1, a, $\boxed{6}$

$$y' = -2x + x^2 + y^2$$

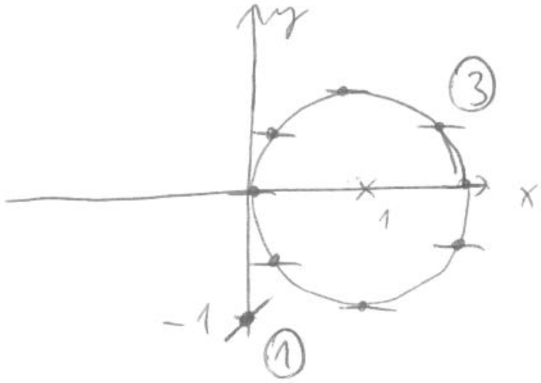
Izoklinák: $-2x + x^2 + y^2 = K$ 2

$K = 0$: $(x-1)^2 + y^2 = 1$

(1,0) kp.-ú, 1 sugarú kör.

Az iránymező itt vízszintes.

(0,−1)-ben $y'(-1) = -2\cdot 0 + 0^2 + (-1)^2 = +1$



b, $\boxed{5}$

$$y'(0) = 0$$

$$y'' = -2 + 2x + 2yy'$$

$$y''(0) = -2 + 2\cdot 0 + 2\cdot 0\cdot 0 = -2 < 0 \Rightarrow$$

a megoldásnak (0,0)-ban lokális maximuma van.

c, $\boxed{4}$

$$y''' = 2 + 2y'^2 + 2yy''$$

$$y'''(0) = 2 + 0 + 0 = 2$$

2, $y' - \frac{5}{x}y = x^5 e^{-3x}$; $x \neq 0$

inhomogén lineáris

(H): $y' = \frac{5}{x}y$ $y \equiv 0$ ✓

$y \neq 0$: $\int \frac{dy}{y} = 5\int \frac{dx}{x}$; $\ln|y| = 5\ln|x| + C$

$$y_{H,ált}(x) = K\cdot x^5, \quad K \in \mathbb{R}$$ 6

(I): $y_{I,part}(x) = K(x)\cdot x^5$ 2

$$y'_{I,part}(x) = K'(x)\cdot x^5 + 5K(x)\cdot x^4$$

Beírva: $K'(x)\cdot x^5 + \cancel{5K(x)x^4} - \cancel{\frac{5}{x}K(x)x^5} = x^5\cdot e^{-3x}$

3 $K'(x) = e^{-3x}$; $K(x) = \int e^{-3x}\,dx = -\frac{1}{3}e^{-3x}$

$y_{I,p}(x) = -\frac{1}{3}e^{-3x}\cdot x^5$ 2 ; $y_{I,ált}(x) = y_{I,part}(x) + y_{H,ált}(x) = x^5\left(K - \frac{e^{-3x}}{3}\right)$ 2

3, $$y' = \frac{y^2-4}{y} \cdot \frac{4}{3x^2+6} \qquad y \neq 0$$

$\boxed{11}$ a, $y = \pm 2$ mo. 2 $\quad y \neq \pm 2$ esetén

$$\int \frac{y\,dy}{y^2-4} = \int \frac{4\,dx}{3x^2+6} \qquad 2$$

$$\int \frac{y\,dy}{y^2-4} = \frac{1}{2}\int \frac{2y\,dy}{y^2-4} = \frac{1}{2}\ln|y^2-4| + C$$

$$\int \frac{4\,dx}{3x^2+6} = \frac{2}{3}\int \frac{dx}{1+\left(\frac{x}{\sqrt{2}}\right)^2} = \frac{2\sqrt{2}}{3}\operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right) + C$$

$$\underline{\underline{\frac{1}{2}\ln|y^2-4| \overset{3}{=} \frac{2\sqrt{2}}{3}\operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right)^{3} + C^{1} \ ; \quad C \in \mathbb{R}}}$$

$\boxed{4}$ b, $y(0) = 1$ esetén $\quad \frac{1}{2}\ln 3 = \frac{2\sqrt{2}}{3}\cdot 0 + C \Rightarrow C = \frac{\ln 3}{2} \quad 2$

$$\frac{1}{2}\ln(4-y^2) = \frac{2\sqrt{2}}{3}\operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right) + \frac{\ln 3}{2}$$

$y(0) = -2$ esetén $\quad y(x) \equiv -2 \quad 2$

$\boxed{4}$ c, $f(x) = \frac{4}{3x^2+6}$; $g(y) = \frac{y^2-4}{y}$

Olyan tartomány esetén egyértelmű a megoldás, ahol f, g folytonos, és $g \neq 0$. Tehát $x \in \mathbb{R}$, $y \in (2, \infty)$ jó.

(Vagy $y \in (0,2)$, $y \in (-2,0)$, $y \in (-\infty,-2)$)

4, $u = x + 3y$; $\quad y = \frac{u-x}{3}$; $\quad y' = \frac{u'}{3} - \frac{1}{3}$

$$y' = \frac{1}{x+3y} \quad \left(y \neq \frac{-x}{3}\right) \Rightarrow \frac{u'}{3} - \frac{1}{3} = \frac{1}{u}^{3} ; \quad u' = \frac{3}{u} + 1$$

* $\int \frac{u}{3+u}\,du = \int dx^{2}$; $\int \frac{u}{3+u}\,dx = \int 1 - \frac{3}{3+u}\,du = u - 3\ln|3+u|$

$u - 3\ln|u+3| = x + C \quad 5$

$$\underline{\underline{x + 3y - 3\ln|x+3y+3| = x + \overset{1}{C}}}$$

* $u = -3$ megoldás
$x + 3y = -3 \quad 2$
$y = -\frac{x}{3} - 1$ mo.

5, a, $\boxed{12}$ $y''' - y'' - 2y' = 20 \sin x$

(H) $\lambda^3 - \lambda^2 - 2\lambda = \lambda(\lambda^2 - \lambda - 2) = \lambda(\lambda - 2)(\lambda + 1) = 0$

$$y_{H,\acute{a}lt}(x) = C_1 + C_2 e^{2x} + C_3 e^{-x} \quad (5) \quad C_1, C_2, C_3 \in \mathbb{R}$$

Nincs rezonancia

(I) $y_{I,p}(x) = A \sin x + B \cos x$ (2) / 0

$y'(x) = A \cos x - B \sin x$ /(-2)

$y''(x) = -A \sin x - B \cos x$ / (-1)

⊕ $y'''(x) = -A \cos x + B \sin x$ / (+1)

$20 \sin x = \sin x (2B + A + B) + \cos x (-2A + B - A)$

$\left.\begin{array}{l} 3B + A = 20 \\ -3A + B = 0 \end{array}\right\} \Rightarrow B = 3A \longrightarrow 9A + A = 20; \quad A = 2, \quad B = 6$

$y_{I,p}(x) = 2 \sin x + 6 \cos x$ (3), $y_{I,\acute{a}lt}(x) = y_{I,p}(x) + y_{H,\acute{a}lt}(x) =$ (2)

$C_1, C_2, C_3 \in \mathbb{R}$ $\quad = 2 \sin x + 6 \cos x + C_1 + C_2 e^{2x} + C_3 e^{-x}$

$\boxed{4}$ b, $y''' - y'' - 2y' = 4\, \mathrm{ch}(3x) + 8x \longleftarrow$ itt van rezonancia!

$y_{I,p}(x) = A e^{3x} + B e^{-3x} + (Cx + D)x \quad \left(\mathrm{ch} 3x = \frac{e^{3x} + e^{-3x}}{2} \right)$

$\boxed{6}$ c, $y = -3 \cos(4x) + 5x^2$

$\lambda_{1,2} = 0 \pm 4i \; ; \; \lambda_{3,4,5} = 0$

$(\lambda + 4i)(\lambda - 4i) \cdot \lambda^3 = (\lambda^2 + 16)\lambda^3 = \lambda^5 - 16\lambda^3 = 0$

$$y^{(5)} - 16 y''' = 0$$

6, a, Ha $a_n > 0$, és $\exists \lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$, akkor $\sum_n a_n = \infty$, (div.)

$\boxed{3}$ ha $a_n > 0$, és $\exists \lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, akkor $\sum_n a_n < \infty$ (konv.)

b, $\underbrace{\sum_{n=1}^{\infty} \frac{5^n n!}{(2n)!}}_{0<a_n}$; $\frac{a_{n+1}}{a_n} = \frac{5^{n+1}(n+1)!}{(2n+2)!} \cdot \frac{(2n)!}{5^n n!} = \frac{5n}{(2n+1)(2n+2)} \to$

$\boxed{6}$ $\longrightarrow 0 < 1 \Rightarrow$ a sor konvergens!

c, $\underbrace{\sum_{n=1}^{\infty} \left(\frac{2n-1}{2n}\right)^{n^2} \frac{1}{6^{n+1}}}_{a_n}$

$\boxed{7}$

$$\sqrt[n]{a_n} = \left(1 - \frac{1}{2n}\right)^n \cdot \frac{1}{6 \cdot \sqrt[n]{6}} \longrightarrow e^{-1/2} \cdot \frac{1}{6} < 1 \Rightarrow \sum_n a_n < \infty \text{ (konv.)}$$

7, $f(n) = -\frac{2}{3} f(n-1) + \frac{1}{3} f(n-2)$

a, $q^2 = -\frac{2}{3} q + \frac{1}{3} \Rightarrow 3q^2 + 2q - 1 = 0$; $q_{12} = \frac{-2 \pm \sqrt{4+12}}{6} = \begin{cases} -1 \\ \frac{1}{3} \end{cases}$

$\boxed{6}$

$$f(n) = A(-1)^n + B\left(\frac{1}{3}\right)^n ; \quad A, B \in \mathbb{R}$$

$\boxed{3}$ b, $\left.\begin{array}{l} A + B = 7 \\ -A + \frac{B}{3} = 1 \end{array}\right\}$ $\frac{4}{3} B = 8 \Rightarrow B = 6,\ A = 1$

$$f(n) = (-1)^n + 6\left(\frac{1}{3}\right)^n$$

$\boxed{2}$ c, $f(n) = o(1)$, ha $\lim_{n\to\infty} f(n) = 0$, tehát $f(n) = B\left(\frac{1}{3}\right)^n$; $B \in \mathbb{R}$

8, $y' - y \cos(4x) = 3\cos(4x)$

$y' = (3+y)\cos(4x)$ $\qquad y \equiv -3$ mo. (2)

$\int \frac{dy}{y+3} = \int \cos(4x)\,dx$ (3) ; $\ln|y+3| = \frac{1}{4}\sin(4x) + C$ (2) ; $C \in \mathbb{R}$

$\left.\begin{array}{l} y(x) = \pm e^C \cdot e^{\frac{1}{4}\sin(4x)} - 3 \\ y(x) = -3 \end{array}\right\}$ $y_{ált}(x) = K \cdot e^{\frac{1}{4}\sin(4x)} - 3$ (2)

$K \in \mathbb{R}$