Kalkulus vizsga 2 2023. jan. 12.

1) (15p)

$$7x+2z=46$$
$$-14x+4y-3z=-106$$
$$21x+8y-10z=74$$

$$\left(\begin{array}{ccc|c} 7 & 0 & 2 & 46 \\ -14 & 4 & -3 & -106 \\ 21 & 8 & -10 & 74 \end{array}\right) \sim \left(\begin{array}{ccc|c} 7 & 0 & 2 & 46 \\ 0 & 4 & 1 & -14 \\ 0 & 8 & -16 & -64 \end{array}\right) \sim \left(\begin{array}{ccc|c} 7 & 0 & 2 & 46 \\ 0 & 4 & 1 & -14 \\ 0 & 0 & -18 & -36 \end{array}\right)$$

II+2I 3 III−2II 3

III−3I 3

$$7x+2z=46 \Rightarrow 7x+2\cdot 2=46 \rightarrow 7x=42 \rightarrow \underline{\underline{x=6}}$$
$$\rightarrow \quad 4y+z=-14 \Rightarrow 4y+2=-14 \rightarrow 4y=-16 \rightarrow \underline{\underline{y=-4}}$$

5

1 $$-18z=-36 \Rightarrow \underline{\underline{z=2}}$$

2.) (15p)

$$\lim_{n\to+\infty}\left(\frac{n+1}{n-10}\right)^{2n-1} = \lim_{n\to+\infty}\left(\left(\frac{n+1}{n-10}\right)^{n}\right)^{2}\cdot\left(\frac{n+1}{n-10}\right)^{-1} =$$ 2

$$= \lim\left(\left(\frac{\cancel{n}}{\cancel{n}}\cdot\frac{1+1/n}{1-\frac{10}{n}}\right)^{n}\right)^{2}\cdot\left(\frac{\cancel{n}}{\cancel{n}}\cdot\frac{1+1/n}{1-\frac{10}{n}}\right)^{-1} =$$ 2

$$= \lim\left(\frac{\overbrace{\left(1+1/n\right)^{n}}^{\to e^{1}}}{\underbrace{\left(1+\frac{-10}{n}\right)^{n}}_{\to e^{-10}}}\right)^{2}\cdot\left(\frac{1+\overset{\to 0}{1/n}}{1-\underset{\to 0}{\frac{10}{n}}}\right)^{-1} = \left(\frac{e^{1}}{e^{-10}}\right)^{2}\cdot 1 = \left(e^{11}\right)^{2} = e^{22}$$

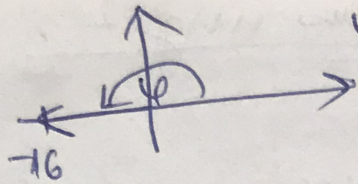
e^{1} 2 3

e^{-10} 2

$$\left(\frac{1}{1}\right)^{-1}=1$$ 2

$$\left(1+\frac{x}{n}\right)^{n} \to e^{x}$$ 2

3) (15p) $z^4=-16$



$\varphi=\pi$

$-16=-16+0\cdot i$

$r=\sqrt{(-16)^2+0^2}=16$

$-16=16\cdot(\cos\pi+i\cdot\sin\pi)$ (1) trigo alak!

negyedik gyökök:

$$z_k=\underbrace{\sqrt[4]{16}}_{2}\cdot\left(\cos\frac{\pi+k\cdot 2\pi}{4}+i\cdot\sin\frac{\pi+k\cdot 2\pi}{4}\right) \quad (2)$$

$k=0,1,2,3$

$$z_0=2\cdot\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right) \quad (1{,}5)$$

$$z_1=2\cdot\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right) \quad (1{,}5)$$

$$z_2=2\cdot\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right) \quad (1{,}5)$$

$$z_3=2\cdot\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right) \quad (1{,}5)$$

4.) a) (10p)

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=$$

behely: $\frac{\sqrt{1}-\sqrt{1}}{0}=\frac{0}{0}\to L'H$ (2) (2)

$$=\lim_{x\to 0}\frac{\frac{1}{2}\cdot(1+x)^{-1/2}-\frac{1}{2}(1-x)^{-1/2}\cdot(-1)}{1}=\underline{\underline{1}} \quad (3)$$

behely: $\frac{\frac{1}{2}\cdot 1^{-1/2}-\frac{1}{2}\,1^{-1/2}\cdot(-1)}{1}=$ (3)

$$=\frac{1/2+1/2}{1}=\underline{\underline{1}}$$

b)
(5p) $\lim\limits_{x\to 0^+} \dfrac{x^4-\sin(3\pi x)}{\sqrt{x^3+4\cos(-\pi x)}} = \underline{\underline{0}}$

Behely: $\dfrac{0^4-\underset{0}{\sin 0}}{\sqrt{0^3+4\cdot\underbrace{\cos 0}_{1}}} = \dfrac{0}{\sqrt{4\cdot 1}} = \dfrac{0}{2} = 0$ (1) (4)

5.)
(20p) $f(x) = \dfrac{x}{(1-2x)^2}$ $\qquad D_f = \mathbb{R}\setminus\{\frac{1}{2}\}$ (2)

$\hookrightarrow (1-2x)^2 \neq 0 \Rightarrow 1-2x\neq 0 \Rightarrow x\neq \frac{1}{2}$

$$f'(x) = \frac{1\cdot(1-2x)^2 - x\cdot 2\cdot(1-2x)\cdot(-2)}{(1-2x)^4} =$$

$$= \frac{\cancel{(1-2x)}\cdot[(1-2x)-2x\cdot(-2)]}{(1-2x)^{\cancel{4}\,3}} =$$

$$= \frac{1-2x+4x}{(1-2x)^3} = \frac{1+2x}{(1-2x)^3} = 0 \Leftrightarrow 1+2x=0$$ (6) (2)

$x = -\frac{1}{2}$

D_f: $-1/2$, $1/2$ (2)

	$(-\infty,-1/2)$	$x=-1/2$	$(-1/2,\ 1/2)$	$(\frac{1}{2},+\infty)$
f'	$-$	0	$+$	$-$
f	$\searrow$	lok. min.	$\nearrow$	$\searrow$

(4)

Tehát lok. min. hely: $x=-1/2$
Mon. fogyó: $(-\infty, -1/2)$-n és $(1/2, +\infty)$-n.
Mon. növő: $(-1/2, 1/2)$-n.

(4)

6.) (20p) $t = e^x \iff x = \underbrace{\ln t}_{g(t)}$ 2 $g'(t) = 1/t$ 2

$$\int f(x)\,dx \overset{4}{=} \int f(g(t)) \cdot g'(t)\,dt$$

$$\int \frac{e^x}{e^x+3}\,dx = \int \frac{\cancel{t}}{t+3} \cdot \frac{1}{\cancel{t}}\,dt \overset{4}{=}$$

$$\int \frac{1}{t+3}\,dt = \ln|t+3| + C \overset{4}{\underset{t=e^x}{=}}$$

$$\overset{4}{=} \ln|e^x+3| + C, \quad C \in \mathbb{R} \text{ tetsz.}$$

7.) (15p) $f(x) = e^{2x-1} - \frac{x^2}{20-x}$

$$f'(x) = 2 \cdot e^{2x-1} - \frac{2x \cdot (20-x) - x^2 \cdot (-1)}{(20-x)^2} =$$

$$= 2 \cdot e^{2x-1} - \frac{40x - 2x^2 + x^2}{(20-x)^2} =$$

$$= 2\,e^{2x-1} - \frac{40x - x^2}{(20-x)^2} \quad 7$$

Érintő egyenlete:

2 $y = f'(x_0) \cdot (x - x_0) + f(x_0)$, $x_0 = 0$

$f(0) = e^{-1} - 0 = e^{-1}$ 2

$f'(0) = 2 \cdot e^{-1} - 0 = 2e^{-1}$ 2

$\boxed{y = 2e^{-1}x + e^{-1}}$ 2