L}}_1=\dfrac{\begin{bmatrix}-3+2 & 1\\ -2 & 2\end{bmatrix}}{-1-(-2)}=\begin{bmatrix}-1 & 1\\ -2 & 2\end{bmatrix}\quad \underline{\underline{L}}_2=\begin{bmatrix}2 & -1\\ 2 & -1\end{bmatrix}$

$$\underline{c}^T e^{\underline{\underline{A}}t}\underline{B}=[1\ 0]\left(e^{-t}\begin{bmatrix}-1 & 1\\ -2 & 2\end{bmatrix}+e^{-2t}\begin{bmatrix}2 & -1\\ 2 & -1\end{bmatrix}\right)\begin{bmatrix}-4\\ -2\end{bmatrix}=$$

$$=2e^{-t}-6e^{-2t}$$

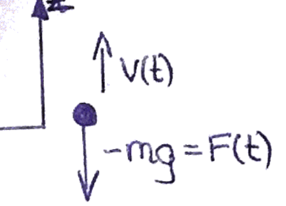
$\Rightarrow h(t)=\delta(t)+\varepsilon(t)\left(2e^{-t}-6e^{-2t}\right)$

c) $y(t)\overset{x(-0)=0}{=}\int_{-0}^{t}\underline{c}^T e^{\underline{\underline{A}}(t-\tau)}\underline{B}\cdot u(\tau)d\tau+Du(t)=\int_{-0}^{t}\left(2e^{-(t-\tau)}-6e^{-2(t-\tau)}\right)\cdot 2\cdot\varepsilon(\tau)d\tau+2\varepsilon(t)=$

$$=\int_{-0}^{t}4e^{\text{[redacted]}-(t-\tau)}d\tau+\int_{-0}^{t}-12e^{-2(t-\tau)}d\tau+2\varepsilon(t)=4e^{-t}\int_{-0}^{t}e^{\tau}-12e^{-2t}\int_{-0}^{t}e^{2\tau}d\tau+2\varepsilon(t)=$$

$$=\left(4e^{-t}\frac{e^{t}-1}{1}-12e^{-2t}\frac{e^{2t}-1}{2}\right)\varepsilon(t)+2\varepsilon(t)=\varepsilon(t)\left(\text{[redacted]}-4e^{-t}+6e^{-2t}\right)$$

3.




$$\dot{\underline{x}} = \begin{bmatrix} \dot{z} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} F(t) =$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} (-mg)$$

$$\underline{x}(-0) = \begin{bmatrix} 0 \\ v_0 \end{bmatrix}$$

$$\underline{x}(t) = e^{\underline{\underline{A}}t} \cdot \underline{x}(-0) + \int_{-0}^{t} e^{\underline{\underline{A}}(t-\tau)} \underline{B} \cdot u(\tau) d\tau = ⊛$$

$$\lambda_1 = 0$$
$$\lambda_2 = 0$$

$$e^{\underline{\underline{A}}t} = \underline{\underline{E}} + \underline{\underline{A}}t + \frac{1}{2!}\underline{\underline{A}}^2 t^2 + \frac{1}{3!}\underline{\underline{A}}^3 t^3 + \dots$$

$$\underline{\underline{A}}^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow e^{\underline{\underline{A}}t} = \underline{\underline{E}} + \underline{\underline{A}}t$$

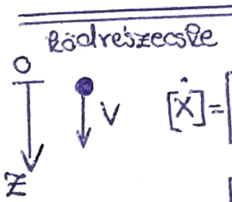
$$⊛ = \underbrace{(\underline{\underline{E}} + \underline{\underline{A}}t)}_{\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}} \underline{x}(-0) + \int_0^t (\underline{\underline{E}} + \underline{\underline{A}}t) \underline{B} \cdot u(\tau) d\tau = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ v_0 \end{bmatrix} + \int_0^t \begin{bmatrix} 1 & (t-\tau) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1/m \end{bmatrix} (-mg) d\tau =$$

$$= \begin{bmatrix} v_0 t \\ v_0 \end{bmatrix} + \int_0^t \begin{bmatrix} -g(t-\tau) \\ -g \end{bmatrix} d\tau = \underline{x}(t) = \varepsilon(t) \left( \begin{bmatrix} v_0 t \\ v_0 \end{bmatrix} + \begin{bmatrix} -\frac{g}{2} t^2 \\ -gt \end{bmatrix} \right)$$

$$\underline{x}(t) = \begin{bmatrix} z(t) \\ v(t) \end{bmatrix} \rightarrow z(t) = v_0 t - \frac{g}{2} t^2$$
$$v(t) = v_0 - gt$$

---

4.




$$[\dot{x}] = \begin{bmatrix} \dot{z} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -B/m \end{bmatrix} \begin{bmatrix} z \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} F(t)$$

$$\begin{bmatrix} \dot{z} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ a \end{bmatrix} \qquad \underline{\underline{A}} \rightarrow \lambda_1 = 0, \ \lambda_2 = -B/m \qquad F(t) \rightarrow mg$$

B - közegellenállási együttható

$$\underline{x}(-0) = \begin{bmatrix} H \\ 0 \end{bmatrix} \qquad \underline{\underline{L}}_1 = \begin{bmatrix} 1 & m/B \\ 0 & 0 \end{bmatrix} \qquad \underline{\underline{L}}_2 = \begin{bmatrix} 0 & -m/B \\ 0 & 1 \end{bmatrix}$$

$$\underline{x}(t) = \left( \underline{\underline{L}}_1 e^{\lambda_1 t} + \underline{\underline{L}}_2 e^{\lambda_2 t} \right) \underline{x}(-0) + \int_0^t \left\{ \underline{\underline{L}}_1 e^{-\lambda_1 t} + \underline{\underline{L}}_2 e^{-\lambda_2 t} \right\} \underline{B} \cdot u(\tau) d\tau =$$

$$= \begin{bmatrix} H \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{mg}{B} \\ 0 \end{bmatrix} t + \begin{bmatrix} \frac{mg}{B} \\ -g \end{bmatrix} (1 - e^{-Bt/m}) \rightarrow z(t) = H - \frac{mg}{B} t + \frac{mg}{B} (1 - e^{-Bt/m})$$

$m = 4{,}2 \cdot 10^{-12}$ kg

$B = 2{,}8 \cdot 10^{-9} \frac{kg}{s}$

# 5. gyakorlat

1. $h(t) = 3\delta(t) + \varepsilon(t)[8 \cdot e^{-0,5t} - 4e^{-0,1t}]$

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |3\delta(t)|\, dt + \int_{-\infty}^{\infty} |\varepsilon(t)(8e^{-0,5t} - 4e^{-0,1t})|\, dt \leq$$

$$\leq \int_{-\infty}^{\infty} |3\delta(t)|\, dt + \int_{-\infty}^{\infty} |8e^{-0,5t}|\, dt + \int_{-\infty}^{\infty} |-4e^{-0,1t}|\, dt = 3 + 8\left[\frac{e^{-0,5t}}{-0,5}\right]_0^{\infty} + 4\left[\frac{e^{-0,1t}}{-0,1}\right]_0^{\infty} = 3 + 8 \cdot 2 + 4 \cdot 10 = 59 < \infty$$

$\Rightarrow$ GV - stabil

2. $h(t) = \varepsilon(t) \cos(3t) \cdot e^{-0,5t}$

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |\varepsilon(t) \cos(3t) \cdot e^{-0,5t}|\, dt \leq \int_{0}^{\infty} |e^{-0,5t}|\, dt = \left[\frac{e^{-0,5t}}{-0,5}\right]_0^{\infty} = \frac{0-1}{-0,5} = 2 < \infty$$

$\Rightarrow$ GV-stabil

3. DI $\lambda^2 - m\lambda + 0,05 = 0$

$1 - m + 0,05 > 0 \rightarrow m < 1,05$

$1 + m + 0,05 > 0 \rightarrow m > -1,05$

$-1,05 < m < 1,05$

$|m| < 1,05$

$|0,05| < 1$

4. FI $\lambda^2 - m\lambda + 0,05 = 0$

$1 > 0$ ✓

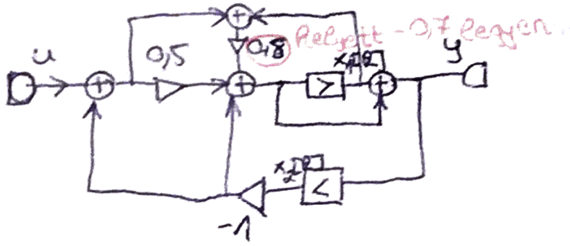
$-m > 0 \rightarrow m < 0$ ✓ $\Rightarrow$ akkor ASZ stabil $\Rightarrow$ GV stabil

$0,05 > 0$ ✓

5. FI $\dot{\underline{x}} = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} u$

$\rightarrow \lambda_1 = -2,\ \lambda_2 = 2 \Rightarrow$ nem ASZ stabil, GV stabilitás nem eldönthető

⑥




$x_1[k+1] = 0{,}5(u - x_2) + 0{,}8(u - x_2 + x_1) - x_2 = 0{,}8x_1 - 2{,}3x_2 + 1{,}3u$

$x_2[k+1] = x_1 + x_1[k+1] = 1{,}8x_1 - 2{,}3x_2 + 1{,}3u$

$y[k] = 1{,}8x_1 - 2{,}3x_2 + 1{,}3u$

$\underline{\underline{A}} = \begin{bmatrix} 0{,}8 & -2{,}3 \\ 1{,}8 & -2{,}3 \end{bmatrix} \quad \underline{B} = \begin{bmatrix} 1{,}3 \\ 1{,}3 \end{bmatrix} \quad \underline{C}^T = [1{,}8 \;\; -2{,}3] \quad D = 1{,}3$

$\begin{vmatrix} \lambda - 0{,}8 & 2{,}3 \\ -1{,}8 & \lambda + 2{,}3 \end{vmatrix} = \lambda^2 + 1{,}5\lambda + 2{,}3 \rightarrow \lambda_{1,2} = -0{,}75 \pm 1{,}32j$

$|\lambda_{1,2}| = \sqrt{0{,}75^2 + 1{,}32^2} > 1 \Rightarrow$ nem ASZ stabil

-0,7-es erősítővel

$x_1[k+1] = -0{,}7x_1 - 0{,}8x_2 - 0{,}2u$

$x_2[k+1] = 0{,}3x_1 - 0{,}8x_2 - 0{,}2u$

$y[k] = 0{,}3x_1 - 0{,}8x_2 - 0{,}2u$

$\underline{\underline{A}} = \begin{bmatrix} -0{,}7 & -0{,}8 \\ 0{,}3 & -0{,}8 \end{bmatrix} \quad \underline{B} = \begin{bmatrix} -0{,}2 \\ -0{,}2 \end{bmatrix} \quad \underline{C}^T = [0{,}3 \;\; -0{,}8] \quad D = -0{,}2$

$\Rightarrow$ karakt. egy.: $\lambda^2 + 1{,}5\lambda + 0{,}8 = 0$

$\lambda_{1,2} = -0{,}75 \pm 0{,}487i$

$|\lambda_{1,2}| = 0{,}894 < 1 \Rightarrow$ ASZ stabil

⑦ Előző feladat:

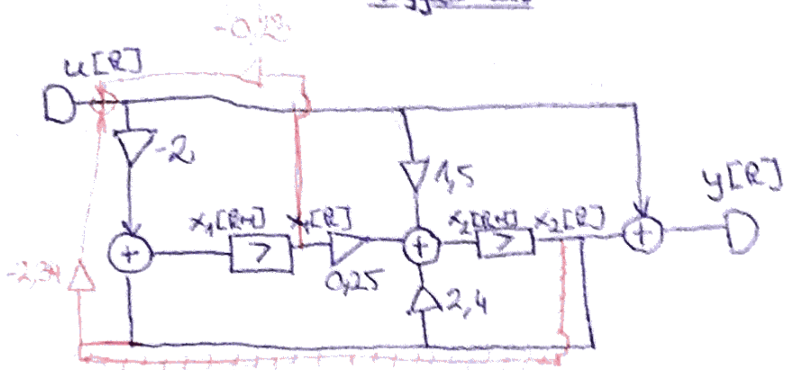
$u[k] = \varepsilon[k]$

| $k$ | $u[k]$ | $x_1[k]$ | $x_2[k]$ | $y[k]$ |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | -0,2 |
| 1 | 1 | -0,2 | -0,2 | -0,1 |
| 2 | 1 | 0,1 | 0,1 | -0,09 |
| 3 | 1 | | | |

# Rendszerelmélet
## 6. gyakorlat



(1.)




Megfigyelhető?
Irányítható

$x_1[k+1] = x_2[k] - 2u[k]$

$x_2[k+1] = 0{,}25x_1[k] + 2{,}4x_2[k+1] + 1{,}5u[k]$

$y[k] = x_2[k] + u[k]$

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 1 \\ 0{,}25 & 2{,}4 \end{bmatrix} \quad \underline{B} = \begin{bmatrix} -2 \\ 1{,}5 \end{bmatrix} \quad \underline{C}^T = [0 \ 1] \quad D = 1$$

$$\underline{\underline{M}}_o = \begin{bmatrix} 0 & 1 \\ 0{,}25 & 2{,}4 \end{bmatrix} \Rightarrow \det(\underline{\underline{M}}_o) = -0{,}25 \Rightarrow \text{megfigyelhető}$$

$$\underline{\underline{M}}_c = \begin{bmatrix} -2 & 1{,}5 \\ 1{,}5 & 3{,}1 \end{bmatrix} \Rightarrow \det(\underline{\underline{M}}_c) = -2 \cdot 3{,}1 - 1{,}5^2 = -8{,}45 \Rightarrow \text{irányítható}$$

$$\varphi_c(\lambda) = \prod_{i=1}^{n} (\lambda - \tilde{\lambda}_i) \overset{\text{Cayley-Hamilton}}{=} 0$$

$$\underline{k}^T = [00\ldots01]\underline{\underline{M}}_c^{-1} \cdot \varphi_c(\underline{\underline{A}})$$

a) Stabilitás? b) $\tilde{\lambda}_1 = -0{,}1$ $\tilde{\lambda}_2 = -0{,}5$ c) Ellenőrzés + hálózatkiegészítés

a) $-\lambda(2{,}4-\lambda) - 0{,}25 = \lambda^2 - 2{,}4\lambda - 0{,}25$

$$\lambda_{1,2} = \frac{2{,}4 \pm \overbrace{\sqrt{2{,}4^2+1}}^{2,6}}{2} \quad \begin{matrix} \nearrow \lambda_1 = 2{,}5 \\ \searrow \lambda_2 = -0{,}1 \end{matrix} \Bigg\} \Rightarrow \text{nem ASZ stabil}$$

b) $$\underline{\underline{M}}_c^{-1} = \frac{\operatorname{adj}(\underline{\underline{M}}_c)}{\det(\underline{\underline{M}}_c)} = \frac{\begin{bmatrix} 3{,}1 & -1{,}5 \\ -1{,}5 & -2 \end{bmatrix}}{-8{,}45} = \begin{bmatrix} -0{,}37 & 0{,}18 \\ 0{,}18 & 0{,}24 \end{bmatrix} \quad \left( \operatorname{adj}(\underline{\underline{M}}_c) = \begin{bmatrix} M_{22} & -M_{12} \\ -M_{21} & M_{11} \end{bmatrix} \right)$$

$$\varphi_c(\lambda) = (\lambda+0{,}1)(\lambda+0{,}5) = \lambda^2 + 0{,}6\lambda + 0{,}05$$

$$\varphi_c(\underline{\underline{A}}) = \underline{\underline{A}}^2 + 0{,}6\underline{\underline{A}} + 0{,}05\underline{\underline{E}} = \begin{bmatrix} 0{,}3 & 3 \\ 0{,}75 & 7{,}5 \end{bmatrix}$$

$$\underline{k}^T = [0 \ 1] \begin{bmatrix} -0{,}37 & 0{,}18 \\ 0{,}18 & 0{,}24 \end{bmatrix} \begin{bmatrix} 0{,}3 & 3 \\ 0{,}75 & 7{,}5 \end{bmatrix} = [0{,}23 \ \ 2{,}34]$$

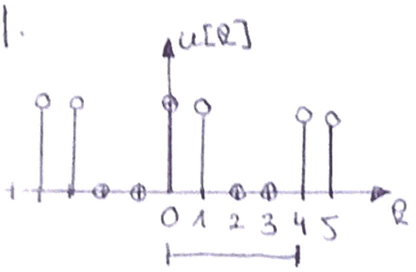
c) $\underline{\underline{\tilde{A}}} = \underline{\underline{A}} - \underline{B} \cdot \underline{Q}^T = \begin{bmatrix} 0 & 1 \\ 0,25 & 2,4 \end{bmatrix} - \begin{bmatrix} -2 \\ 1,5 \end{bmatrix} \begin{bmatrix} 0,23 & 2,34 \end{bmatrix} = \begin{bmatrix} 0,46 & 5,68 \\ -0,1 & -1,11 \end{bmatrix} = \underline{\underline{\tilde{A}}}$

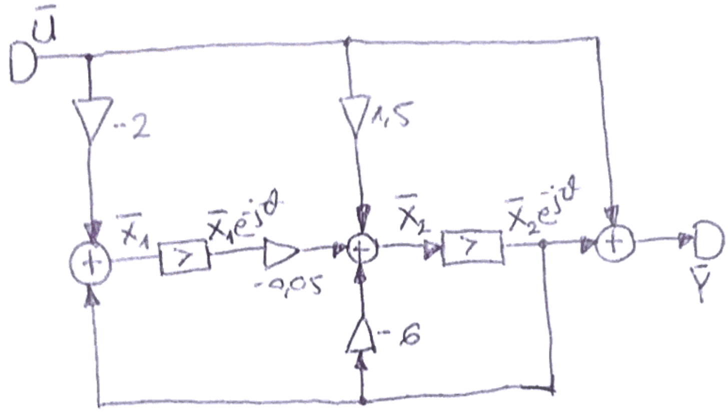
$\det|\underline{\underline{\tilde{A}}} - \lambda \underline{\underline{E}}| = \ldots = \begin{cases} \nearrow \lambda_1 = -0,11 \\ \searrow \lambda_2 = -0,54 \end{cases}$ előírt: $\lambda_1 = -0,1$, $\lambda_2 = -0,5$ ✓

# Rendszerelmélet

## 9. gyakorlat

(1.) Dl.




$L=4$

$M=\frac{L}{2}-1=1$

$$x[k]=X_0+\sum_{p=1}^{M}X_p\cos[p\Theta k+\xi_p]+X_{\frac{L}{2}}(-1)^k$$

$e^{j\alpha}=\cos\alpha+j\sin\alpha$

1. Sorfejtés

a) $U_p^c=\frac{1}{L}\sum_{k=0}^{3}u[k]\cdot e^{-jp\Theta k}=$ $\qquad \Theta=\frac{2\pi}{L}=\frac{\pi}{2}$

$$=\frac{1}{4}\sum_{k=0}^{3}u[k]\cdot e^{-jp\frac{\pi}{2}k}$$

$u[0]=1 \quad u[1]=1 \quad u[2]=0 \quad u[3]=0$

$$U_0^c=\frac{1}{4}(1+1+0+0)=\frac{1}{2}$$

$$U_1^c=\frac{1}{4}\left(e^{-j\frac{\pi}{2}\cdot 0}+e^{-j\frac{\pi}{2}\cdot 1}+0+0\right)=\frac{1}{4}-\frac{1}{4}j=\sqrt{2}\cdot 0{,}25e^{-j\frac{\pi}{4}}$$

$$U_2^c=\frac{1}{4}\left(1+e^{-j\cdot 2\cdot\frac{\pi}{2}\cdot 1}\right)=0$$

$U_0=\frac{1}{2} \quad U_1=2\cdot\left|\sqrt{2}\cdot 0{,}25\cdot e^{-j\frac{\pi}{4}}\right|=\sqrt{2}\cdot 0{,}5 \quad \xi_1=-\frac{\pi}{4} \quad U_2=0 \quad \xi_2=0$

$$u[k]=\frac{1}{2}+\sqrt{2}\cdot 0{,}5\cdot\cos\left(\frac{\pi}{2}k-\frac{\pi}{4}\right)$$

$$H(e^{j\vartheta})=\frac{\bar{Y}}{\bar{U}}=\frac{1+2{,}1e^{-j\vartheta}+0{,}15e^{-j2\vartheta}}{1+0{,}6e^{-j\vartheta}+0{,}05e^{-j\vartheta 2}}$$

$$H(e^{j\vartheta})\Big|_{\vartheta=0}=\frac{1+2{,}1+0{,}15}{1+0{,}6+0{,}05}=1{,}9\dot{6}$$

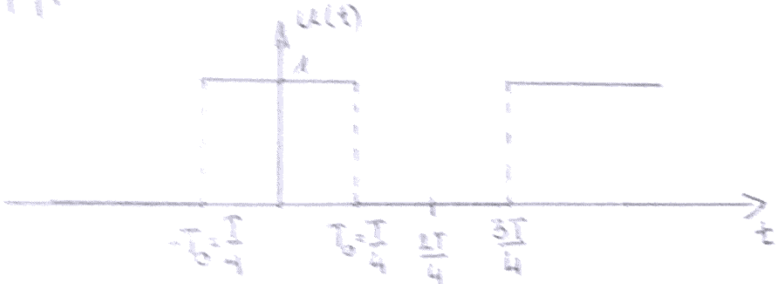
$$H(e^{j\vartheta})\Big|_{\vartheta=\frac{\pi}{2}}=\frac{1+2{,}1(-j)+0{,}15\cdot(-1)}{1+0{,}6(-j)-0{,}05}\approx 2{,}03\cdot e^{-j0{,}63}$$

$\bar{U}_0=\frac{1}{2} \quad \bar{U}_1=\frac{\sqrt{2}}{2}e^{-j\frac{\pi}{4}}$

$\bar{Y}_{\bullet}=\bar{U}_0\cdot H(e^{j\vartheta})\Big|_{\vartheta=0}+\bar{U}_1\cdot H(e^{j\vartheta})\Big|_{\vartheta=\frac{\pi}{2}}=0{,}984+1{,}44e^{-j1{,}42} \quad \Longrightarrow$

$y[k] = 0{,}384 + 1{,}44\cos\left(\frac{\pi}{2}k - 1{,}42\right)$

2) FI:




$$U_p^c = \frac{1}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}} \underbrace{u(t)}_{1}\, e^{-jp\Omega t}\,dt = \frac{1}{T}\left[\frac{e^{-jp\Omega t}}{-jp\Omega}\right]_{-T/4}^{T/4} = \frac{1}{-jp\Omega T}\left(e^{-jp\Omega\frac{T}{4}} - e^{jp\Omega\frac{T}{4}}\right) =$$

$$\Omega = \frac{2\pi}{T}$$

$$= \frac{1}{jp2\pi}\left(e^{jp\frac{\pi}{2}} - e^{-jp\frac{\pi}{2}}\right) = \frac{\sin\left(p\frac{\pi}{2}\right)}{p\pi} = \frac{1}{2}\,\frac{\sin\left(p\frac{\pi}{2}\right)}{p\frac{\pi}{2}}$$

$U_0^c = \frac{1}{2}$ $\quad U_0 = \frac{1}{2}$

$U_1^c = \frac{1}{\pi}$ $\quad U_1 = \frac{2}{\pi}$ $\quad \xi_1 = \varnothing$

$U_2^c = 0$ $\quad U_3 = \frac{2}{3\pi}$ $\quad \xi_3 = \pi$ $\quad \left(-\frac{1}{3\pi} = \frac{1}{3\pi}\cdot e^{j\pi}\right.$

$U_3^c = -\frac{1}{3\pi}$

$U_4^c = 0$ $\quad U_5 = \frac{2}{5\pi}$ $\quad \xi_5 = \varnothing$

$U_5^c = \frac{1}{5\pi}$

$$\tilde{u}(t) = \frac{1}{2} + \frac{2}{\pi}\cos(\Omega t) + \frac{2}{3\pi}\cos(3\Omega t + \pi) + \frac{2}{5\pi}\cos(5\Omega t)$$

# Rendszerelmélet

## 10. heti gyakorlat

Parseval-tétel:

$$P_x = \int_{\langle T \rangle} |x(t)|^2 dt = \sum_{p=-\infty}^{\infty} |X_p^c|^2$$

$$x(t) = \cos^2(\Omega t) = \frac{1}{2} + \frac{1}{2}\cos(2\Omega t) \rightarrow X_0^c = \frac{1}{2} \quad X_1^c = \frac{1}{4}$$

Spektrum: $P_x = \sum_{p=-1}^{1} |X_p^c|^2 = \left|\frac{1}{4}\right|^2 + \left|\frac{1}{2}\right|^2 + \left|\frac{1}{4}\right|^2 = \frac{3}{8}$

Időtartomány: $P_x = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$

$$x^2(t) = \left(\frac{1}{2} + \frac{1}{2}\cos(2\Omega t)\right)^2 = \frac{1}{4} + \frac{1}{2}\cos(2\Omega t) + \frac{1}{4}\cos^2(2\Omega t) =$$

$$= \frac{1}{4} + \frac{1}{2}\cos(2\Omega t) + \frac{1}{8} + \frac{1}{8}\cos(4\Omega t) = \frac{3}{8} + \frac{1}{2}\cos(2\Omega t) + \frac{1}{8}\cos(4\Omega t)$$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)dt = \frac{1}{2\pi}\int_{-\pi}^{\pi}\Big[\frac{3}{8} + \underbrace{\frac{1}{2}\cos(2\Omega t)}_{\emptyset} + \underbrace{\frac{1}{8}\cos(4\Omega t)}_{\emptyset}\Big]dt =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{3}{8}dt = \frac{3}{8}$$

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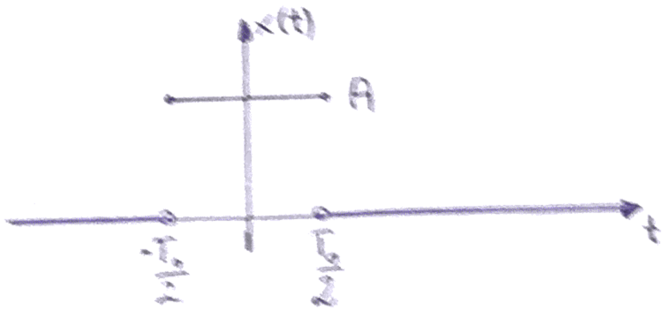



$$X(j\omega) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = A\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} e^{-j\omega t} dt = A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}} =$$

$$= A\left(\frac{e^{-j\omega\frac{T_0}{2}} - e^{j\omega\frac{T_0}{2}}}{-j\omega}\right) = 2\frac{T_0}{2}A\frac{e^{j\omega\frac{T_0}{2}} - e^{-j\omega\frac{T_0}{2}}}{2j\omega\frac{T_0}{2}} = T_0 A\frac{\sin(\omega\frac{T_0}{2})}{\omega\frac{T_0}{2}}$$

$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(j\frac{\omega}{a}\right)$$

$\frac{T_0}{2}$ széles impulzus spektruma: $X_{\frac{T_0}{2}}(j\omega) = \frac{T_0 A}{2} \frac{\sin\left(\omega \frac{T_0}{4}\right)}{\omega \frac{T_0}{4}}$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = A^2 T_0$$

---

$T_0$ széles impulzus

$$u(t) = x(t) \cdot \cos(\omega_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2}\left[X(j(\omega-\omega_0)) + \frac{1}{2} X(j(\omega+\omega_0))\right)$$

$$\mathcal{F}\{u(t)\} = U(j\omega) = \frac{1}{2} T_0 A \frac{\sin\left((\omega-\omega_0)\frac{T_0}{2}\right)}{(\omega-\omega_0)\frac{T_0}{2}} + \frac{1}{2} T_0 A \frac{\sin\left((\omega+\omega_0)\frac{T_0}{2}\right)}{(\omega+\omega_0)\frac{T_0}{2}}$$

---

$$f(t) = \varepsilon(t) e^{-\alpha t} \quad (\alpha > 0)$$

10 % $\varepsilon = 0{,}1$

$$F(j\omega) = \int_0^{\infty} e^{-\alpha t} \cdot e^{-j\omega t} dt = \int_0^{\infty} e^{-(\alpha+j\omega)t} dt = \left[\frac{e^{-(\alpha+j\omega)t}}{-(\alpha+j\omega)}\right]_0^{\infty} = \frac{1}{\alpha+j\omega}$$

$$|F(j\omega)| = \frac{1}{\sqrt{\alpha^2+\omega^2}}$$

Max: $\omega = 0$-nál $= \frac{1}{\sqrt{\alpha^2}} = \frac{1}{\alpha}$

$$|F(j\omega_2)| = \varepsilon \cdot |F(j\omega)|_{max} = 0{,}1 \frac{1}{\alpha} \qquad \frac{1}{\sqrt{\alpha^2+\omega_2^2}} = \varepsilon \cdot \frac{1}{\alpha} \Rightarrow$$

$$\Rightarrow \omega_2 = \sqrt{99\alpha^2} \approx 10\alpha$$

Sávszélesség: $B = |\omega_0 - \omega_2| = 10\alpha$

---

$$x(t) = \cos^2(\omega_0 t) = \cos(\omega_0 t) \cdot \cos(\omega_0 t)$$

$$\mathcal{F}\{\cos^2(\omega_0 t)\} = \mathcal{F}\left\{\frac{1}{2}(1+\cos(2\omega_0 t))\right\} = \frac{1}{2}\left[2\pi\delta(\omega) + \pi\delta(\omega-2\omega_0) + \pi\delta(\omega+2\omega_0)\right]$$

# Rendszerelmélet

## 12. Reti gyakorlat

$$H(s) = \frac{1}{s^2+2s+2} = \frac{Y(s)}{U(s)} \quad \text{stabil} = ? \rightarrow \text{GV-stab.}$$

$$R(t) = ?$$

$$\mathrm{Re}\{p_i\} < 0$$

$$H(s) = \frac{1}{s^2+2s+2} = \frac{1}{(s-(-1+j))(s-(-1-j))} = \frac{A}{s-(-1+j)} + \frac{A^*}{s-(-1-j)}$$

$$p_{1,2} = \frac{-2 \pm \sqrt{4-8}}{2} \begin{matrix} \nearrow -1+j \\ \searrow -1-j \end{matrix} \Bigg\} \Rightarrow \text{GV-stabil}$$

$$A = \frac{1}{s-(-1-j)}\Bigg|_{s=-1+j} = \frac{1}{-1+j-(-1-j)} = \frac{1}{2j} = -\frac{1}{2}j = \frac{1}{2}e^{-j\frac{\pi}{2}}$$

$$A^* = \frac{1}{2}e^{+j\frac{\pi}{2}}$$

$$H(s) = \frac{\frac{1}{2}e^{-j\frac{\pi}{2}}}{s-(-1+j)} + \frac{\frac{1}{2}e^{+j\frac{\pi}{2}}}{s-(-1-j)}$$

$$x(t) = \varepsilon(t)\, 2|A| e^{\alpha t}\cos(\omega t + \varphi) \Rightarrow R(t) = \varepsilon(t) e^{-t}\cos\left(t - \frac{\pi}{2}\right)$$

$$A = |A|e^{j\varphi}$$

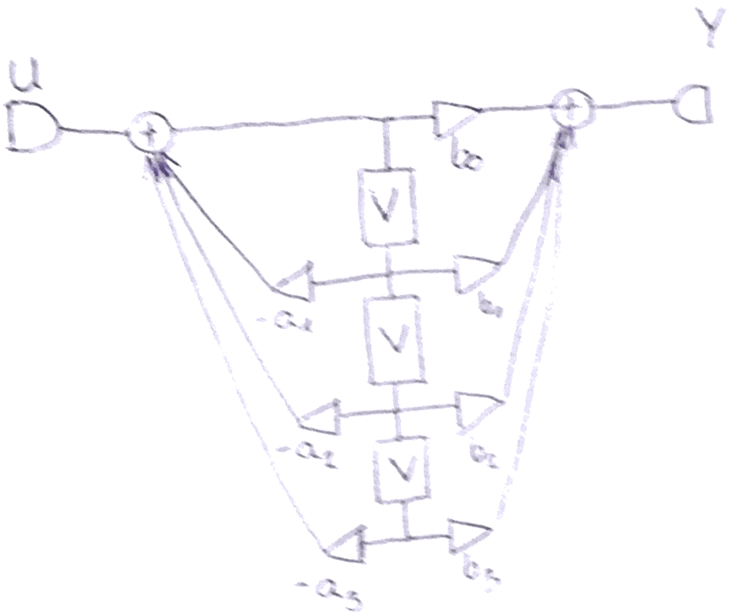
$$p = \alpha + j\omega$$

$$\alpha = -1$$

$$\omega = 1$$

$$|A| = \frac{1}{2}$$

$$\varphi = -\frac{\pi}{2}$$




$a_0 = 1 \qquad b_0 = 1$

$a_1 = 5 \qquad b_1 = 8$

$a_2 = 6 \qquad b_2 = 13$

$a_3 = 0 \qquad b_3 = 0$

$$H(j\omega) = \frac{b_0 (j\omega)^n + b_1 (j\omega)^{n-1} + \dots + b_n}{(j\omega)^n + a_1 (j\omega)^{n-1} + \dots + a_n}$$

$$H(j\omega) = \frac{(j\omega)^2 + 8j\omega + 13}{(j\omega)^2 + 5j\omega + 6}$$

$$H(s) = \frac{s^2 + 8s + 13}{s^2 + 5s + 6}$$

$s^2 + 5s + 6 = 0$

$$\frac{-5 \pm \sqrt{25-24}}{2} \left.\begin{array}{l} \nearrow -2 \\ \searrow -3 \end{array}\right\} \text{GW}$$

$$u(t) = \varepsilon(t-3) \longrightarrow \tilde{u}(t) = \varepsilon(t) \longrightarrow \mathcal{L}\{\tilde{u}(t)\} = \frac{1}{s}$$

$$Y(s) = H(s) \cdot U(s) = \frac{s^2 + 8s + 13}{s^2 + 5s + 6} \cdot \frac{1}{s} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$A = \frac{13}{6}$

$B = -\frac{1}{2}$

$C = -\frac{2}{3}$

$$\Rightarrow \tilde{y}(t) = \varepsilon(t) \left( \frac{13}{6} - \frac{1}{2} e^{-2t} - \frac{2}{3} e^{-3t} \right)$$

$y(t) = \tilde{y}(t-3)$

$$f_a[k] = k \cdot 0{,}5^k \cdot \varepsilon[k-4] = (k-4+4)\,\underbrace{0{,}5^{k-4+4}}_{0{,}5^{k-4} \cdot 0{,}5^4} \cdot \varepsilon[k-4] =$$

$$= (k-4) \cdot \underbrace{0{,}5^4 \cdot 0{,}5^{k-4}}_{0{,}5^k \cdot 0{,}5^{k-5}} \cdot \varepsilon[k-4] + 4 \cdot 0{,}5^4 \cdot 0{,}5^{k-4} \cdot \varepsilon[k-4]$$

$$\Rightarrow F_a(z) = 0{,}5^5 \cdot z^{-4} \cdot \frac{z}{(z-0{,}5)^2} + 4 \cdot 0{,}5^4 \cdot z^{-4} \frac{z}{z-0{,}5}$$

$$f_b[k] = \varepsilon[k] \cdot 0{,}7^k \cdot \cos[5k]$$

$$F_b(z) = \frac{1 - 0{,}7\cos(5)z^{-1}}{1 - 2 \cdot 0{,}7\cos(5)z^{-1} + 0{,}49z^{-2}}$$

$$F_a(z) = \frac{1 - z^{-1} + z^{-2}}{1 - z^{-1} + 0{,}5z^{-2}} = \frac{z^2 - z + \overset{0{,}5+0{,}5}{1}}{z^2 - z + 0{,}5} = 1 + \frac{0{,}5}{z^2 - z + 0{,}5} =$$

$$= 1 + \frac{0{,}5}{(z-(0{,}5+j0{,}5))(z-(0{,}5-j0{,}5))} = 1 + \frac{A}{z-(0{,}5+j0{,}5)} + \frac{A^*}{z-(0{,}5-j0{,}5)} =$$

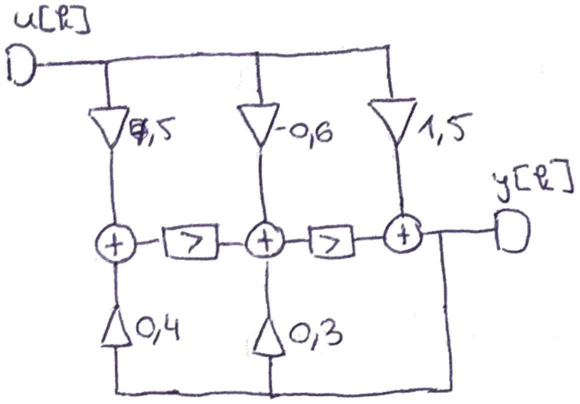
$$A = -0{,}5j \quad A^* = +0{,}5j$$

$$= 1 + \frac{-0{,}5j}{z-(0{,}5+j0{,}5)} + \frac{0{,}5j}{z-(0{,}5-j0{,}5)} = 1 + z^{-1}\left(\frac{-0{,}5jz}{z-(0{,}5+j0{,}5)} + \frac{0{,}5jz}{z-(0{,}5-j0{,}5)}\right)$$

$$f_a[k] = \delta[k] + \varepsilon[k-1]\left(-0{,}5j \cdot \underbrace{(0{,}5+j0{,}5)}_{0{,}5\sqrt{2}e^{j\frac{\pi}{4}}}{}^{k-1} + 0{,}5j\underbrace{(0{,}5-j0{,}5)}_{0{,}5\sqrt{2}e^{-j\frac{\pi}{4}}}{}^{k-1}\right) =$$

$$= \delta[k] + \varepsilon[k-1]\left((0{,}5\sqrt{2})^{k-1}\left(-0{,}5je^{+j\frac{\pi}{4}(k-1)} + 0{,}5je^{-j\frac{\pi}{4}(k-1)}\right)\right) =$$

$$= \delta[k] + \varepsilon[k-1]\left((0{,}5\sqrt{2})^{k-1}\sin[\tfrac{\pi}{4}(k-1)]\right)$$




$$H(z)=\frac{1,5-0,6z^{-1}+7,5z^{-2}}{1-0,3z^{-1}-0,4z^{-2}}=\frac{1,5z^2-0,6z+7,5}{z^2-0,3z-0,4}=\frac{1,5z^2-0,6z+7,5}{(z-0,8)(z+0,5)}$$

$$\left.\begin{matrix} p_1=0,8 \\ p_2=-0,5 \end{matrix}\right\} \text{GV stabil} \Rightarrow \text{ASZ}$$

(hálózatból)

# Rendszerelmélet

## 14. Heti gyakorlat

$$x(t) = a \cdot \cos(\omega t + \varphi)$$

AM - DSB

AM - DSB - SC

$$s_v(t) = \cos(\omega_v t)$$

$$s_m(t) = 2 \cdot \cos(\omega_1 t) + \cos(\omega_2 t)$$

$$s_{sc}(t) = s_v(t) \cdot s_m(t) = \frac{1}{2} \cdot (2 \cdot \cos[(\omega_v + \omega_1)t] + 2\cos[(\omega_v - \omega_1)t] + \cos[(\omega_v + \omega_2)t] +$$

$$+ \cos[(\omega_v - \omega_2)t])$$

$$S_{sc}(j\omega) = 2\pi \cdot \Big( \frac{\delta(\omega - (\omega_v + \omega_1)) + \delta(\omega + (\omega_v + \omega_1))}{2} + \frac{\delta(\omega - (\omega_v - \omega_1)) + \delta(\omega - (\omega_v - \omega_1))}{2}$$

$$+ \frac{1}{2} \cdot \frac{\delta(\omega - (\omega_v + \omega_2)) + \delta(\omega + (\omega_v + \omega_2))}{2} + \frac{1}{2} \cdot \frac{\delta(\omega - (\omega_v - \omega_2)) + \delta(\omega + (\omega_v - \omega_2))}{2}$$

---

1. $R(t) = \varepsilon(t)(5 \cdot e^{-2t} + 2e^{-4t})$ → lecsengő ⇒ GV

Ha kauzális

a) $H(j\omega) = ?$

$$R(t) \to H(s) \to H(j\omega)$$

GV✓

$$\Rightarrow H(s) = \frac{5}{s+2} + \frac{2}{s+4} = \frac{5s + 20 + 2s + 4}{s^2 + 6s + 8} =$$

$$= \frac{7s + 24}{s^2 + 6s + 8} \Rightarrow H(j\omega) = \frac{7(j\omega) + 24}{(j\omega)^2 + 6(j\omega) + 8}$$

b) $u(t) = 5 + 5 \cdot \cos(4t)$

$y(t) = ?$

$$H(j0) = \frac{7(j0) + 24}{(j0)^2 + 6(j0) + 8} = 3 \qquad H(j4) = \frac{28j + 24}{-16 + 24j + 8} \approx \frac{36{,}88 e^{j0{,}86}}{25{,}3 e^{j1{,}89}} \approx 1{,}46 e^{-j1{,}03}$$

$$U_0 = 5$$

$$U_1 = 5 \cdot e^{j0}$$

$$Y_0 = U_0 \cdot H(j0) = 15$$

$$Y_1 = U_1 \cdot H(j4) = 5 \cdot e^{j0} \cdot 1{,}46 e^{-j1{,}03} = 7{,}3 e^{-j1{,}03}$$

$$\Bigg\} \; y(t) = 15 + 7{,}3\cos(4t - 1{,}03)$$

c) $u(t) = \varepsilon(t)\cdot e^{-4t}$

$y(t) = ?$

$$GV \Rightarrow H(s) = \frac{7s+24}{(s+2)(s+4)} \quad , U(s) = \frac{1}{s+4}$$

$$Y(s) = U(s)\cdot H(s) = \frac{7s+24}{(s+2)(s+4)^2} = \frac{C_1}{s+2} + \frac{C_2}{(s+4)^2} + \frac{C_3}{s+4}$$

$$C_1 = \left.\frac{7s+24}{(s+4)^2}\right|_{s=-2} = 2,5$$

$$C_2 = \left.\frac{7s+24}{s+2}\right|_{s=-4} = 2$$

$$C_3: \quad 7s+24 = 2,5(s+4)^2 + 2(s+2) + C_3(s+4)(s+2)$$

$$7s+24 = \underbrace{(2,5+C_3)}_{=\varnothing}s^2 + \underbrace{(22+6\cdot C_3)}_{=7}s + \underbrace{(44+8C_3)}_{=24}$$

$$\Rightarrow 2,5 + C_3 = \varnothing \rightarrow C_3 = -2,5$$

$$\text{Ell.: } 22 + 6(-2,5) \overset{?}{=} 7$$

$$7 = 7$$

$$44 - 8\cdot 2,5 = 24$$

$$24 = 24$$

$$Y(s) = \frac{2,5}{s+2} + \frac{2}{(s+4)^2} - \frac{2,5}{s+4}$$

$$y(t) = 2,5\,\varepsilon(t)\cdot e^{-2t} + 2\varepsilon(t)\cdot t\cdot e^{-4t} - 2,5\cdot\varepsilon(t)\cdot e^{-4t}$$

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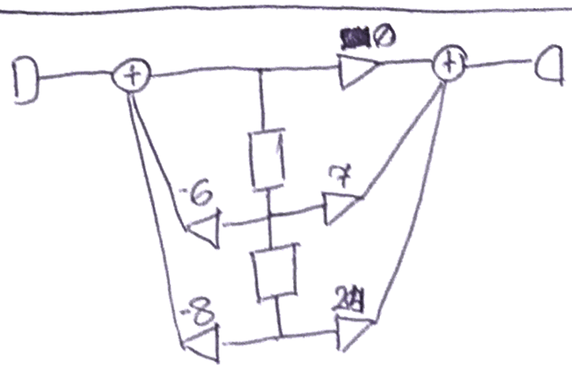
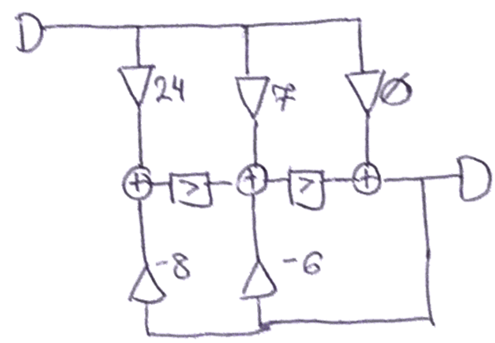
d)



2. $H(e^{j\vartheta}) = \dfrac{1-e^{-2j\vartheta}}{1-0{,}25\,e^{-2j\vartheta}} \rightarrow \dfrac{z^2-1}{z^2-0{,}25}$

$p_{1,2}: \; z^2-0{,}25=0$

$p_1=0{,}5 \quad p_2=-0{,}5$

a) stabilitás?

DI: $|p_i|<1$ ✓

$\Rightarrow$ GV

realizáció nem ismert $\Rightarrow$ ASZ ismeretlen

b) $u[k]=2\delta[k]+2\delta[k-1]$

$y[k] \rightarrow Y(e^{j\vartheta}) = ? =$

$= 2\cdot H(e^{j\vartheta}) + 2H(e^{j\vartheta})\cdot e^{-j\vartheta} =$

$= 2\cdot\dfrac{1-e^{-2j\vartheta}}{1-0{,}25e^{-2j\vartheta}} + 2\cdot\dfrac{1-e^{-2j\vartheta}}{1-0{,}25e^{-2j\vartheta}}\cdot e^{-j\vartheta} = 2\left(\dfrac{1-e^{-2j\vartheta}+e^{-j\vartheta}-e^{-3j\vartheta}}{1-0{,}25e^{-2j\vartheta}}\right)$

c) $u[k]=2\delta[k]+2\cdot\delta[k-1] \quad 0\le k\le 3$

$u[k+4]=u[k]$

$L=4 \rightarrow \Theta=\frac{2\pi}{L}=\frac{\pi}{2} \quad M=\frac{L}{2}-1=1$

$u[k]=U_0+\sum_{p=1}^{M} U_p\cdot\cos(p\Theta k+\xi_p)+U_{\frac{L}{2}}\cdot(-1)^k$

$U_2$

$U_p^c=\frac{1}{L}\sum_{k=\langle L\rangle} u[k]\cdot e^{-jp\Theta k}$

$U_0=U_0^c$

$U_p=2|U_p^c|$

$\xi_p=\mathrm{arc}\{U_p^c\}$

$U_{\frac{L}{2}}=U_{\frac{L}{2}}^c$

$U_0^c=\frac{1}{4}\left(u[0]+u[1]+u[2]+u[3]\right)=\frac{1}{4}(2+2+\varnothing+\varnothing)=1$

$U_1^c=\frac{1}{4}\left(2\cdot e^{-j\varnothing}+2\cdot e^{-j\frac{\pi}{2}}\right)=\frac{1}{2}-\frac{1}{2}j=\frac{\sqrt{2}}{2}e^{-j\frac{\pi}{4}}$

$U_{\frac{L}{2}}^c=U_2^c=\frac{1}{4}(2+2e^{-j\pi})=\varnothing$

$U_0=U_0^c=1$

$U_1=2|U_1^c|=2\cdot\left|\frac{\sqrt{2}}{2}e^{-j\frac{\pi}{4}}\right|=\sqrt{2} \quad \xi_1=-\frac{\pi}{4}$

$U_2=\varnothing$

$u[k]=1+\sqrt{2}\cos\left(\frac{\pi}{2}k-\frac{\pi}{4}\right)+\varnothing$

$Y_1=1\cdot\dfrac{1-1}{1-0{,}25}=\varnothing$

$Y_2=\bar{U}_1\cdot\dfrac{1-e^{-j\pi}}{1-0{,}25e^{-j\pi}}=\dfrac{2}{1{,}25}=1{,}6$

$\Rightarrow y[k]=1{,}6\cdot\sqrt{2}\cos\left(\frac{\pi}{2}k-\frac{\pi}{4}\right)$

d) $h[k] = ? = \mathcal{Z}^{-1}\{H(z)\}$

$$H(z) = \frac{z^2-1}{z^2-0{,}25} = \frac{z^2-0{,}25-0{,}75}{z^2-0{,}25} = 1 + \frac{-0{,}75}{z^2-0{,}25} = 1 + \frac{A}{z+0{,}5} + \frac{B}{z-0{,}5}$$

$$A = \left.\frac{-0{,}75}{z-0{,}5}\right|_{z=-0{,}5} = 0{,}75 \quad B = -0{,}75$$

$$\Rightarrow H(z) = 1 + \left(\frac{0{,}75z}{z+0{,}5} + \frac{-0{,}75z}{z-0{,}5}\right)z^{-1} \Rightarrow \delta[k] + 0{,}75 \cdot \varepsilon[k-1](-0{,}5)^{k-1} - 0{,}75 \cdot \varepsilon[k-1]\cdot(0{,}5)^{k-1}$$

e) kauzalitás?

kauzális, nincs $k+\dots$-os tag

0-ban és pozitív tagban belépő

Kiskérdések:

1) $h[k] = \delta[k+2] + 10 \cdot \varepsilon[k] \cdot (0{,}2)^k$

$\hookrightarrow$ nem kauzális $\Rightarrow \nexists H(z)$

2) $x(t) = 5\varepsilon(t)e^{-t} \xrightarrow{\mathcal{F}} \frac{5}{1+j\omega}$ $\qquad \varepsilon(t)\cdot e^{-\alpha t} \to \frac{1}{\alpha+j\omega}$

$B = ?$

$\frac{|X(j\omega)|\big|_{max}}{50}$ $\qquad |X(j\omega)| = \frac{5}{\sqrt{1^2+\omega^2}}$ $\quad |X(j\omega)|\big|_{max} = \frac{5}{1}$

$\omega_2 \quad \frac{5}{\sqrt{1^2+\omega_2^2}} = \frac{5}{50}$

$1+\omega_2^2 = 2500$

$\omega_2 = \pm 49{,}99$

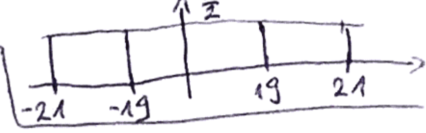
$B = \omega_2 - \omega_1 = 49{,}99$

$* = -1 \cdot \cos(t)\sin(20t) =$ $\qquad \cos(y)\cdot\sin(x) = \frac{\sin(x-y)+\sin(x+y)}{2}$

$= -1 \cdot \frac{\sin(19t)+\sin(21t)}{2}$

$U(j\omega) = -1 \cdot \frac{1}{2}\left(2\pi \cdot \frac{\delta(\omega-19)-\delta(\omega+19)}{2j} + 2\pi\frac{\delta(\omega-21)-\delta(\omega+21)}{2j}\right)$

$= -\frac{\pi}{2j}\left(\delta(\omega-19)-\delta(\omega+19)+\delta(\omega-21)-\delta(\omega+21)\right)$




3) $H(j\omega) = \frac{j\omega+2}{j\omega+4}$ $\qquad u(t) = 5 \cdot \varepsilon(t) \cdot e^{-2t}$ $\qquad y(t) = ?$

$\hookrightarrow \frac{5}{2+j\omega} = U(j\omega)$ $\quad Y(j\omega) = \frac{5j\omega+10}{(j\omega)^2+6(j\omega)+8} = \frac{5}{j\omega+4} \to$

$\to y(t) = 5 \cdot \varepsilon(t) \cdot e^{-4t}$

4) $x(t)$ belépő $X(s) = \frac{6}{s^2+s+4}$ $\quad \lim_{t\to\infty} x'(t) = ?$ $\quad x'(t) \to sX(s) + x(-0)$ $\quad \lim_{s\to\infty} s^2X(s) = \dots$

(with $x(-0) = \varnothing$)

5) $u(t) = \cos(t+\pi)\cdot\sin(20t)$ $\quad *$