1, [12] a, [10] $y'(x) = \frac{1}{3}(3y+2x)^2 + \frac{7}{3}$; $u(x) = 3y + 2x$ (3)

$y' = \left(\frac{u-2x}{3}\right)' = \frac{1}{3}u' - \frac{2}{3}$

$\frac{1}{3}u' - \frac{2}{3} = \frac{1}{3}u^2 + \frac{7}{3}$

$u' = \frac{du}{dx} = u^2 + 9$ (1) $\Rightarrow \int \frac{du}{u^2+9} = \int dx$ (1) $\Rightarrow \frac{1}{9}\,\mathrm{arctg}\Big(\underbrace{\frac{u}{3}}_{y+\frac{2}{3}x}\Big)\cdot 3$ (3) $= x + C$ (1)

$y_{ált}(x) = \mathrm{tg}(3(x+C)) - \frac{2}{3}x$ (1)

b, [2] $1 = \mathrm{tg}(3C) - 0 \Rightarrow 3C = \frac{\pi}{4}$; $C = \frac{\pi}{12}$; $y_{kezd}(x) = \mathrm{tg}\left(3x + \frac{\pi}{4}\right) - \frac{2x}{3}$ (2)

2, [10] (H): $y' = -4(x+1)y \Rightarrow \int \frac{dy}{y} = -4\int (x+1)\,dx$

$\ln|y| = -2x^2 - 4x + C$; $y_{H,ált}(x) = K e^{-2x^2-4x}$; $K \in \mathbb{R}$ (5)

$y_{I,p}(x) = K(x)e^{-2x^2-4x}$ (1) ; $K'(x)e^{-2x^2-4x} + \cancel{K(x)\cdot(-4x-4)e^{-2x^2-4x}} +$ (1)

$+ \cancel{4(x+1)K(x)e^{-2x^2-4x}} = e^{-2x^2}$

$K'(x) = e^{4x}$ (1) ; $K(x) = \int e^{4x}dx = \frac{1}{4}e^{4x}$; $y_{I,p}(x) = \frac{1}{4}e^{-2x^2}$ (1)

$y_{I,ált}(x) = K e^{-2x^2-4x} + \frac{1}{4}e^{-2x^2}$; $K \in \mathbb{R}$ (1)

3, [8] $f(n+1) = -f(n) + 12f(n-1)$; $q^{n+1} = -q^n + 12q^{n-1}$

$q^2 + q - 12 = (q+4)(q-3) = 0$ (2) ; $f(n) = A\,3^n + B(-4)^n$ (2)

$\left.\begin{array}{l} f(0) = -3 \Rightarrow A + B = -3 \\ f(1) = 26 \Rightarrow 3A - 4B = 26 \end{array}\right\}$ (2) $7A = 14$; $A = 2$; $B = -5$

$f(n) = 2\cdot 3^n - 5\cdot(-4)^n$ (2)

4, $\boxed{10}$ (H) $\lambda^2-2\lambda+10=0$; $\lambda_{1,2}=1\pm\sqrt{1-10}=1\pm 3i$; ②

$y_{H,\acute{a}lt}(x)=C_1 e^x \sin(3x)+C_2 e^x \cos(3x)$; $C_1, C_2 \in \mathbb{R}$ ②

$y_{I,p}(x)=A\sin(3x)+B\cos(3x)$ ② $/\cdot 10$

$y'_{I,p}(x)=3A\cos(3x)-3B\sin(3x)$ $/\cdot(-2)$

$\oplus$ $y''_{I,p}(x)=-9A\sin(3x)-9B\cos(3x)$ $/\cdot 1$

$4\cos(3x)=\underbrace{(10A+6B-9A)}_{A+6B}\sin(3x)+\underbrace{(10B-6A-9B)}_{B-6A}\cos(3x)$

$\left.\begin{array}{r} A+6B=0 \\ -6A+B=4 \end{array}\right\}$ ② $\Longrightarrow 37B=4$; $B=\frac{4}{37}$; $A=-\frac{24}{37}$ $y_{I,p}(x)=\frac{-24}{37}\sin(3x)+\frac{4}{37}\cos(3x)$ ①

$y_{I,\acute{a}}(x)=C_1 e^x\sin(3x)+C_2 e^x\cos(3x)-\frac{24}{37}\sin(3x)+\frac{4}{37}\cos(3x)$ ①

5, a, $\boxed{5}$ $\frac{1}{k(k+1)}=\frac{1}{k}-\frac{1}{k+1}$; ② Teleszkópikus

$S_k=\left(1-\cancel{\tfrac{1}{2}}\right)+\left(\cancel{\tfrac{1}{2}}-\cancel{\tfrac{1}{3}}\right)+\dots+\left(\cancel{\tfrac{1}{k}}-\frac{1}{k+1}\right)=1-\frac{1}{k+1}$ ② $\longrightarrow 1=S$ ①

b, $\boxed{5}$ $\sum_{n=1}^{\infty}\frac{(-3)^{n+3}+2^{2n+1}}{5^n}=-27\sum_{n=1}^{\infty}\left(-\frac{3}{5}\right)^n+2\sum_{n=1}^{\infty}\left(\frac{4}{5}\right)^n=$ ③

$=-27\left(\frac{-3}{5}\right)\frac{1}{1+3/5}+2\cdot\frac{4}{5}\cdot\frac{1}{1-4/5}=\frac{27\cdot 3}{\cancel{5}}\cdot\frac{\cancel{5}}{8}+\frac{2\cdot 4}{\cancel{5}}\cdot\cancel{5}=$ ②

$=18+\frac{1}{8}$