2014. 03. 13. Mat. an. 2. I. ZH, β változat -1-

1, $\boxed{12}$ $y' = \dfrac{\operatorname{arctg}(3x)}{\operatorname{ch}(2y)}$ Szeparálható. $\int \operatorname{ch}(2y)\,dy = \int \operatorname{arctg}(3x)\,dx$ ②

$$\int \operatorname{arctg}(3x)\,dx = \int \underset{u'}{1} \cdot \underset{v}{\operatorname{arctg}(3x)}\,dx = x \operatorname{arctg}(3x) - \frac{1}{6}\int \frac{3x \cdot 6}{1+9x^2}\,dx =$$

f'/f alak

$$u = x \quad v' = \frac{3}{1+9x^2} \quad \Bigg| = x \operatorname{arctg}(3x) - \frac{1}{6}\ln(1+9x^2) + C$$ ⑥

$$\int \operatorname{ch}(2y)\,dy = \frac{1}{2}\operatorname{sh}(2y) + \tilde{C}$$ ③

Tehát: $\underline{\underline{\frac{1}{2}\operatorname{sh}(2y) = x \operatorname{arctg}(3x) - \frac{1}{6}\ln(1+9x^2) + C}}$ ①

2, $\boxed{12}$ $y' = \dfrac{x^2}{x^2+1} + \dfrac{2}{x}y$ elsőrendű lineáris

$$(H):\ y' = \frac{2}{x}y \implies \int \frac{dy}{y} = 2\int \frac{dx}{x} \implies \ln|y| = 2\ln|x| + C$$

$$\implies \underline{y_{H,ált}(x) = Kx^2 ;\ K \in \mathbb{R}}$$ ⑤

$$y_{I,p}(x) = K(x)\,x^2 ; \quad \text{Beírva: } K' \cdot \cancel{x^2} + \cancel{2xK} = \frac{\cancel{x^2}}{x^2+1} + \cancel{\frac{2}{x} \cdot x^2 K}$$

$$K'(x) = \frac{1}{1+x^2} ;\ K(x) = \operatorname{arctg} x ;\ \underline{y_{I,p}(x) = x^2 \cdot \operatorname{arctg} x}$$ ⑤

②

$$y_{I,ált}(x) = y_{H,ált}(x) + y_{I,p}(x) = \underline{\underline{x^2(K + \operatorname{arctg} x)}} ;\ K \in \mathbb{R}$$

B

3, $x^2 y' = y^2 - 4xy + 4x^2$; $y' = \left(\frac{y}{x}\right)^2 - 4\frac{y}{x} + 4$

16

$u = \frac{y}{x}$; $y(x) = u(x) \cdot x$; $y' = u + xu'$

$u + xu' = u^2 - 4u + 4$; $u' = \frac{u^2 - 5u + 4}{x} = \frac{(u-1)(u-4)}{x}$ 5

$u \equiv 1$ ill. $u \equiv 4$, azaz $y = x$, ill. $y = 4x$ megoldás 1

$$\int \frac{du}{(u-1)(u-4)} = \int \frac{dx}{x}$$

$$\frac{1}{(u-1)(u-4)} = \frac{A}{u-1} + \frac{B}{u-4} = \frac{(A+B)u - 4A - B}{(u-1)(u-4)} ; \quad \left.\begin{array}{l} A + B = 0 \\ -4A - B = 1 \end{array}\right\} \begin{array}{l} A = -\frac{1}{3} \\ B = \frac{1}{3} \end{array}$$

$$\int \frac{du}{(u-1)(u-4)} = \int \frac{-1/3}{u-1}\, du + \frac{1}{3}\int \frac{1}{u-4}\, du = -\frac{1}{3}\ln|u-1| + \frac{1}{3}\ln|u-4| + C =$$

$$= \frac{1}{3}\ln\left|\frac{u-4}{u-1}\right| + C = \frac{1}{3}\ln\left|\frac{y-4x}{y-x}\right| + C$$ 5 1

$\int \frac{dx}{x} = \ln|x| + \tilde{C}$; 2 $\ln\left|\frac{y-4x}{y-x}\right| = 3\ln|x| + C$

$\frac{y-4x}{y-x} = K \cdot x^3$; $y - 4x = Kx^3 y - Kx^4$; $y = \frac{4x - Kx^4}{1 - Kx^3}$ 2

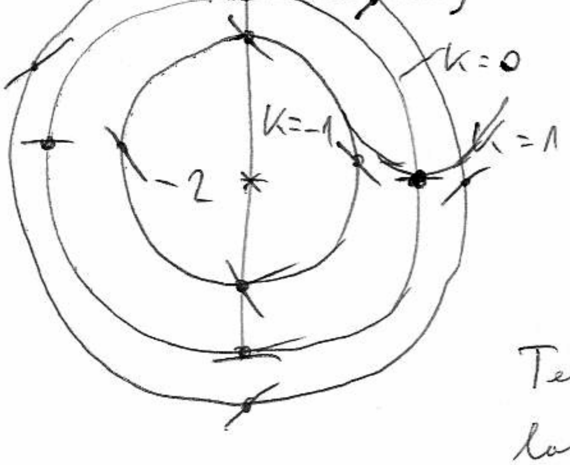
— o —

4, $y' = y^2 + 4y + x^2$; izoklinák: $y^2 + 4y + x^2 = K$;

10

$(y+2)^2 + x^2 = K + 4$ 2 kör

a,



b, Legyen y a $(2,-2)$ ponton átmenő megoldás! 1

$y' = y^2 + 4y + x^2 \Rightarrow y'(2) = 4 - 8 + 4 = 0$

$y'' = 2yy' + 4y' + 2x \Rightarrow y''(2) = 0 + 0 + (+4) =$
$= +4 > 0$ 2

Tehát $y(x)$-nek a $(2,-2)$-ben lokális minimuma van. 2

5, $y''' - 9 y' = 4 e^{3x}$

$\boxed{14}$ (H) $\lambda^3 - 9\lambda = \lambda(\lambda+3)(\lambda-3) = 0$

$$\boxed{y_{H,ált}(x) = C_1 + C_2 e^{3x} + C_3 e^{-3x}\,;\quad C_1, C_2, C_3 \in \mathbb{R}}$$ 5

(I) Külső rezonancia!

$y_{I,p}(x) = A x e^{3x}$ 3 $\quad / \cdot 0$

$y'_{I,p}(x) = A e^{3x} + 3Ax e^{3x} \quad / \cdot (-9)$

$y''_{I,p}(x) = 6A e^{3x} + 9Ax e^{3x} \quad / \cdot 0$

$+ \quad y'''_{I,p}(x) = 27A e^{3x} + 27Ax e^{3x} \quad / \cdot (1)$

$$4e^{3x} = Ae^{3x}(27-9) + \cancel{Axe^{3x}}\underbrace{(27-27)}_{0}$$

$$4 = 18A \Rightarrow A = \frac{2}{9}\,;\quad \boxed{y_{I,p}(x) = \frac{2}{9} x e^{3x}}$$ 4

$$y_{I,ált}(x) = y_{H,ált}(x) + y_{I,p}(x) = \underline{\underline{C_1 + C_2 e^{3x} + C_3 e^{-3x} + \frac{2}{9} x e^{3x}}}$$ 2

— o —

6, $(3x+2)\,\mathrm{ch}(5x) = \frac{1}{2}(3x+2)e^{5x} + \frac{1}{2}(3x+2)e^{-5x}\,;\quad \lambda_1 = \lambda_2 = 5$

$\boxed{12}$ $\quad \lambda_3 = \lambda_4 = -5$

Kar. pol.:

$$(\lambda+5)^2(\lambda-5)^2 = (\lambda-25)^2 = \lambda^4 - 50\lambda^2 + 625$$

$\underline{y^{(4)} - 50y'' + 625y = 0}$ 7

$$\underline{y_{H,ált}(x) = C_1 e^{5x} + C_2 x e^{5x} + C_3 e^{-5x} + C_3 x e^{-5x}}$$ 5

$\underline{C_1, C_2, C_3, C_4 \in \mathbb{R}}$

B

7, [10]

$$f(n+2) = -\frac{5}{3} f(n+1) + \frac{2}{3} f(n)$$

$$q^2 = -\frac{5}{3} q + \frac{2}{3} \Rightarrow 3q^2 + 5q - 2 = 0 \quad ②$$

$$q_{1/2} = \frac{-5 \pm \sqrt{25+24}}{6} = \frac{-5 \pm 7}{6} = \begin{cases} \frac{1}{3} \\ -2 \end{cases} \quad ②$$

$$\underline{f(n) = A\left(\frac{1}{3}\right)^n + B(-2)^n} \quad ②$$

$$\left.\begin{array}{l} f(0) = +1 \Rightarrow A + B = +1 \\ f(1) = 5 \Rightarrow \frac{1}{3}A - 2B = 5 \end{array}\right\} /2$$

$$+ \quad \frac{7}{3}A = 7 \; ; \; A = 3 ; \; B = -2 \quad ②$$

$$\underline{\underline{f(n) = 3\left(\frac{1}{3}\right)^n - 2(-2)^n}} \quad ②$$

8, [14] a, $\displaystyle\sum_{n=0}^{\infty} \underbrace{\left(\frac{3n+1}{3n+5}\right)^{n^2+2n}}_{a_n > 0}$ Gyökkritériummal:

$$\sqrt[n]{a_n} = \left(\frac{3n+1}{3n+5}\right)^{n+2} = \underbrace{\left(\frac{3n+1}{3n+5}\right)^2}_{\downarrow \; 1} \cdot \frac{\left(1+\frac{1/3}{n}\right)^n}{\left(1+\frac{5/3}{n}\right)^n} \xrightarrow{n\to\infty} 1 \cdot \frac{e^{1/3}}{e^{5/3}} = e^{-4/3}$$

$$e^{-4/3} < 1 \Rightarrow \sum_n a_n \text{ konvergens.} \quad ⑧$$

b, $\displaystyle\sum_{n=0}^{\infty} \underbrace{\frac{(2n)!\, 4^{n-2}}{(3n)!}}_{a_n}$ Hányados kritériummal:

$$\frac{a_{n+1}}{a_n} = \frac{(2n+2)! \cdot 4^{n-1}}{(3n+3)!} \cdot \frac{(3n)!}{(2n)!\, 4^{n-2}} = \frac{(2n+1)(2n+2)\cdot 4}{(3n+1)(3n+2)(3n+3)} \to$$

$$\to 0 < 1 \Rightarrow \sum_n a_n \text{ konvergens!} \quad ⑥$$