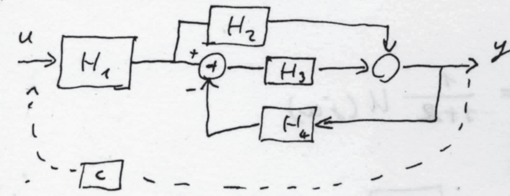
Szabtech 1. fel. nagy. old.

1, Blokkdiagram algebra

2, Közelítő bode diagram rajzolás

3, Átviteli fv. $\Leftrightarrow$ SS

1, Könyv. 60 – 80 old



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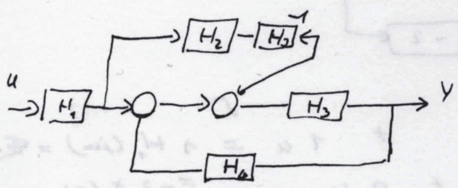

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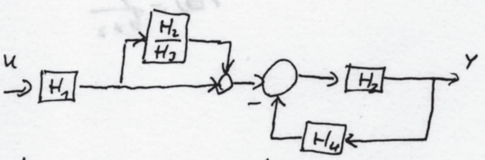
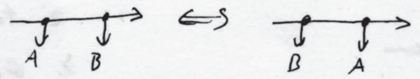


$$H_1 \cdot \left(\frac{H_2}{H_3} + 1\right) \left(\frac{H_3}{1 + H_3 H_4}\right) = H$$ zárt alakú átviteli fv.

Alapképzések

1,



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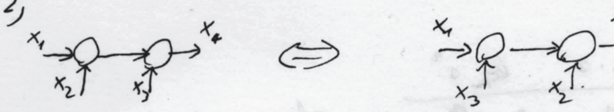

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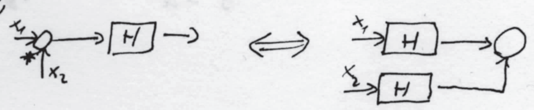

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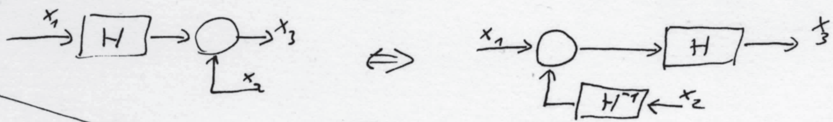

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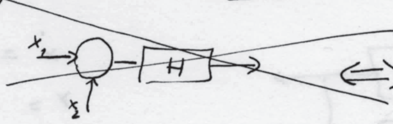
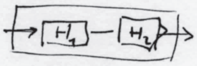
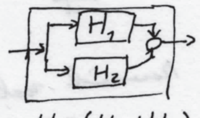




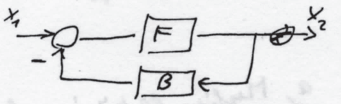
$H = H_1 H_2$

Soros kapcsolás



$H = (H_1 + H_2)$

párhuzamos kapcsolás



$$H = \frac{F}{1 + FB}$$

Visszacsatolt kapcsolás

2)

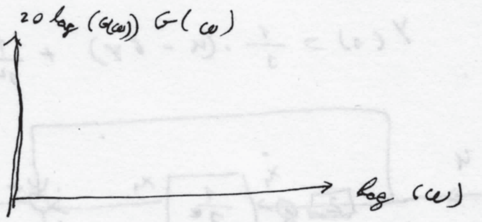
$$H(s) = \frac{K}{s^i} \frac{(1 + sT_1)\ldots(1 + 2\xi T_2 +}{(1 + sT_4)\ldots(} \cdot e^{-sT_H}$$

$u = 2 \sin(5t + 10) \Rightarrow Y = 3 \sin(5t + \varphi) + T_{tranziens}$

A_u φ_u A_y φ_y

$$A_y = \boxed{G(\omega)} A_u$$

$$\varphi_Y = \varphi_u + \varphi(\omega)$$



Pl: $H(s) = \frac{2}{s} \frac{(1 + 3s)}{(1 + 7s)} \frac{(1 + 10s)}{(1 + 2 \cdot 2 \cdot 11 s - 11^2 s^2)}$

K

$e^i = 1$

$k = 2$

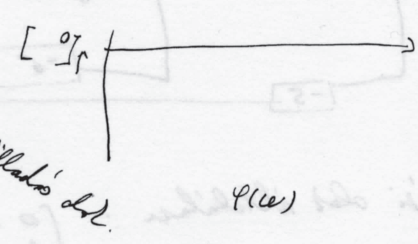
$T_1 = 3 \Rightarrow \omega_1 = \frac{1}{3}$

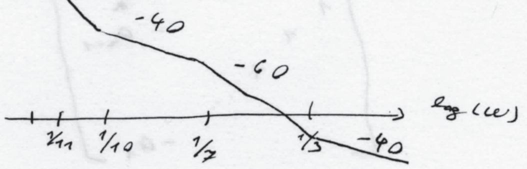
$T_2 = 7 \Rightarrow \omega_2 = \frac{1}{7}$

$T_3 = 10 \Rightarrow \omega_3 = 1/10$

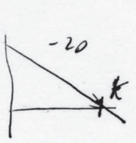
$T_4 = 11 \Rightarrow \omega_4 = \frac{1}{11}$

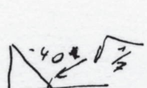
kezdő szög -20 db/dek

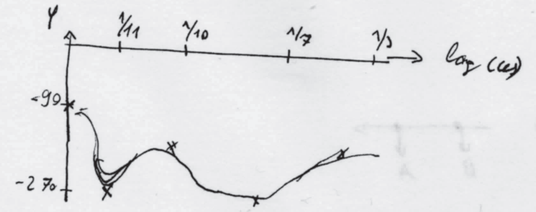




$\frac{2}{s} \Rightarrow 2\left(\frac{1}{s}\right)$







1. időállandós alak
2. törési frekvenciák meghatározása
3. $G(\omega)$ és $\varphi(\omega)$ felrajzolása
rendszámfokat ... és K ...

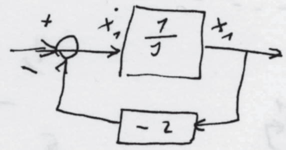
3)

$$H(s) = \frac{b_0 + b_1 \cdot s \ldots b_m s^n}{1 + a_1 s \ldots a_n s^n}$$

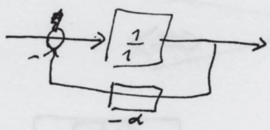
pl.: $H(s)\ \dfrac{1}{s - a}$

$$Y(j\omega) = \frac{1}{s+2} U(j\omega)$$

$$\dot{x} = \underline{A}\, x + \underline{B}\, x$$
$$Y = \underline{C}\, x \quad Du$$



$$H(s) = \frac{1}{1+\alpha} \Rightarrow$$



$$\dot{x} = -2x + 1u = s H_1(j\omega) = $$
$$Y = 1 \cdot x_1 + 0u$$
$$= -2 x_1(s) + 1 \cdot u\, s$$
$$Y(s) = \frac{Y}{s+2}$$

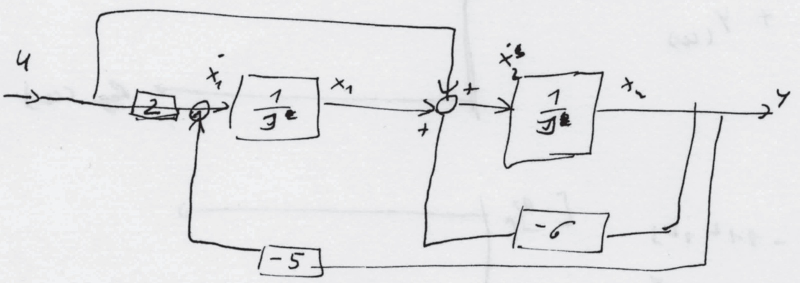
a) Megfigyelhetőségi alakra rendszer realizációt

$$\frac{Y(s)}{U(s)} = H(s) = \frac{s+2}{s^2 + 6s + 5} = \frac{s+2}{(s+1)(s+5)} = \frac{2}{5} \cdot \frac{1 + 0,5s}{(1+s)\ (1+0,2s)}$$

1. osztok s^2

$$\Rightarrow \frac{0 + \frac{1}{s} + \frac{2}{s^2}}{1 + \frac{6}{s} + \frac{5}{s^2}} = \frac{Y(s)}{U(s)} \Rightarrow Y(s) + Y(s) \cdot \frac{6}{s} + \frac{5}{s^2} \cdot Y(s) = \frac{u}{s} + \frac{2u}{s^2}$$

$$Y(s) = \frac{1}{s} \cdot (u - 6Y) + \frac{1}{s^2} (2u - 5Y)$$



$$\begin{matrix}\dot{x}_1 \\ \dot{x}_2\end{matrix} = \begin{bmatrix} 0 & -5 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad 0 \cdot u$$

Megfigyelhetőségi alak általában:

$$\begin{bmatrix} 0 & 0 & 0 & \ldots & -a_n \\ 1 & & & & -a_{n-1} \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & -a_1 \end{bmatrix}$$

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Feladatmegoldó 2.

Állapottereş...

$$\dot{x}(t) = A\,x(t) + b\,u(t)$$
$$y(t) = c\,x(t) + d\,x(t)$$
$$Y(s) = H(s)\cdot U(s)$$

Pl: Kocsi

$m\ddot{p} = F \xrightarrow{\text{Lap}} m\,s^2 P(s) = F(s) \qquad P(s) = \frac{1}{m\cdot s^2}\,F(s)$

$$x = \begin{bmatrix} p \\ v \end{bmatrix} \qquad \dot{x} = \begin{bmatrix} v \\ a \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} p \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} F$$

$$p = [1 \;\; 0]\begin{bmatrix} p \\ v \end{bmatrix} + [0]\,F$$

Diagonális kanonikus alak : A diag.

Megfigyelhetőségi kanonikus alak : $H(s) = \dfrac{b_1 s^{n-1} + \dots + b_n}{a_0 \dots}$ $\Rightarrow A: \begin{bmatrix} -a_1 & 1 & 0 & 0 & \dots \\ \vdots & 0 & 1 & 0 & \\ & 0 & 0 & 1 & \\ -a_n & \vdots & & & 1 \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \quad c = [1\;0\;0\;0\dots]$

Irányíthatósági — " — : — " — $\Rightarrow A \begin{bmatrix} -a_1 & \dots & a_n \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B: \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad c = [b_1 \dots b_n]$

$$\boxed{H(s) = \frac{s+2}{(s+1)(s+5)} = \frac{s+2}{s^2+6s+5} = \frac{2}{5}\cdot\frac{1+6+5s}{(1+s)(1+0{,}2s)}}$$

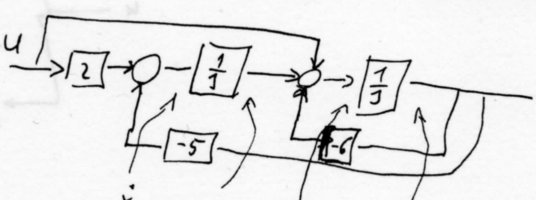
Megf. alak:

$$H(s) = \frac{\frac{1}{s} + \frac{2}{s^2}}{\frac{6}{s} + \frac{5}{s^2}} \qquad Y(s)\left(1 + \frac{6}{s} + \frac{5}{s^2}\right) = \left[\frac{1}{s} + \frac{2}{s^2}\right] U(s)$$

$$Y(s) = \frac{1}{s^2}\left[2U(s) - 5Y(s)\right] + \frac{1}{s}\left[U(s) - 6Y(s)\right]$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -6 & 1 \\ -5 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = [1 \;\; 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0\cdot u$$

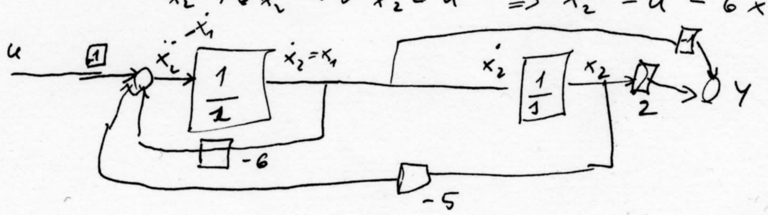


$$M_o = \begin{bmatrix} c^T \\ c^T A \\ c^T A^2 \\ \vdots \end{bmatrix} = \text{teljes rangú} \Rightarrow \text{megfigyelhető}$$

Irányíthatósági alak : $f(s) = \dfrac{1}{s^2+6s+5}$ (csak a nevező van ~~felírva~~ hozzá rendelve)

$$Y = (s+2)\cdot\underbrace{f(s)\,U(s)}_{x_2}$$

$$s^2 x_2(s) + 6s\,x_2(s) + 5x_2(s) = U$$
$$\Downarrow$$
$$\ddot{x}_2 + 6\dot{x}_2 + 5x_2 = u \Rightarrow \ddot{x}_2 = u - 6\dot{x}_2 - 5x_2$$

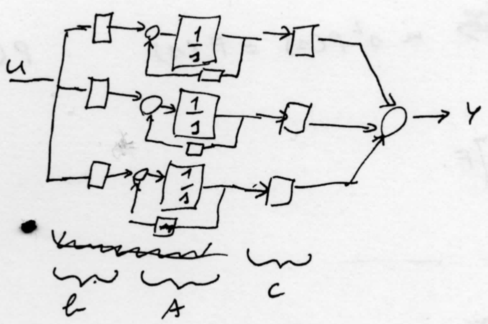


$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -6 & -5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$c = [1 \;\; 2]$ $\qquad M_c = [b \;\; bA \;\; bA^2 \dots]$ ha teljes rangú akkor irányítható

Diagonális alak.

$\dot{x} = \begin{smallmatrix} \diagdown & & \\ & A & \\ & & \diagdown \end{smallmatrix} x \quad \Rightarrow$



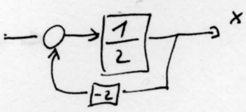
$\dot{x}_1 = -2x_1 + u(t)$

$y = x_1$

$(s+2)X_1(s) = U(s)$

$X_1(s) = \boxed{\frac{1}{s+2}} \cdot U(s)$

$\Downarrow$



Pl:

$$\frac{s+2}{s^2+6s+5} = \cancel{s^2+6s+5} = \frac{s+2}{(s+1)(s+5)}$$

$$= \frac{\alpha}{s+1} + \frac{\beta}{s+5} = \frac{\alpha(s+5)+\beta(s+1)}{(s+1)(s+5)}$$

$$s(\alpha+\beta) + 1\cdot(5\alpha+\beta) = s+2$$

$\alpha + \beta = 1 \Rightarrow \beta = 1-\alpha$

$5\alpha + \beta = 2$

$4\alpha = 1 \Rightarrow \alpha = \frac{1}{4} \Rightarrow \beta = \frac{3}{4}$

$$H(s) = \frac{1}{4}\cdot\frac{1}{s+1} + \frac{3}{4}\cdot\frac{1}{s+5}$$

$Y(s) = H(s)\cdot U(s)$

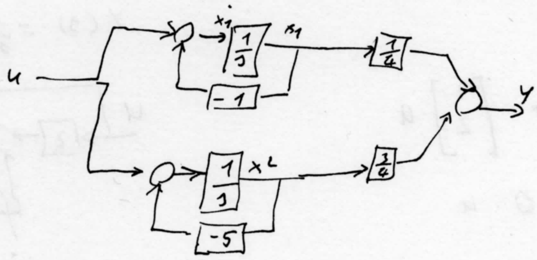
$A = \begin{bmatrix} -1 & 0 \\ 0 & -5 \end{bmatrix} \quad b: \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$c^T = [\frac{1}{4} \;\; \frac{3}{4}] \quad d: 0$

$s x_1 = -1 x_1 + u$

$(s+1)x = u$

$x_1 = \frac{1}{s+1} u$



Jordan alak

$$H(s) = \frac{1}{(s+1)^2(s+2)} = \frac{\alpha}{(s+1)} + \frac{\beta}{(s+1)^2} + \frac{\gamma}{(s+2)} =$$

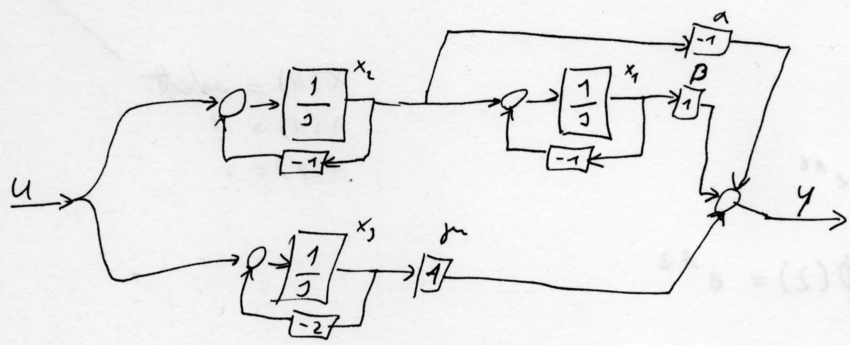
$$= \frac{\alpha(s+1)\cdot(s+2) + \beta\cdot(s+2) + \gamma(s+1)^2}{(s+1)^2\cdot(s+2)} = \frac{1}{\dots}$$

$$\alpha(s^2+3s+2) + \beta(s+2) + \gamma(s^2+2s+1) =$$

$$= \underbrace{s^2(\alpha+\gamma)}_{0} + \underbrace{s(3\alpha+2\gamma)}_{0} + \underbrace{2\alpha+2\beta+\gamma}_{1} = 1$$

$$\left.\begin{aligned} \alpha + \gamma &= 0 \\ 3\alpha + 2\gamma &= 0 \\ 2\alpha + 2\beta + \gamma &= 1 \end{aligned}\right\} \quad \alpha = -1 \Rightarrow \beta = 1 \Rightarrow \gamma = 1$$

$$H(s) = \underset{(\alpha)}{\frac{-1}{s+1}} + \underset{(\beta)}{\frac{1}{(s+1)^2}} + \underset{(\gamma)}{\frac{1}{s+2}}$$



$$A: \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \qquad b: \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$c^T: [-1 \ 1 \ 1] \qquad d: 0$$

Jordan ~~al~~ alak miatt az egyes blokkoknál b vektornál minden utolsó elem lesz csak zh.-n lehet

$\dot{x} = Ax + Bu$
$y = c^T x$

$x^0(0)$, $u(t)$

$$x(t) = e^{At} \cdot x(0) + \int_0^t e^{A(t-\tau)} b\, u(\tau)\, d\tau$$

$x' = ax$
$(e^{ax})' = a e^{ax}$ } ! szerint így alak

időtartomány leírás alapja egy állapotegyenletek

Ha A diag

$$e^{\begin{bmatrix} a_{11} & & \\ & a_{22} & \\ & & a_{33} \end{bmatrix} t} = \begin{bmatrix} e^{a_{11}} & & \\ & e^{a_{22}} & \\ & & e^{a_{33}} \end{bmatrix}$$

e-re így alakítunk át
állapot átmeneti mátrix

$x(0) =$ adott
$x(1) = ?$
$x(7) = ?$

Ha A diag

$x(2) = e^{A2} \cdot x(0)$
$x(7) = e^{A7} \cdot x(0) =$
$= e^{A(7-2)} \cdot \cancel{e^{A(2)}} \cdot x(2)$
$= e^{A(7-2)} e^{A(2)} \cdot x(0)$

$\phi(t) = e^{At}$

$\phi(2) = e^{A \cdot 2}$

$\phi(1)$, $x(0)$ } $x(7)$
$\phi(1) = e^{A}$

$x(7) = e^A \cdot e^A \cdot e^A \cdot e^A \cdot e^A \cdot e^A \cdot x(0)$

dinamikus rendszer félcsoport tulajdonsága

$\phi(2)$, $\phi(3)$, $x(0)$ } $x(7) = [\phi(2)]^2 \phi(3) \cdot x(0)$

szabályozástechnika III. f.m.

1. Stabilitás

$H(s) = \frac{1}{s^n}\,\frac{(1+sT)\ldots}{(1+sT)\ldots}$ rendszer stabil, ha a karakterisztikus egyenlet valós részei ("nevező" = 0 egyenlet) legyen kisebb mint 0.

$H(s) = \frac{\alpha}{s+\beta} \Rightarrow \mathcal{L}^{-1}\ \alpha e^{-\beta t}$

ha ez negatív, → e-ados tag lecsengő

0)

$H(s) = \frac{2}{s+4}$

$s+4=0$

$s=-4<0$ stabil

1)

$H(s)=\frac{2+s}{s^2+s+1}$

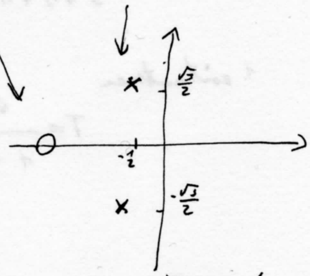
zérus: $s+2=0$, $s=-2$

pólusok

$s^2+s+1=0$

$\Delta = 1^2-4=-3 \nearrow \frac{-1\pm\sqrt{3}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

stabil

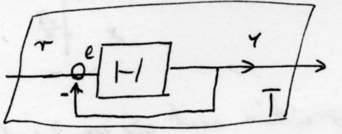


ha x -ek[illegible] nem stabil a rendszer

2)

$H(s) = \frac{\cancel{(s-2)}(s+2)}{\cancel{(s-2)}(\ldots)}$ elvárás egyszerűsítésük

Merev negatív visszacsatolás



$T = \frac{H}{1+H}$ stabilitás: $1+H$ vizsgáljuk.

e: hibajel, azaz hogy mennyivel tér el r és y

Eltérő:

$H(s) = \frac{s+2}{s^3+2s^2+2s+1}$

pólusok mind negatívak ⇒ stabil rendszer

$s^3+2s^2+2s+1=0 \Rightarrow \overset{s=-1}{} s^3+2s^2+2s+1 = (s+1)(s^2+s+1)$

Próbálással vizsgál meg

$s=1$ nem jó

$s=-1$ jó

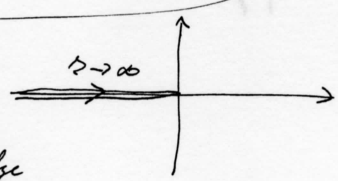
$$\begin{array}{l|l|l}
s^3+2s^2+2s+1 & s+1 & s^2+s+1 \\
-(s^3+s^2+0s+0) & & \\
\quad s^2 \quad 2s \quad +1 & & \\
-(s^2 \quad s+0) & & \\
\quad 0+s+1 & & \\
\quad\quad 0\ \checkmark & &
\end{array}$$

~~maradék~~

Gyökhelygörbe

$H(s)=\frac{1}{1+sk} \Rightarrow s=-\frac{1}{k}$

k paraméter alapján nézzük hogy változik a pólusok helye



pl:

$$H(s) = \frac{1 \cdot K}{s+5}$$

$$\Downarrow$$

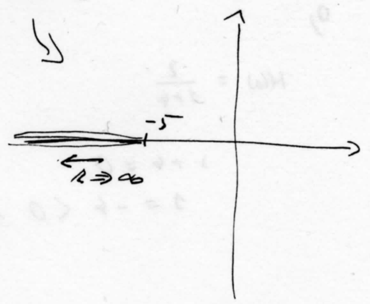
$$T = \frac{K \cdot \frac{1}{s+5}}{1 + K \cdot \frac{1}{s+5}}$$

→ 1. $$\frac{K \cdot \frac{1}{s+5}}{\frac{s+5+K}{s+5}} = \frac{K}{s+5} \cdot \frac{s+5}{s+5+K} = \frac{K}{s+5+5} \} = 0$$

→ $$1 + K \cdot \frac{1}{s+5} = 0 = \frac{s+5+K}{s+5} = 0 \Rightarrow s + K + 5 = 0$$

$$\Downarrow$$

$$s = -5 - K$$



pl:

$$H(s) = \frac{s+2}{s^2+s+1}$$ + negatív visszacsatolás

1. zárt rendszer

$$T = \frac{K \frac{s+2}{s^2+s+1}}{1 + K \cdot \frac{s+2}{s^2+s+1}} \Rightarrow 1 + K \frac{s+2}{s^2+s+1} = 0$$

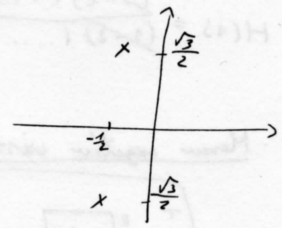
$$s^2 + s + 1 + K(s+2) = 0$$

$$s^2 + (K+1)s + 2K + 1 = 0$$

$$\Delta = (K+1)^2 - 4(2K+1)$$

$$s_{1,2} = \frac{-K-1 \pm \sqrt{\Delta}}{2}$$

K = 0 eset



Δ pozitív eset ⇒ van 2 valós pólus

$$\Delta = K^2 + 2K + 1 - 8K - 4 =$$

$$= K^2 - 6K - 3$$

$$\Delta_2 = 36 + 12 \quad 48$$

$$K_{1,2} = \frac{6 \pm \sqrt{48}}{2} \quad \begin{cases} \approx 6 > 0 \\ \approx -\frac{1}{2} \end{cases}$$

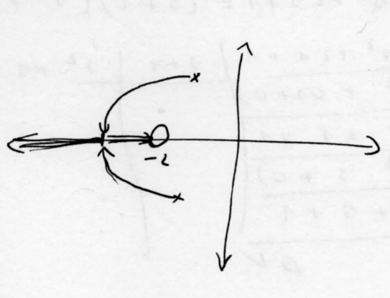
$$\Delta = \underbrace{K^2 - 6K + 9}_{(K-3)^2} - 9 - 3$$

$$\Delta = (K-3)^2 \left(1 - \frac{12}{(K-3)^2}\right)$$

$$\Delta_{1,2} = \frac{-K-1 \pm (K-3)\sqrt{\dots}}{2}$$ → 1 ha K nagy

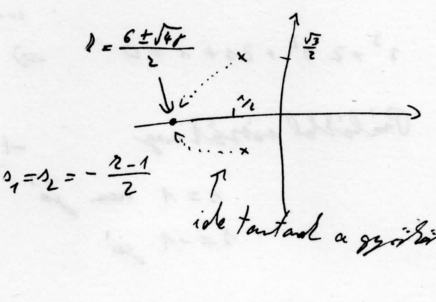
$$s_1 = \frac{-K-1+K-3}{2} = -2$$

$$s_2 = -\infty$$

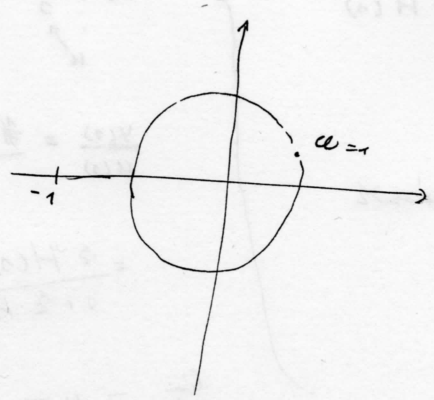


Valós szakasza a gyökhely görbének : $\#(p > x) - \#(z > 0) =$ páratlan

(K H(s) + (-1) visszacsatolás típusú esetben)



- H stabil
- ⊖ visszacsatolás } H nyquist diagramja nem veszi körbe a -1 pontot



Pl: $H(s) = \frac{s+2}{s+5}$

$u = A \cdot \sin(\omega t + \xi_0)$

$Y_t(t) = G(\omega) A \sin(\omega t + \varphi_0 + \varphi_z)$

$H(s) = \frac{s+2}{s+5}\Big|_{s=j\omega} = H(j\omega)$

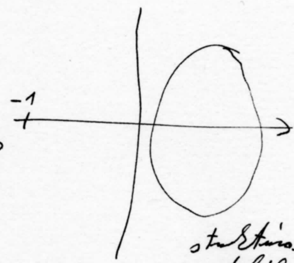
$\omega = 1 \Rightarrow H(s) = \frac{(2+1)(5-i)}{(5+i)(5-i)} = \frac{10+3i+1}{25+i} = \frac{11+3i}{26}$

Pl:

$$H(\omega) = \frac{i\omega+2}{i\cdot\omega+5} = \frac{(i\omega+2)(i\omega-5)}{(i\omega+5)(i\omega-5)} = \frac{-\omega^2-3i\omega+10}{\underset{((i\omega)^2)}{-\omega^2}\ \underset{(5^2)}{-25}} =$$

$$= \frac{\omega^2+3i\omega+10}{\omega^2+5^2} = \underbrace{\frac{\omega^2+10}{\omega^2+5^2}}_{>0 \text{ mindig}} + \frac{3\omega i}{\omega^2+5^2}$$

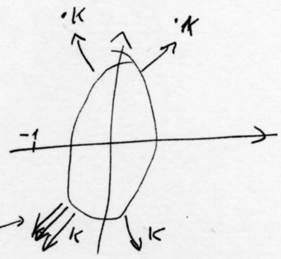
>0 mindig ⟶ pozitív térfélen járkál. ⟹



strukturálisan stabil rendszer

$$H(\omega) = \frac{i\omega+2}{i\omega-5} = \ldots$$

$$= \frac{-\omega^2+7i\omega+10}{-\omega^2-25} = \frac{\omega^2-10}{\omega^2+5^2} + \frac{-7\omega i}{\omega^2+5^2} \Longrightarrow$$

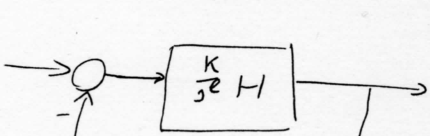


tegyük vissza az erősítést → K

∃ K, hogy a rendszer nincsen stabil

Nyquist diagram nyílt körnél nézi a zárt kör stabilitását
Zárt körnél z/p-ket kell nézni

Statikus jelkövetés

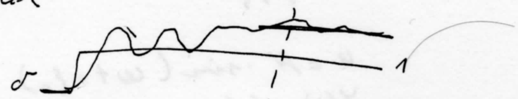


$\frac{K}{s^e}$	Hiba a kiírásra: K	K/s	K/s²
	$1/(K+1)$	0	0
x	∞	$\frac{1}{K}$	0
x^2	∞	∞	$\frac{1}{K}$

pl:

$$L = \frac{2}{s} \cdot H(s) \Rightarrow T(s) = \frac{\frac{2}{s} \cdot H(s)}{1 + \frac{2}{s} H(s)}$$

Végérték



↳ beállás után nézzük

$$Y(s) = T(s) \cdot U(s)$$

egységugrás

$$U(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s} \cdot \frac{2 \cdot H(s)}{s + 2 \cdot H(s)}$$

u

$$\frac{Y(s)}{U(s)} = \frac{\frac{1}{\cancel{s}} \cdot \frac{2 \cdot H(s)}{s + 2 \cdot H(s)}}{\frac{1}{\cancel{s}}} =$$

$$= \frac{2\, H(s)}{s + 2 \cdot H(s)}$$

$$\lim_{s \to 0} \quad - \text{''} - \quad = \frac{2}{2} = 1 \Rightarrow \text{hiba 0-hoz tart}$$