1, $\boxed{10}$ $y' + (\sin x)\, y = \sin x$ $\qquad y(0) = 3$

$y \equiv 1$ mo.

Szeparálható: $\dfrac{dy}{dx} = \sin x\,(1-y)$ (1) $\Longrightarrow \displaystyle\int \frac{dy}{1-y} = \int \sin x\, dx \Longrightarrow -\ln|1-y| = -\cos x + c$ (2) (2)

$$1 - y = \tilde{A}\cdot e^{\cos x}$$

$$y_{ált}(x) = A\cdot e^{\cos x} + 1\ ;\ A \in \mathbb{R}.$$ (2)

Kezdeti felt.: $3 = A\cdot e^{1} + 1 \Longrightarrow A = \dfrac{2}{e}\ ;\ y(x) = \dfrac{2}{e}\, e^{\cos x} + 1$ (3)

A feladat megoldható az inhomogén lineáris egyenletre tanult módszerrel is.

2, $\boxed{10}$ $y^{(4)} + 3y^{(3)} + 2y'' = x$

(H) $\lambda^4 + 3\lambda^3 + 2\lambda^2 = \lambda^2(\lambda^2 + 3\lambda + 2) = \lambda^2(\lambda+2)(\lambda+1) = 0$

$\Longrightarrow \lambda_{1,2} = 0,\ \lambda_3 = -2,\ \lambda_4 = -1$ (2) $\Longrightarrow y_{H,ált}(x) = C_1 e^{-2x} + C_2 e^{-x} + C_3 x + C_4$ (2)

Belső rez.

külső rez. miatt $\qquad C_i \in \mathbb{R}$ $(i = 1,2,3,4)$

Inhomogén: $y_{I,p}(x) = (Ax + B)x^2 = Ax^3 + Bx^2$ (2)

$$y''_{I,p}(x) = 6Ax + 2B \qquad /\cdot 2$$

$$y'''_{I,p}(x) = 6A \qquad /\cdot 3$$

$$y^{(4)}_{I,p}(x) = 0 \qquad /\cdot 1$$

(+)

$$x = 18A + 12Ax + 4B \Longrightarrow 12A = 1 \Longrightarrow A = \frac{1}{12}$$

$$18A + 4B = 0 \Longrightarrow B = -\frac{9A}{2} = -\frac{3}{8}$$

$$y_{I,p}(x) = \frac{1}{12}x^3 - \frac{3}{8}x^2$$ (3)

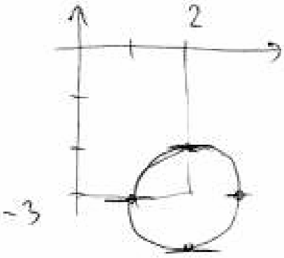
$$y_{I,ált}(x) = y_{H,ált}(x) + y_{I,p}(x) = C_1 e^{-2x} + C_2 e^{-x} + C_3 x + C_4 - \frac{3}{8}x^2 + \frac{1}{12}x^3$$ (1)

3, $\boxed{8}$

a, $y' = \sqrt{(2-x)^2+(3+y)^2} - 1$

$K = 0 = \sqrt{(2-x)^2+(3+y)^2} - 1$

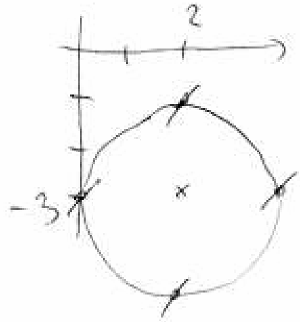
$\Downarrow$ $1 = (x-2)^2+(y+3)^2$ kör, $R=1$




(2)

$K = 1 = \sqrt{(2-x)^2+(3+y)^2} - 1$

$\Downarrow$ $4 = (x-2)^2+(y+3)^2$ $R=2$, kör




(2)

b, Legyen $y(x)$ a $(3;-3)$ pontban átmenő megoldás!

$y'(3) = \sqrt{(2-3)^2+(3-3)^2} - 1 = 0$ (1)

$y''(x) = \dfrac{1}{2\sqrt{(x-2)^2+(y+3)^2}} \cdot \left(2(x-2)+2(y+3)\cdot y'\right)$ (2)

$y''(3) = \frac{1}{2}\cdot\left(2\cdot(3-2)+0\right) = 1 > 0 \Rightarrow$ lokális minimuma (1) van a megoldásnak P. ben.

— o —

4, a, $\boxed{5}$ $f(n+1) = \frac{7}{3} f(n) - \frac{2}{3} f(n-1)$

$q^{n+1} = \frac{7}{3} q^n - \frac{2}{3} q^{n-1}$ (1) $\Rightarrow 3q^2 - 7q + 2 = 0$ (1)

$q_{1,2} = \dfrac{7 \pm \sqrt{49-24}}{6} = \dfrac{7\pm 5}{6} = \begin{cases} 2 \\ \frac{1}{3}\end{cases}$ (2)

$f_{ált}(n) = A\cdot 2^n + B\cdot\left(\frac{1}{3}\right)^n$ (1)

$\boxed{2}$ b, $f(n)$ korlátos, ha $A = 0$, pl.) $f(n) = \left(\frac{1}{3}\right)^n$ (2)

c, $\boxed{5}$ $f(0) = A + B = 5$ $\quad /\cdot(-2)$

$f(1) = 2A + \frac{1}{3}B = 7$ $\quad (+)$

$\underbrace{\left(-2+\frac{1}{3}\right)}_{-\frac{5}{3}} B = -3 \Rightarrow B = \frac{9}{5}$ ; $A = 5 - \frac{9}{5} = \frac{16}{5}$

$f(n) = \frac{16}{5}\cdot 2^n + \frac{9}{5}\cdot\left(\frac{1}{3}\right)^n$

5, a, $\boxed{6}$ $\displaystyle\sum_{n=1}^{\infty} \underbrace{\frac{\sqrt{n^6-3n^5}}{\sqrt[3]{n^{12}+1}+n^2}}$

$$a_n \sim \frac{n^3}{n^4} = \frac{1}{n} \implies \text{Sejtés: div.}$$

$$a_n = \frac{\sqrt{n^6-3n^5}}{\sqrt[3]{n^{12}+1}+n^2} \underset{\uparrow}{\geq} \frac{\sqrt{n^6-\frac{1}{2}n^6}}{\sqrt[3]{n^{12}+n^{12}}+n^4} = \frac{1}{\sqrt{2}} \cdot \frac{n^3}{\sqrt[3]{2}\cdot n^4+n^4} =$$

ha $n > N_0$

$$= \frac{1}{\sqrt{2}(1+\sqrt[3]{2})} \cdot \frac{1}{n} = b_n$$

$\sum_n b_n = \infty$, így a minoráns krit. alapján az eredeti sor is divergens.

b, $\boxed{4}$ $\displaystyle\sum_{n=1}^{\infty} \underbrace{\sqrt{\frac{n+3}{3n+1}}}_{a_n}$ $\qquad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{\frac{1+3/n}{3+1/n}} = \frac{1}{\sqrt{3}} \neq 0,$

nem teljesül a konv. szükséges feltétele, így a sor divergens.