1 ideális PID

$$W_{PID}(s) = A_P\left(1 + \frac{1}{sT_i} + sT_d\right) = \frac{A_P}{T_i}\cdot\frac{1+sT_i+s^2T_iT_d}{s}$$

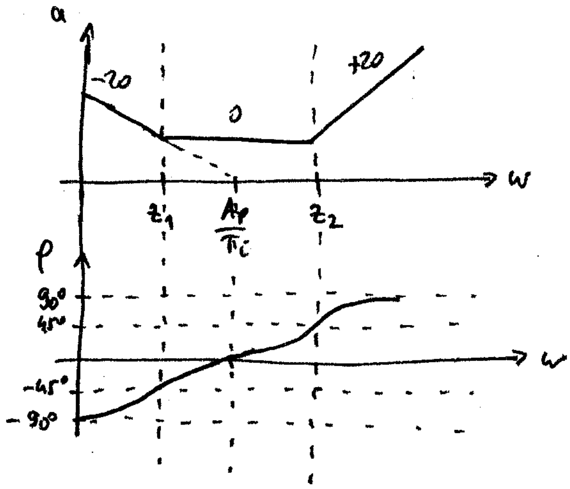
$$z_{1,2} = \frac{-T_i \pm \sqrt{T_i^2 - 4T_iT_d}}{2\cdot T_i\cdot T_d} \rightarrow \text{valós, ha } T_i^2 - 4\cdot T_i\cdot T_d \geq 0$$

$$\hookrightarrow \underline{T_d \leq \frac{T_i}{4}}$$

$$p_1 = 0$$

2 ideális PID

$$W_{PID}(s) = \frac{A_P}{T_i}\cdot\frac{1+sT_i+s^2\cdot T_i\cdot T_d}{s}$$



3 közelítő PID

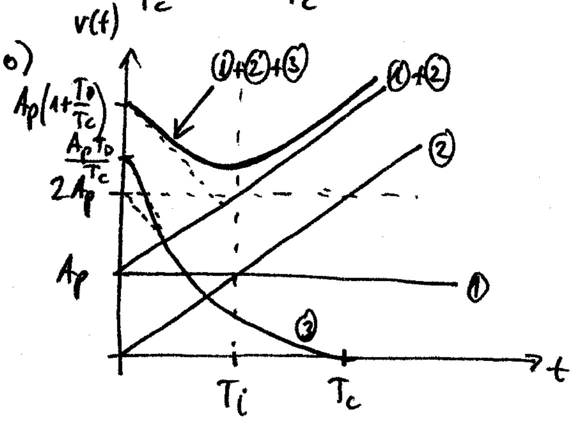
$$W_{PID}(s) = A_P\left(1 + \frac{1}{sT_i} + \frac{sT_D}{1+sT_c}\right)$$

$$v(t) = \mathcal{L}^{-1}\left\{\frac{W_{PID}(s)}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{A_P}{s} + \frac{A_P}{T_i}\cdot\frac{1}{s^2} + \frac{A_P}{s}\cdot\frac{sT_D}{1+sT_c}\right\}$$

$$\frac{A_P}{s} \rightarrow A_P \qquad \frac{A_P}{T_i}\cdot\frac{1}{s^2} \rightarrow \frac{A_P}{T_i}\cdot t \qquad \frac{A_PT_D}{T_c}\cdot\frac{1}{s+\frac{1}{T_c}} \rightarrow \frac{A_PT_D}{T_c}\cdot e^{-\frac{t}{T_c}}$$

$$v(t) = \underbrace{A_P}_{①} + \underbrace{\frac{A_P}{T_i}\cdot t}_{②} + \underbrace{\frac{A_PT_D}{T_c}e^{-\frac{t}{T_c}}}_{③}$$

$(t \geq 0)$



4 közelítő PID

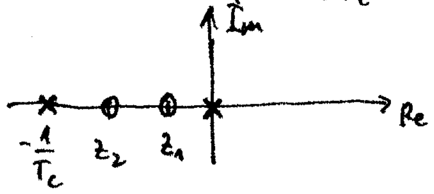
$$W_{PID}(s) = A_P\left(1 + \frac{1}{sT_i} + \frac{sT_D}{1+sT_c}\right) = \frac{A_P}{T_i}\left(\frac{1+s(T_i+T_c)+s^2T_i(T_c+T_d)}{s(1+sT_c)}\right)$$

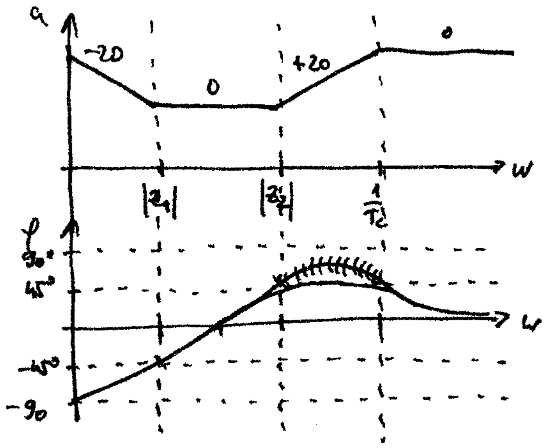
$$z_{1,2} = \frac{-(T_i+T_c) \pm \sqrt{(T_i+T_c)^2 - 4T_i(T_c+T_d)}}{2\cdot T_i(T_c+T_d)} \rightarrow \text{valós, ha } (T_i+T_c)^2 - 4T_i(T_c+T_d) \geq 0$$

$$\underline{\text{ha } T_D \leq \frac{(T_i-T_c)^2}{4\cdot T_i}}$$

5

$$W_{PID}(s) = A_p\left(1+\frac{1}{sT_i}+\frac{sT_D}{1+sT_c}\right) = \frac{A_p}{T_i}\cdot\frac{1+s(T_i+T_c)+s^2T_i(T_c+T_d)}{s(1+sT_c)}$$





- gyors működés
- körerősítést $\frac{A_p}{T_i}$-szeresére növeli
- típusszámot 1-el növeli
- adott fázistartalék elérése
- statikus pontosság

6

$$a_n\cdot s^n + a_{n-1}\cdot s^{n-1} + \dots + a_1\cdot s + a_0 = 0 \qquad a_n > 0$$

Hurwitz: 1. $\forall a_i > 0$

2. $\forall \Delta_i > 0$ → aldeterminánsok

$$\begin{matrix} a_{n-1} & a_{n-3} & a_{n-5} & \dots \\ a_n & a_{n-2} & a_{n-4} & \dots \\ 0 & a_{n-1} & a_{n-3} & \\ 0 & a_n & a_{n-2} & \\ \vdots & & & \end{matrix}$$

$$\Delta 1 = |a_{n-1}|$$

$$\Delta 2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix}$$

$\vdots$

7

$$W_0(s) = \frac{b_{n-1}s^{n-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}$$

$$W(s) = \frac{W_0(s)}{1+W_0(s)} = \frac{b_{n-1}s^{n-1} + \dots + b_1 s + b_0}{a_n s^n + (a_{n-1}+b_{n-1})s^{n-1} + \dots + (a_1+b_1)s + (a_0+b_0)}$$

$\mathrm{Re}\{s_i\} < 0$ → zárt rendszer stabil

8

$$W_0(s) = \frac{25(s+0{,}1)}{s(s+1)(s+5)} \qquad W(s) = \frac{W_0(s)}{1+W_0(s)} = \cancel{\frac{25s+2{,}5}{s^3+6s^2+5s}} \quad \frac{s^3+6s^2+30s+2{,}5}{s^3+6s^2+5s}$$

$$\Delta 3 = \begin{vmatrix} 6 & 2{,}5 & 0 \\ 1 & 30 & 0 \\ 0 & 6 & 2{,}5 \end{vmatrix}$$

mivel $\det(\Delta 3) > 0$ → STABIL

9 Nyquist-stabilitáskritérium, P db labilis pólus

zárt rendszer stabil, ha a teljes Ny-görbe a −1 pontot az óramutató járásával ellentétes irányban P-szer veszi körül

$$\mathrm{EKVSZ}\left(W_0(j\omega), -1\right) = P$$

10 Nyquist-kritérium

$s_{1,2} = 1 \pm j10 \rightarrow$ 2 pólus $p = 2$

$\underline{EKVSZ\left(W_0(j\omega), -1\right) = 2}$

11 Bode-stabilitáskrit.

$KVSZ\left(W_0(j\omega), -1\right) = 0 \rightarrow$ ne vegye körbe a -1 pontot

ω_c – ahol az amplitúdógörbe metszi a 0dB-es tengelyt

↳ $a_{dB}(\omega_c) = \varnothing$

$|W(j\omega_c)| = 1$

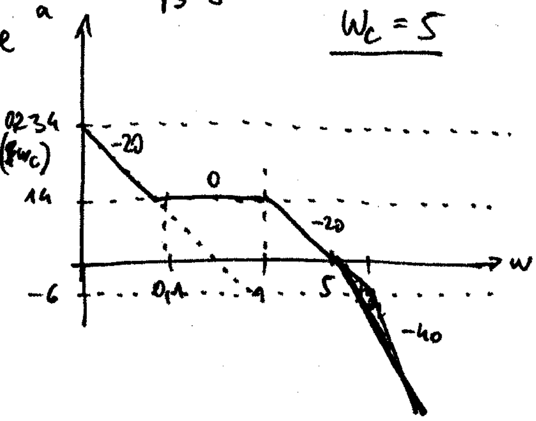
$\varphi_t = 180° + \varphi(\omega_c) \rightarrow$ ha $\varphi_t > 0 \rightarrow$ STABIL

12 $W_0(s) = \dfrac{25(s+0{,}1)}{s(s+1)(s+5)} = \dfrac{0{,}5}{s} \cdot \dfrac{1+10s}{(1+s)(1+0{,}2s)}$

$p_1 = 0$ $\quad z = 0{,}1$
$p_2 = 1$
$p_3 = 5$

$k = 0{,}5 \qquad 20 \log k = -6 \rightarrow \omega = 1$-nél

$\underline{\omega_c = 5}$



$\varphi(\omega_c) = 180° - 90 + \operatorname{arctg}(10\omega_c) - \operatorname{arctg}(\omega_c) - \operatorname{arctg}(0{,}2\omega_c)$

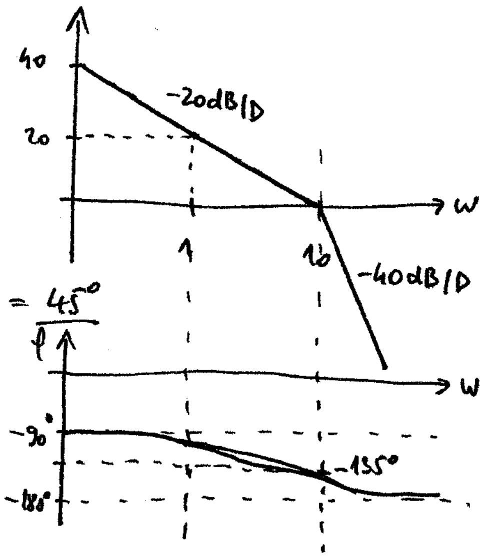
mivel $\underline{\varphi_t > 0}$, ezért a rsz STABIL

13 $W_0(s) = \dfrac{10}{s(1+0{,}1s)}$

$k = 10$

$\underline{\omega_c = 10}$

$\underline{\varphi_t} = 180° - 90° - \operatorname{arctg}(0{,}1\omega_c) = \underline{45°}$



$\cancel{\dfrac{10}{0{,}1s^2+s} = \dfrac{1}{1+2\xi Ts+T^2s^2}}$

$\cancel{T^2 = 0{,}1}$

$W(s) = \dfrac{W_0(s)}{1+W_0(s)} = \dfrac{1}{1+0{,}1s+0{,}01s^2}$

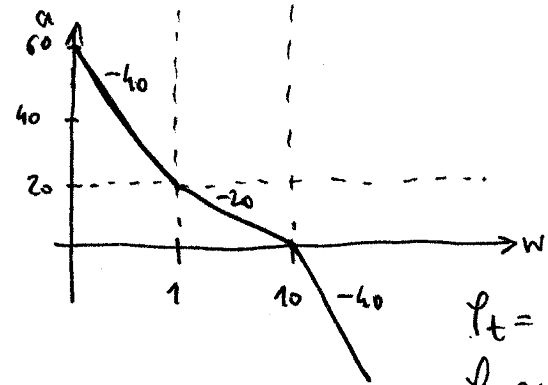
$1 + 0{,}1s + 0{,}01s^2 = 1 + 2\xi \cdot T \cdot s + T^2 \cdot s^2$

$T = 0{,}1 \rightarrow \underline{\omega_0 = 10}$

$\underline{\xi = 0{,}5}$

14 $W_0(s) = \dfrac{10(1+s)}{s^2(1+0{,}1s)}$

$k = 10$



$\underline{\omega_c = 10}$

$\varphi_t = 180° - 90° - 90° + \operatorname{arctg}\omega_c - \operatorname{arctg} 0{,}1\omega_c$

$\underline{\varphi_t \approx 80°}$

15

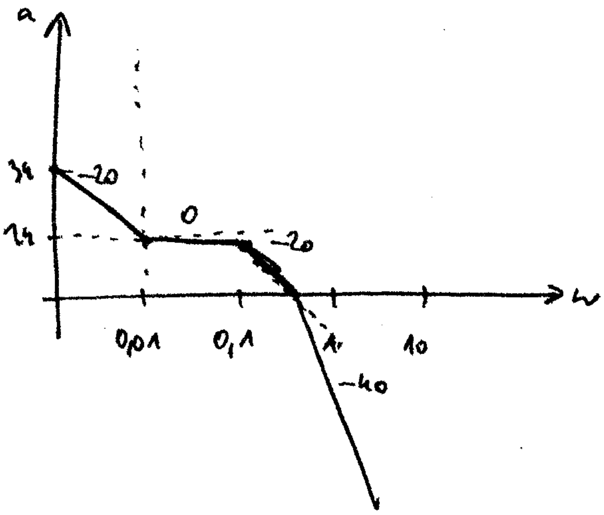
$$W_0(s) = \frac{0{,}05\,(1+100s)}{s(1+10s)(1+2s)}$$

$\underline{k = 0{,}05}$

$\underline{w_c = 0{,}5}$

$$\varphi(t) = 180° - 90° + \operatorname{arctg}(0{,}01 w_c) - \operatorname{arctg}(0{,}1 w_c) - \operatorname{arctg}(0{,}5 w_c)$$

$$\underline{\varphi(t)} = 90° + 1 - 9 - 44{,}1 = \underline{37{,}9°}$$



16

$$W_p(s) = \frac{A}{(1+sT_1)(1+sT_2)(1+sT_3)}$$

$A = 10$

$T = 100, 10, 1$

PI: $W_{PI} = \frac{A_p}{T_i} \cdot \frac{1+sT_i}{s} \rightarrow T_i = T_1$

PD közelítő: $W_{PD} = A_p \cdot \frac{1+s(T_d+T_c)}{1+sT_c}$ → 2. leglassabb pólus kiejtése

$T_d + T_c = T_2$

PID közelítő: $$W_{PID} = \frac{A_p}{T_i} \cdot \frac{1 + s(T_i+T_c) + s^2 \cdot T_i^2 (T_d+T_c)}{s(1+sT_c)}$$

$\underline{T_1 \text{ és } T_2 \text{ kiejtése}}$

17

$$W_0(s) = \frac{K}{s} e^{-sT_h}$$

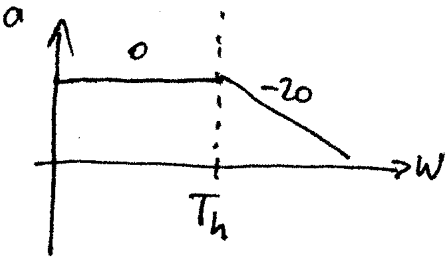
$\varphi_t = 45°$

$\varphi(w) = -w \cdot T_h$

$$\varphi_t = 180° - 90° - w_c \cdot T_h \left(\frac{180}{\pi}\right) = 45°$$

$$\underline{w_c = \frac{\pi}{180} \cdot \frac{45°}{T_h}}$$

$\underline{k = w_c}$



18

$$W_p(s) = \frac{A}{1+sT} e^{-sT_h}$$

$\varphi_t = 45°$

$K = \frac{A \cdot A_p}{T_i}$

PI: $A_p\left(1 + \frac{1}{sT_i}\right) = \frac{A_p}{T_i} \cdot \frac{1+sT_i}{s}$

$\underline{T_i = T}$

$$W(s) = \overset{K=}{\frac{A \cdot A_p}{T_i}} \cdot \frac{1}{s} e^{-sT_h} = \frac{k}{s} e^{-sT_h}$$

$$45° = 180° - 90° - w_c \cdot T_h \left(\frac{180}{\pi}\right)$$

$$\underline{w_c = \frac{\pi}{180} \cdot \frac{45°}{T_h}}$$

$\underline{K = w_c}$

$$\underline{w_c = \frac{A \cdot A_p}{T_i}}$$

$$T_i = \frac{A \cdot A_p}{w_c}$$