1. Elemi algebrai azonosságok

Műveletek hatványokkal: tegyük fel, hogy $a, b \in \mathbb{R}^+$, $\alpha, \beta \in \mathbb{N}^+$, ekkor

$a^\alpha \cdot b^\alpha = (a \cdot b)^\alpha$	$a^\alpha \cdot a^\beta = a^{\alpha+\beta}$	$(a^\alpha)^\beta = a^{\alpha \cdot \beta}$
$a^{-\alpha} = \frac{1}{a^\alpha}$	$\frac{a^\alpha}{a^\beta} = a^{\alpha-\beta}$	$a^0 = 1$
$\sqrt[\alpha]{a} = a^{\frac{1}{\alpha}}$	$\sqrt[\alpha]{a^\beta} = a^{\frac{\beta}{\alpha}} = (\sqrt[\alpha]{a})^\beta$	$\sqrt[\alpha]{a \cdot b} = \sqrt[\alpha]{a} \cdot \sqrt[\alpha]{b}$

2. Műveletek logaritmussal

Tegyük fel, hogy $a \in \mathbb{R}^+ \setminus \{1\}$, $b, c, d \in \mathbb{R}^+$, $\alpha \in \mathbb{R}$ ekkor definíció szerint $\log_a b = c \Leftrightarrow a^c = b$, és

$\log_a(c \cdot d) = \log_a c + \log_a d$	$\log_a(b^\alpha) = \alpha \log_a b$
$\log_a(\frac{b}{c}) = \log_a b - \log_a c$	$\log_a 1 = 0$, $\log_a a = 1$

3. Trigonometrikus azonosságok

Tegyük fel, hogy $\alpha, \beta \in \mathbb{R}$, ekkor $\sin^2\alpha + \cos^2\alpha = 1$, továbbá

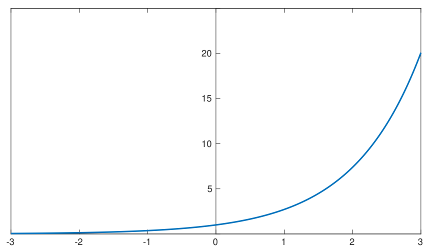
$\sin(-\alpha) = -\sin\alpha$	$\cos(-\alpha) = \cos(\alpha)$
$\sin(\alpha + \pi) = -\sin(\alpha)$	$\cos(\alpha + \pi) = -\cos\alpha$
$\sin\left(\alpha + \frac{\pi}{2}\right) = \cos\alpha$	$\cos\left(\alpha + \frac{\pi}{2}\right) = -\sin\alpha$
$\sin(2\alpha) = 2\sin\alpha\cos\alpha$	$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$
$\sin^2\alpha = \frac{1-\cos(2\alpha)}{2}$	$\cos^2\alpha = \frac{1+\cos(2\alpha)}{2}$
$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$	$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$	$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

4. Nevezetes szögek szögfüggvényei

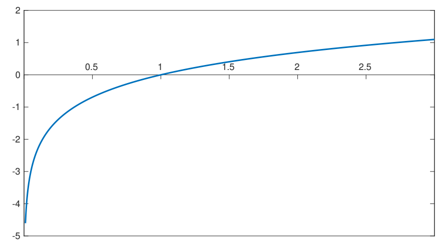
Tegyük fel, hogy $k \in \mathbb{Z}$, ekkor

$\sin\frac{\pi}{6} = \frac{1}{2}$	$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$	$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\cos\frac{\pi}{3} = \frac{1}{2}$	$\tan\frac{\pi}{3} = \sqrt{3}$
$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\tan\frac{\pi}{4} = 1$
$\sin\left(\frac{\pi}{2} + k\pi\right) = (-1)^k$	$\cos\left(\frac{\pi}{2} + k\pi\right) = 0$	$\tan\left(\frac{\pi}{2} + k\pi\right)$ nem ért.
$\sin(k\pi) = 0$	$\cos(k\pi) = (-1)^k$	$\tan(k\pi) = 0$

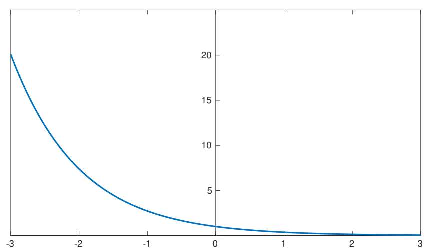
5. Elemi függvények



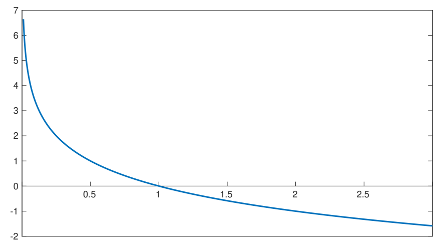
$f(x) = a^x$, $a > 1$
$D_f = \mathbb{R}$
$R_f = \mathbb{R}^+$
$\lim_{x\to\infty} a^x = \infty$
$\lim_{x\to-\infty} a^x = 0$
szig. mon. nő, folytonos $\mathbb{R}$-en
inverze : $\log_a x$
$(a^x)' = \ln(a)\ a^x$



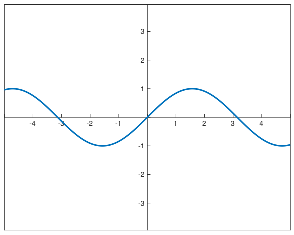
$f(x) = \log_a(x)$, $a > 1$
$D_f = \mathbb{R}^+$
$R_f = \mathbb{R}$
$\lim_{x\to\infty} \log_a(x) = \infty$
$\lim_{x\to0+} \log_a(x) = -\infty$
szig. mon. nő, folytonos $\mathbb{R}^+$-en
inverze : a^x
$(\log_a x)' = \frac{1}{\ln(a)\cdot x}$



$f(x) = a^x$, $0 < a < 1$
$D_f = \mathbb{R}$
$R_f = \mathbb{R}^+$
$\lim_{x\to\infty} a^x = 0$
$\lim_{x\to-\infty} a^x = \infty$
szig. mon. csökk., folytonos $\mathbb{R}$-en
inverze : $\log_a x$
$(a^x)' = \ln(a)\ a^x$

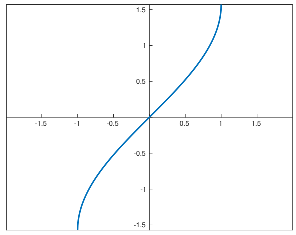


$f(x) = \log_a(x)$, $0 < a < 1$
$D_f = \mathbb{R}^+$
$R_f = \mathbb{R}$
$\lim_{x\to\infty} \log_a(x) = -\infty$
$\lim_{x\to0+} \log_a(x) = \infty$
szig. mon. csökk., folytonos $\mathbb{R}^+$-en
inverze : a^x
$(\log_a x)' = \frac{1}{\ln(a)\cdot x}$



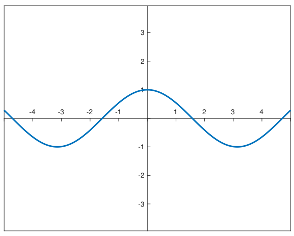
$f(x) = \sin(x)$,
$D_f = \mathbb{R}$
$R_f = [-1, 1]$
folytonos $\mathbb{R}$-en, páratlan, 2π-periódikus
inverze : $\arcsin(x)$

$(\sin x)' = \cos(x)$



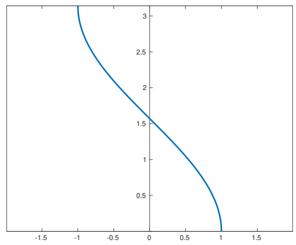
$f(x) = \arcsin(x)$
$D_f = [-1, 1]$
$R_f = [-\frac{\pi}{2}, \frac{\pi}{2}]$
szig. mon. nő, folytonos $[-1, 1]$-en, páratlan
inverze : $\sin(x)$

$(\arcsin x)' = \dfrac{1}{\sqrt{1 - x^2}}$



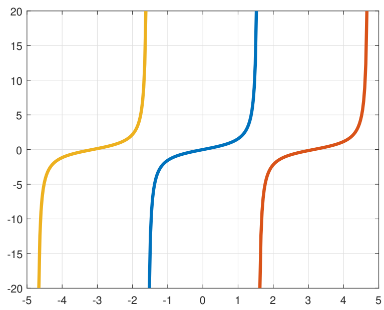
$f(x) = \cos(x)$,
$D_f = \mathbb{R}$
$R_f = [-1, 1]$
folytonos $\mathbb{R}$-en, páros, 2π-periódikus
inverze : $\arccos(x)$

$(\cos x)' = -\sin(x)$



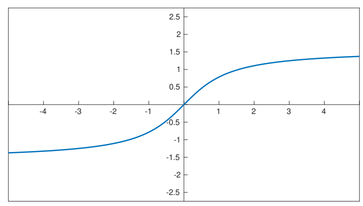
$f(x) = \arccos(x)$
$D_f = [-1, 1]$
$R_f = [0, \pi]$
szig. mon. csökken, folytonos $[-1, 1]$-en
inverze : $\cos(x)$

$(\arccos x)' = \dfrac{-1}{\sqrt{1 - x^2}}$



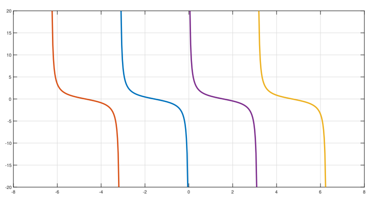
$f(x) = \tan(x)$,
$D_f = \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$
$R_f = \mathbb{R}$
folytonos $\mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$-en, páratlan, π-periódikus

inverze : $\arctan(x)$
$(\tan x)' = 1 + \tan^2(x) = \frac{1}{\cos^2(x)}$



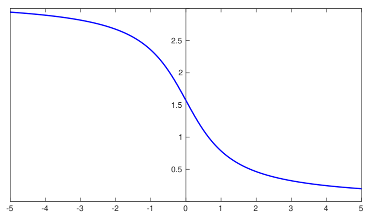
$f(x) = \arctan(x)$
$D_f = \mathbb{R}$
$R_f = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
szig. mon. nő, folytonos $\mathbb{R}$-en, páratlan

$\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$, $\lim_{x\to-\infty} \arctan x = \frac{-\pi}{2}$
inverze : $\tan(x)$
$(\arctan x)' = \frac{1}{1+x^2}$

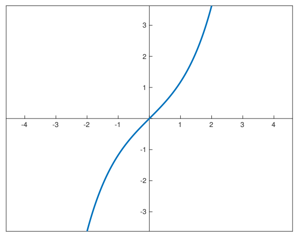


$f(x) = \text{cotan}(x)$,
$D_f = \mathbb{R} \setminus \{k\pi\}$
$R_f = R$
folytonos $\mathbb{R} \setminus \{k\pi\}$-en, páratlan, π-periódikus

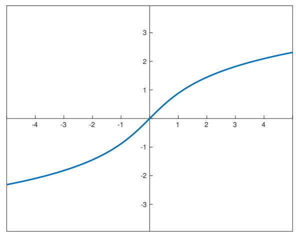
inverze : $\text{arccotan}(x)$
$(\text{cotan}\, x)' = -1 - \text{cotan}^2(x) = -\frac{1}{\sin^2(x)}$



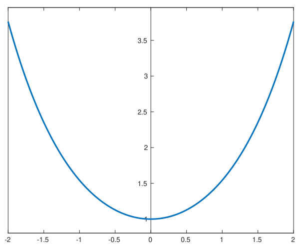
$f(x) = \text{arccotan}(x)$
$D_f = \mathbb{R}$
$R_f = (0, \pi)$
szig. mon. csökken, folytonos $\mathbb{R}$-en
$\lim_{x\to\infty} \text{arccotan}\, x = 0$, $\lim_{x\to-\infty} \text{arccotan}\, x = \pi$
inverze : $\text{cotan}(x)$
$(\text{arccotan}\, x)' = \frac{-1}{1+x^2}$



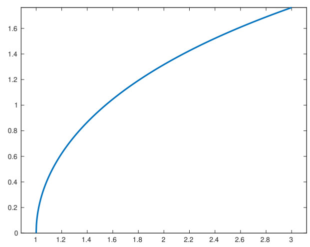
$f(x) = \sinh(x)$,
$D_f = \mathbb{R}$
$R_f = \mathbb{R}$
$\lim_{x\to\infty} \sinh x = \infty$
$\lim_{x\to-\infty} \sinh x = -\infty$
szig. mon. nő, folytonos $\mathbb{R}$-en, páratlan
inverze : $\text{arsinh}(x)$
$(\sinh x)' = \cosh(x)$



$f(x) = \text{arsinh}(x)$
$D_f = \mathbb{R}$
$R_f = \mathbb{R}$
$\lim_{x\to\infty} \text{arsinh}\, x = \infty$
$\lim_{x\to-\infty} \text{arsinh}\, x = -\infty$
szig. mon. nő, folytonos $\mathbb{R}$-en, páratlan
inverze : $\sinh(x)$
$(\text{arsinh}\, x)' = \frac{1}{\sqrt{x^2+1}}$



$f(x) = \cosh(x)$,
$D_f = \mathbb{R}$
$R_f = [1, \infty)$
$\lim_{x\to\infty} \cosh x = \infty$
$\lim_{x\to-\infty} \cosh x = \infty$
folytonos $\mathbb{R}$-en, páros
inverze : $\text{arcosh}(x)$
$(\cosh x)' = \sinh(x)$



$f(x) = \text{arcosh}(x)$
$D_f = [1, \infty)$
$R_f = \mathbb{R}^+$
$\lim_{x\to\infty} \text{arcosh}\, x = \infty$

szig. mon. nő, folytonos $[1, \infty)$-en
inverze : $\cosh(x)$
$(\text{arcosh}\, x)' = \frac{1}{\sqrt{x^2-1}}$