1, $y' = \operatorname{ch}(\underbrace{x+y}_{u(x)})$ $\quad u'(x) = 1 + y'$; $\quad y' = u' - 1$

$\boxed{30}$

$u' - 1 = \operatorname{ch} u$ szeparálható $\Rightarrow \int \frac{du}{1+\operatorname{ch} u} = \int dx$ $\boxed{10}$

$\int dx = x + C$ (3)

$\boxed{12}$

$$\int \frac{du}{1+\operatorname{ch} u} = 2\int \frac{du}{2+e^u+e^{-u}} = 2\int \frac{dz}{z(2+z+1/z)}\Bigg|_{z=e^u} = 2\int \frac{dz}{z^2+2z+1}\Bigg|_{z=e^u} =$$

$z = e^u$; $dz = e^u du = z\,du$

$$= 2\int (z+1)^{-2}\,dz\Big|_{z=e^u} = \frac{-2}{z+1} + C\Bigg|_{z=e^u} = \frac{-2}{e^u+1} + C$$

Tehát $\frac{-2}{e^u+1} = x + C \Rightarrow \frac{-2}{e^{x+y}+1} = x + C$ (5)

2, $y' + \frac{2}{x} y = 3x^2$; $y(1) = 4$ elsőrendű lineáris

$\boxed{20}$

(H): $y' + \frac{2}{x} y = 0 \underset{y=0\checkmark}{\Rightarrow} \int \frac{dy}{y} = -\int \frac{2}{x}\,dx \Rightarrow \ln|y| = -2\ln|x| + C$

$\Rightarrow y_{H,\text{ált}}(x) = K \cdot \frac{1}{x^2}$; $K \in \mathbb{R}$. $\boxed{7}$

$y_{I,p}(x) = K(x) \cdot \frac{1}{x^2}$; $\quad y_{I,p}'(x) = K'(x) \cdot \frac{1}{x^2} + K(x) \cdot \frac{-2}{x^3}$

Beírva: $\frac{K'}{x^2} - \cancel{\frac{2K}{x^3}} + \cancel{\frac{2}{x} \cdot \frac{K}{x^2}} = 3x^2 \Rightarrow K'(x) = 3x^4$

$\Rightarrow K(x) = \int 3x^4\,dx = \frac{3}{5} x^5$; $\quad y_{I,p}(x) = \frac{3}{5} \cdot x^3$

$\boxed{8}$

$y_{I,\text{ált}}(x) = \frac{K}{x^2} + \frac{3}{5} x^3$; (2)

Kezdeti felt.:

$4 = \frac{K}{1^2} + \frac{3}{5} 1^3 \Rightarrow K = 4 - \frac{3}{5} = \frac{17}{5} \Rightarrow y(x) = \frac{17}{5} x^{-2} + \frac{3}{5} x^3$ (3)

3, a, [8] $5e^{3x} - 7\sin(3x) \Rightarrow \lambda_1 = 3;\ \lambda_{2,3} = \pm 3i$ ③

$$(\lambda-3)\underbrace{(\lambda+3i)(\lambda-3i)}_{\lambda^2+9} = \lambda^3 - 3\lambda^2 + 9\lambda - 27 \quad ②$$

$$\boxed{y''' - 3y'' + 9y' - 27y = 0} \quad ①$$

[6]

$$\underline{y_{ált}(x) = Ae^{3x} + B\sin(3x) + C\cos(3x)} \quad ②$$

— o —

b, [8] $xe^{-2x};\ 3x-4 \Rightarrow \lambda_{1,2} = -2;\ \lambda_{3,4} = 0$

$$(\lambda+2)^2\cdot\lambda^2 = \lambda^4 + 4\lambda^3 + 4\lambda^2 \Rightarrow \underline{y^{(4)} + 4y''' + 4y'' = 0}$$

[6]

$$\underline{y_{ált}(x) = Ae^{-2x} + Bxe^{-2x} + Cx + D} \quad ②$$

— o —

c, [9] $2\,\mathrm{ch}(5x) = e^{5x} + e^{-5x}$ ② $;\ 7xe^{5x} \Rightarrow \lambda_{1,2} = 5;\ \lambda_3 = -5$

[5]

$$\underbrace{(\lambda-5)^2}_{\lambda^2-10\lambda+25}(\lambda+5) = \lambda^3 - 5\lambda^2 - 25\lambda + 125 \Rightarrow \underline{y''' - 5y'' - 25y' + 125y = 0}$$

$$\underline{y_{ált}(x) = Ae^{5x} + Bxe^{5x} + Ce^{-5x}} \quad ②$$

— o —

4, a, [9] $a_n = \dfrac{2n+3}{\sqrt{n^5 - 8n^3 - 5}} \underset{\substack{\uparrow \\ n\gg 1 \text{ esetén}}}{\approx} \dfrac{2n}{n^{5/2}} = \dfrac{2}{n^{3/2}}$, és $\sum_n \dfrac{1}{n^{3/2}} < \infty$,

Sejtés: konv. $\Rightarrow$ majoráns krit. ③

de $a_n = \dfrac{2n+3}{\sqrt{n^5 - 8n^3 - 5}} \underset{\uparrow}{\le} \dfrac{2n + 3n}{\sqrt{n^5 - \frac{1}{2}n^5}} = \dfrac{5n}{\sqrt{\frac{1}{2}n^5}} = 5\sqrt{2}\cdot n^{-3/2} =: b_n$

ha $\frac{1}{2}n^5 > 8n^3 + 5$,

azaz ha $n > N_0 \in \mathbb{N}$ ⑤

$\sum_n b_n = 5\sqrt{2}\sum_n \dfrac{1}{n^{3/2}} < \infty$ $(3/2 > 1)$, tehát a majoráns krit. értelmében az eredeti $\sum_n a_n$ sor is <u>konvergens</u>. ①

4/b, i, [8] $3f(n+1)=4f(n)+4f(n-1)$ $f(n)=q^n$

$3q^2=4q+4$; $3q^2-4q-4=0$; ③ $q_{1,2}=\dfrac{4\pm\sqrt{16+48}}{6}$ = 2 ; $-\frac{2}{3}$ ③ (8)

$f_{\acute{a}ll}(n)=A\cdot 2^n+B\left(-\frac{2}{3}\right)^n$ ②

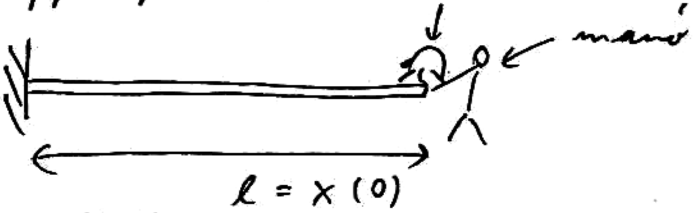
ii, [8] $\sum_n f(n)$ konvergens $\Rightarrow A=0$; $f(n)=B\left(-\frac{2}{3}\right)^n$ ③

$f(0)=\underline{\underline{B=3}}$ ① ; $f(1)=3\cdot\left(-\frac{2}{3}\right)^1=\underline{\underline{-2}}$ ①

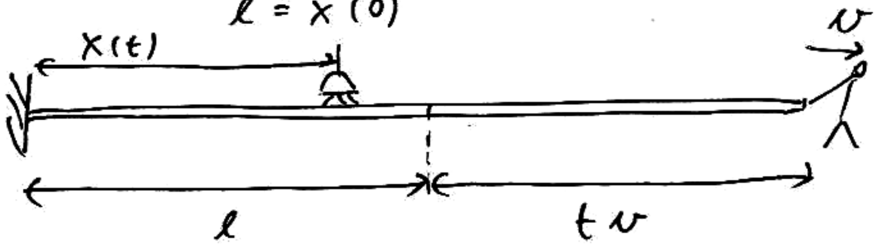
$S=\sum_{n=0}^{\infty}3\left(-\frac{2}{3}\right)^n=3\cdot\dfrac{1}{1-(-2/3)}=3\cdot\dfrac{3}{5}=\underline{\underline{\dfrac{9}{5}}}$ ③

[MSC] Jelölje $x(t)$ a katicabogár távolságát a rögzített végponttól az idő függvényében!

Kezdeti helyzet:
$t=0$



t időpillanatban:



A kötél sebessége az $x(t)$ helyen: $v\cdot\dfrac{x(t)}{l+tv}$;

a katica sebessége a kötélhez képest: $-v$

$\dot{x}(t)=\dfrac{v}{l+tv}\,x(t)-v$ ③ elsőrendű, lineáris d.e.

(H) $\int\dfrac{dx}{x}=\int\dfrac{v}{l+tv}\,dt \Rightarrow \ln x=\ln(l+tv)+C \Rightarrow \underline{x_H(t)=K(l+tv)}$

$x_{I,p}(t)=K(t)(l+tv)$; $\dot{x}_{I,p}(t)=\dot{K}(t)(l+tv)+K(t)\cdot v$

Beírva: $\dot{K}(l+tv)+\cancel{Kv}=\dfrac{v}{\cancel{l+tv}}\cancel{K(l+tv)}-v$; $\dot{K}(t)=\dfrac{-v}{l+tv}$

$K(t)=-\int\dfrac{v}{l+tv}\,dt=-\ln(l+tv)$; $x_{I,p}(t)=-(l+tv)\ln(l+tv)$

IMSC (folyt.)

$$x_{\text{ált}}(t) = (\ell + tv)\left(K - \ln(\ell + tv)\right) \quad ②$$

Kezd. felt.: $x(0) = \ell \Rightarrow \ell(K - \ln \ell) = \ell \Rightarrow K = \ln \ell + 1$

$$x(t) = (\ell + tv)\left(1 + \ln\left(\frac{\ell}{\ell + tv}\right)\right) = (\ell + tv)\left(1 - \ln\left(\frac{\ell + tv}{\ell}\right)\right)$$

A katica T időpillanatban eléri a falat: $x(T) = 0$

$$\Rightarrow 1 - \ln\left(\frac{\ell + Tv}{\ell}\right) = 0 \Rightarrow \frac{\ell + Tv}{\ell} = e \Rightarrow T = \frac{\ell}{v}(e - 1)$$

Tehát a katica $T = \frac{\ell}{v}(e-1) = \frac{100\,\text{cm}}{1\,\frac{\text{cm}}{\text{s}}} \cdot (2.71 - 1) = 171\,\text{s}$ alatt éri el a falat. ①

Egyenlet felírása: 3 IMSC pont

Ált. megoldás : 2 —"—

Kezd. felt. és T : 1 —"—

 $-\beta-$ (Csak végeredmények)

1, $u = y - x$ helyettesítéssel szeparálható: $\frac{du}{ch\,u - 1} = dx$ $\boxed{10}$
$\boxed{30}$

$\int dx = x + c$ ③

$z = e^{u}$ helyettesítés

$$\int \frac{du}{ch\,u-1} = \ldots = \frac{-2}{e^{u}-1} + c \quad \boxed{12}; \text{ megoldás: } \underline{\underline{\frac{-2}{e^{x+y}-1} = x + c}} \quad \boxed{5}$$

2, $y' + \frac{3}{x} y = 2x^3$ elsőrendű lin.
$\boxed{20}$

$\underline{y_{H,ált}(x) = K \cdot x^{-3}}$ $\boxed{7}$ ... $\underline{y_{I,p}(x) = \frac{2}{7} x^4}$ $\boxed{8}$

$\underline{y_{ált} = K \cdot x^{-3} + \frac{2}{7} x^4}$ ②; kezd. felt. $K = \frac{19}{7}$ ③

3, a, $y_{ált}(x) = Ax + B + C e^{-3x} + D x e^{-3x}$ ②
$\boxed{8}$

$\lambda^2 (\lambda + 3)^2 = \lambda^4 + 6\lambda^3 + 9\lambda^2$; egyenlet... } $\boxed{6}$

$\boxed{8}$ b, $y_{ált}(x) = A \cdot e^{2x} + B\cos(2x) + C\sin(2x)$ ②

$(\lambda - 2)\underbrace{(\lambda + 2i)(\lambda - 2i)}_{\lambda^2 + 4} = \lambda^3 - 2\lambda^2 + 4\lambda - 8$; egyenlet... } $\boxed{6}$

$\boxed{9}$ c, $sh(4x) = \frac{e^{4x} - e^{-4x}}{2}$ ② ; $y_{ált}(x) = A e^{4x} + B x e^{4x} + C e^{-4x}$ ②

$\underbrace{(\lambda - 4)^2}_{\lambda^2 - 8\lambda + 16} \cdot (\lambda + 4) = \lambda^3 - 4\lambda^2 - 16\lambda + 64$; egyenlet } $\boxed{5}$

4, a, $a_n \sim \frac{n^2}{n^{5/2}} = \frac{1}{n^{1/2}} \Rightarrow \sum_n \frac{1}{\sqrt{n}} = \infty \Rightarrow$ minoráns } $\boxed{3}$
$\boxed{9}$

$$a_n = \frac{n^2 - 8n}{\sqrt{n^5 + 8n^3 - 5}} \underset{\uparrow}{\geq} \frac{n^2 - \frac{1}{2}n^2}{\sqrt{n^5 + 8n^5}} = \frac{1}{6} n^{-1/2} = b_n ; \sum_n b_n = \infty \Rightarrow \sum_n a_n = \infty \quad \boxed{5}$$

ha $n > N_0 \in \mathbb{N}$ minoráns kr. $\Rightarrow$ $\underline{\text{divergens}}$ ①

b, i, $2q^2 = 5q + 3$ ③; $q_1 = 3$; $q_2 = -\frac{1}{2}$ ③; $f_{ált}(n) = A \cdot 3^n + B(-\frac{1}{2})^n$ ②
$\boxed{8}$

$\boxed{8}$ ii, $A = 0$, $f(n) = B(-\frac{1}{2})^n$ ③; $B = 2$ ①; $f(1) = -1$ ①; $S = \sum_{n=0}^{\infty} 2\left(\frac{-1}{2}\right)^n = 2 \cdot \frac{1}{1 + \frac{1}{2}} = \frac{4}{3}$ ③