2016. 06. 16. EMT megold.:

1, Hengerkondenzátor

$r_1 = 3\ mm$

$r_2 = 6\ mm$

$l = 35\ mm$

$\varepsilon_r = 4$

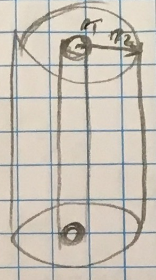
$Q_1 = 0$

$Q_2 = 0$

$\rho = 3 \frac{\mu C}{m^3}$

$U = ?$

$\varepsilon_0 = 8{,}85 \cdot 10^{-12} \frac{As}{Vm}$



Gauss:

$$\oint \vec{E}\, d\vec{A} = \frac{1}{\varepsilon_0 \varepsilon_r} \cdot \int_V \rho\, dV$$

$$E \cdot 2\cancel{\pi} r \cancel{l} = \frac{1}{\varepsilon_0 \varepsilon_r} \cdot \rho \cdot (r^2 - r_1^2)\cancel{\pi}\cancel{l}$$

$$E(r) = \frac{1}{2\varepsilon_0 \varepsilon_r} \cdot \rho \cdot \frac{(r^2 - r_1^2)}{r}$$

$$E(r) = \frac{1}{2\varepsilon_0 \varepsilon_r} \cdot \rho \cdot \left(r - \frac{r_1^2}{r}\right)$$

$$U = \int_{r_1}^{r_2} \frac{1}{2\varepsilon_0 \varepsilon_r} \cdot \rho \left(r - \frac{r_1^2}{r}\right) dr =$$

$$= \frac{1}{2\varepsilon_0 \varepsilon_r} \rho \int_{r_1}^{r_2} r - \frac{r_1^2}{r}\, dr = \frac{1}{2\varepsilon_0 \varepsilon_r} \rho \left(\int_{r_1}^{r_2} r\, dr - r_1^2 \int_{r_1}^{r_2} \frac{1}{r} dr \right.$$

$$= \frac{\rho}{2\varepsilon_0 \varepsilon_r} \cdot \left(\left(\frac{r_2^2 - r_1^2}{2} \right) - r_1^2 \ln \frac{r_2}{r_1} \right) \cong 0{,}3077\ V \approx 0{,}31\ V$$

2, $\phi_1 = -20\ V \leftarrow P_1$

$\phi(P_2) = \phi_2 = ?$

$$\int_{P_1}^{P_2} \vec{E}\, d\vec{l} = -30\ V$$

$$-\int_{P_1}^{P_2} \vec{E}\, d\vec{l} = \phi_2 - \phi_1$$

$$30 = \phi_2 - (-20)$$

$$30 = \phi_2 + 20$$

$$\phi_2 = 10\ V$$

3/

$\vec{E} = (30; 40; 0)\ \frac{kV}{m}$

$\vec{D} = (0,84; 1,12; 0)\ \mu\frac{As}{m^2}$

$w_e = 0,07 \frac{J}{m^3}$

lineáris $\Leftrightarrow$ $\varepsilon\vec{E} = \vec{D}$ ha $E = 0 \Leftrightarrow D = 0$ minden irányban

$D_x = 0$, de $E_x \neq 0$ $\Rightarrow$ nemlineáris

4/

$$\left.\begin{matrix} I_1 = 2\ A \\ I_2 = 0 \end{matrix}\right\} \Rightarrow \begin{matrix} \Psi_1 = 2\ Wb \\ \Psi_2 = 0,6\ Wb \end{matrix}$$

$$\left.\begin{matrix} I_1 = 0 \\ I_2 = 0,5\ A \end{matrix}\right\} \Rightarrow \Psi_1 = ?$$

kölcsönös induktivitás: $L_{21} = \left.\frac{\Psi_2}{I_1}\right|_{I_2=0}$

$L_{21} = \frac{0,6}{2} = 0,3\ H$

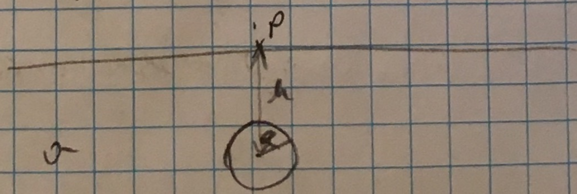
$L_{12} = \left.\frac{\Psi_1}{I_2}\right|_{I_1=0}$

$0,3 = \frac{\Psi_1}{0,5}$

$\Psi_1 = 0,15\ Wb$

5/ $\sigma = 0,1 \frac{S}{m}$

$\sigma_{kv} = 0$



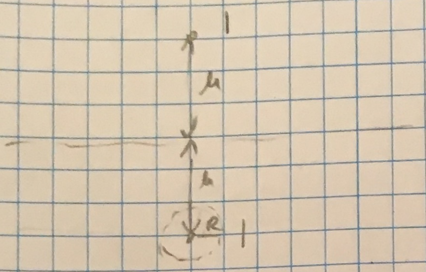
$h = 5\ m$

$R = 20\ cm$

$I = 100\ A$

$\Phi(P) = ?$

Áramtükrözés:



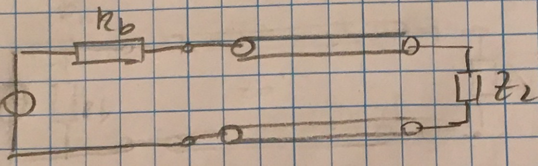
$$\phi(r) = \frac{1}{4\pi\sigma} \cdot \frac{1}{r}$$ ← elektrosztatikai analógiából

Kis gyűrű közelítéssel:

$$\phi(P) = \frac{2I}{4\pi\sigma} \cdot \frac{1}{h} = \frac{2 \cdot 100}{4\pi \cdot 0,1} \cdot \frac{1}{5} \approx 31,83\ V \approx 31,8\ V$$

6,

$l = 4\ m$
$Z_0 = 75\ \Omega$
$R_b = 10\ \Omega$
minimum generátor
$\lambda = 2,5\ m$



$$P = max \iff Z_{bez} = Z_0$$

$$\beta = \frac{2\pi}{\lambda}$$

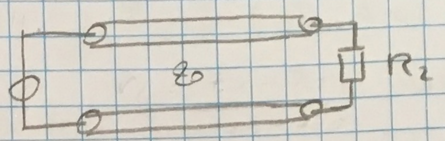
$$\beta l = \frac{2\pi}{\lambda} \cdot l = 3,2\pi$$

$$Z_{bez} = Z_0 \cdot \frac{R_b + j Z_0 \, tg(\beta l)}{Z_0 + j R_b \, tg(\beta l)} = \frac{75 \cdot (10 + j75\, tg(3,2\pi))}{75 + j10\, tg(3,2\pi)}$$

$$Z_{bez} = 15,1366 + j53,0244\ \Omega$$

7.

$Z_0 = 75\,\Omega$
$R_2 = 50\,\Omega$
$U_{max} = 12\,V$



①

$P = P_{beeső} - P_{reflektált} = P^+ - P^-$ ← a forrás által leadott teljesítmény

$$P = \frac{1}{2Z_0}\left(|U^+|^2 - |U^-|^2\right) = \frac{1}{2Z_0}\cdot\left(|U_2^+| - |U_2^-|^2\right)$$

$$U_{max} = (1 + |r_2|)\,U_2^+$$

$$r_2 = \frac{R_2 - Z_0}{R_2 + Z_0} = -0{,}2$$

$\Rightarrow U_1^+ = 10\,V$ $\Big\}$ $P = 0{,}64\,W$

$$|U_2^-| = |r_2 \cdot U_2^+| = 10 \cdot 0{,}2 = 2\,V \quad \Leftarrow \quad r_2 = \frac{U_2^-}{U_2^+} \quad \leftarrow \text{lezárás}$$

② $R_2 = Z_0 \Rightarrow$ illesztett lezárás $\Rightarrow r_2 = 0$

$$P = \frac{1}{2Z_0}\cdot|U_2^+|$$

$$|U_2^+| = \sqrt{2Z_0 \cdot P} = 9{,}8\,V$$

8.

$$E(x,y,z,t) = \hat{e}_y \cdot 20\,\frac{V}{m}\cdot\cos(2\pi f t + \beta x)$$ Ceugó

x irányban terjed

$f = 100\,MHz$

$H(2,3,-1,0) = ?$

$$H(x,y,z,t) = -\hat{e}_z\cdot\frac{E_0}{Z_0}\cdot\cos(2\pi f t + \beta x)$$

$Z_0 = 377\,\Omega$

$$\lambda = \frac{c}{f} = 3\,m$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{3}$$

$$H(2,3,-1,0) = -\hat{e}_z \cdot 0{,}05305\cdot\cos\left(\frac{2\pi}{3}\cdot 2\right) \mathrel{\hat{=}} 0{,}02653\,\frac{mA}{m}$$

9.

$\sigma \to \infty \Rightarrow Z_{02} \to \infty$ $\quad Z_{01} = 377\,\Omega$

$E_1^+ = 53\,\frac{V}{m}$ $\quad \frac{E_1^+}{H_1^+} = Z_{01} \longrightarrow H_1^+ = \frac{E_1^+}{Z_0} = 0{,}14058\,\frac{A}{m}$

$K = ?$

$E_1 t = E_2 t \Rightarrow E_1^+ + E_1^- = E_2^+$ $\qquad E_2^+ = 0$

$$\frac{E_1^+}{Z_{01}} - \frac{E_1^-}{Z_{01}} - \underbrace{\frac{E_2^+}{Z_{02}}}_{0} = K \qquad \Downarrow \qquad E_1^+ = -E_1^-$$

$$\frac{2E_1^+}{Z_{01}} = K = 0{,}28116\,\frac{A}{m}$$

10.

$D_{dB} = 3,52 \qquad D_{dB} = 10 \lg(D) \qquad 3,75 = 10 \lg D$

$P_S = 2\,kW \qquad 0,375 = \lg D$

$E(r = 500\,m)$ ← maximális sugárzási irány) $\qquad D = 2,25$

$$D \triangleq \frac{S_{max}}{S_{átl}} = \frac{S_{max}}{\frac{P_S}{4r^2\pi}} \longrightarrow S_{max} = D\,\frac{P_S}{4r^2\pi} = 1,4324\,\frac{mW}{m^2}$$

$$\vec{S} = \frac{1}{2}\,\vec{E} \times \vec{H}^* = \frac{1}{2}\left(\overline{E_\vartheta} \cdot \vec{e_\vartheta}\right) \times \left(\overline{H_\varphi}^* \cdot \vec{e_\varphi}\right)$$

$$\vec{S} = \overline{S_r} \cdot \vec{e_r}$$

$$\overline{S_r} = \frac{\overline{E_\vartheta} \cdot \overline{H_\varphi}^*}{2}$$

$$\frac{\overline{E_\vartheta}}{\overline{H_\varphi}} = Z_0 = 377\,\Omega$$

$$\overline{H_\varphi} = \frac{\overline{E_\vartheta}}{Z_0}$$

$$S_r = \frac{|\overline{E_\vartheta}|^2}{2 Z_0}$$

$$\sqrt{2 Z_0 S_{max}} = |\overline{E_\vartheta}| \triangleq 1,039\,\frac{V}{m} \triangleq 1,04\,\frac{V}{m}$$