Kalkulus vizsga 3

2023. jún. 19.

1.) G. elim:
(15p)

$$\begin{pmatrix} 4 & 4 & -1 \\ -4 & 1 & 0 \\ 8 & 13 & -3 \end{pmatrix} \sim \begin{pmatrix} 4 & 4 & -1 \\ 0 & 5 & -1 \\ 0 & 5 & -1 \end{pmatrix} \sim \begin{pmatrix} 4 & 4 & -1 \\ 0 & 5 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

II + I (3)
III − 2·I (3)

III − II (3)

rang: $r(A) = 2$
G. elim. után nem nulla sorok száma (2)

(2) det $A = 4 \cdot 5 \cdot 0 = 0$
G. elim. után a főátlóbeli elemek szorzata

(2) Mivel det $A = 0 \Rightarrow \nexists\, A^{-1}$.

2.)
(15p)

$$\lim_{n\to+\infty} \left(\frac{n-2}{n+4}\right)^{3+n} = \lim_{n\to+\infty} \left(\frac{n-2}{n+4}\right)^{3} \cdot \left(\frac{n-2}{n+4}\right)^{n} = \quad (2)$$

(2)

$$= \lim \underbrace{\left(\frac{1-\frac{2}{n}}{1-\frac{4}{n}}\right)^{3}}_{\to \left(\frac{1}{1}\right)^3 = 1} \cdot \frac{\left(1-\frac{2}{n}\right)^{n}}{\left(1+\frac{4}{n}\right)^{n}} = 1 \cdot \frac{e^{-2}}{e^{4}} = e^{-2-4} = e^{-6} \quad (3)$$

(2) $\frac{2}{n} \to 0$, $\frac{4}{n} \to 0$

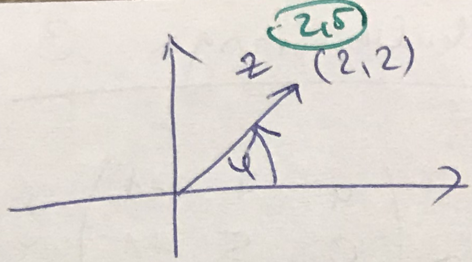
(2) $\left(1-\frac{2}{n}\right)^{n} \to e^{-2}$

(2) $\left(1+\frac{4}{n}\right)^{n} \to e^{4}$

$$\left(1+\frac{x}{n}\right)^{n} \longrightarrow e^{x} \quad (2)$$

1A

3.) (15p) $z = 2i + 2$



$\varphi = \frac{\pi}{4} = 45°$ 2

$r = \sqrt{2^2+2^2} = \sqrt{8}$ 2

trigo. alak: 2

$z = \sqrt{8}\cdot\left(\cos\frac{\pi}{4} + i\cdot\sin\frac{\pi}{4}\right) = r\cdot(\cos\varphi + i\sin\varphi)$

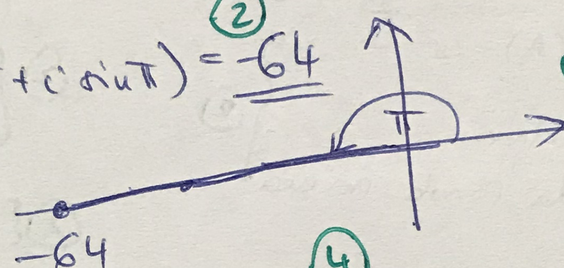
2

$z^4 = r^4\cdot(\cos 4\varphi + i\cdot\sin 4\varphi) = \sqrt{8}^4\cdot(\cos\pi + i\cdot\sin\pi)$

2

$z^4 = 64\cdot(\cos\pi + i\sin\pi) = \underline{\underline{-64}}$

2,5



4

4.) (15p) $\lim\limits_{x\to -5}\dfrac{x^2+x-20}{2x+10} = \lim\limits_{x\to -5}\dfrac{2x+1}{2} = \underline{\underline{-\frac{9}{2}}}$ 3

behely: $\dfrac{-10+1}{2}$

behely: $\dfrac{25-5-20}{-10+10} = \dfrac{0}{0} \Rightarrow L'H$ 4

4

5.) (20p) $f(x) = \dfrac{x}{(1-2x)^2}$ $D_f = R \setminus \{\frac{1}{2}\}$ 2

hányados der: $\dfrac{f'g - fg'}{g^2}$

$f'(x) = \dfrac{1\cdot(1-2x)^{\cancel{2}} - x\cdot 2\cancel{(1-2x)}\cdot(-2)}{(1-2x)^{\cancel{4}\,3}} = \dfrac{1-2x+4x}{(1-2x)^3}$

$f'(x) = \dfrac{1+2x}{(1-2x)^3}$ 3

$f''(x) = \dfrac{2\cdot(1-2x)^{\cancel{3}\,1} - (1+2x)\,3\cdot\cancel{(1-2x)^2}\cdot(-2)}{(1-2x)^{\cancel{6}\,4}} =$

$= \dfrac{2(1-2x) - (1+2x)\cdot(-6)}{(1-2x)^4} = \dfrac{2-4x+6+12x}{(1-2x)^4} = \dfrac{8x+8}{(1-2x)^4}$

Tehát $f''(x) \overset{4}{=} \dfrac{8(x+1)}{(1-2x)^4} \overset{?}{=} 0$

$\Updownarrow$

$x=-1$ (1)

Konvexitás táblázat:

D_f : -1, $1/2$ (2)

	$(-\infty,-1)$	$x=-1$	$(-1,1/2)$	$x=1/2$	$(1/2,+\infty)$
f''	$\ominus$	0	$\oplus$	X	$\oplus$
f	$\cap$ konkáv	inflexiós pont	$\cup$ konvex	X	$\cup$ konvex

(8)

parciális integrálás: $\int f\cdot g' = f\cdot g - \int f'\cdot g$ (3)

6.) (20p) $\int \underbrace{(5+x)}_{f(x)}\cdot \underbrace{e^{2x}}_{g'(x)} dx = \underbrace{(5+x)\cdot \frac{e^{2x}}{2}}_{f\cdot g} (2) - \int \underbrace{1\cdot \frac{e^{2x}}{2}}_{f'\cdot g} (2) dx \ominus$

(1) $f(x)$	$g'(x)$ (1)
$f'(x)=1$ (2)	$g(x)=\int e^{2x} = \frac{e^{2x}}{2}$ (3)

lin. hely. int. $\int f(ax+b) = \frac{1}{a}\cdot F(ax+b)$ (2), $\int f = F$.

$\ominus (5+x)\cdot\frac{e^{2x}}{2} - \underbrace{\int \frac{e^{2x}}{2} dx}_{\frac{e^{2x}}{4}} (3) = (5+x)\cdot\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + C$ (1)

7.) $\lim\limits_{n\to+\infty} \dfrac{n^5-2n^2-5n^7}{1-100n^2-n^3}$ dom (2) dom (2) $= \lim \boxed{\dfrac{n^7}{n^3}} \cdot \dfrac{\frac{1}{n^2}-\frac{2}{n^5}-5}{\frac{1}{n^3}-\frac{100}{n}-1} =$ (4)

$\frac{n^7}{n^3} = n^4 \to +\infty$ (3); $\frac{1}{n^2}\to 0$, $\frac{2}{n^5}\to 0$, $\frac{1}{n^3}\to 0$, $\frac{100}{n}\to 0$

$= +\infty \cdot \dfrac{-5}{-1} = +\infty\cdot 5 = \underline{\underline{+\infty}}$

(2) (2)