# Rendszerelmélet

2017.09.26.

## 1. HF

(1.) $x[k] = 1{,}39 \cdot \sin(0{,}3\pi k + 1{,}9) \qquad L = ?$

$$\frac{0{,}3\pi}{2\pi} = \frac{M}{L} = \frac{0{,}3}{2} = \frac{3}{20} \Rightarrow \boxed{L = 20}$$

(2.) $h[k] = -2 \cdot (0{,}1 \cdot (-7))^k \cdot \cos\left(\frac{2\pi}{7} \cdot k + 0{,}4\right) \cdot \varepsilon[k]$

$u[k] = 3 \cdot \delta[k] + 9 \cdot (0{,}1 \cdot 3)^{k+1} \cdot \varepsilon[k]$

$y[2] = ?$

$$y[2] = u[2] \cdot h[0] + u[1] \cdot h[1] + u[0] \cdot h[2] =$$

$$= \Big(\underbrace{3 \cdot \delta[2]}_{\emptyset} + \underbrace{9 \cdot (0{,}1 \cdot 3)^{2+1}}_{0{,}243} \cdot \underbrace{\varepsilon[2]}_{1}\Big) \cdot \Big(\underbrace{-2 \cdot (0{,}1 \cdot (-7))^0}_{-2} \cdot \underbrace{\cos\left(\frac{2\pi}{7} \cdot 0 + 0{,}4\right)}_{0{,}921} \cdot \underbrace{\varepsilon[0]}_{1}\Big) +$$

$$\Big(\underbrace{3 \cdot \delta[1]}_{\emptyset} + \underbrace{9 \cdot (0{,}1 \cdot 3)^{1+1}}_{0{,}81} \cdot \underbrace{\varepsilon[1]}_{1}\Big) \cdot \Big(\underbrace{-2 \cdot (0{,}1 \cdot (-7))^1}_{1{,}4} \cdot \underbrace{\cos\left(\frac{2\pi}{7} \cdot 1 + 0{,}4\right)}_{0{,}27} \cdot \underbrace{\varepsilon[1]}_{1}\Big) +$$

$$\Big(\underbrace{3 \cdot \delta[0]}_{3} + \underbrace{9 \cdot (0{,}1 \cdot 3)^{0+1}}_{2{,}7} \cdot \underbrace{\varepsilon[0]}_{1}\Big) \cdot \Big(\underbrace{-2 \cdot (0{,}1 \cdot (-7))^2}_{-0{,}98} \cdot \underbrace{\cos\left(\frac{2\pi}{7} \cdot 2 + 0{,}4\right)}_{-0{,}585} \cdot \underbrace{\varepsilon[2]}_{1}\Big) =$$

$$= (\emptyset + 0{,}243)(-2 \cdot 0{,}921) + (\emptyset + 0{,}81) \cdot (1{,}4 \cdot 0{,}27) + (3 + 2{,}7)(-0{,}98 \cdot (-0{,}585)) =$$

$$\boxed{= 3{,}126}$$

3.

$$h[k]=\varepsilon[k]\cdot 15{,}5\cdot 0{,}2^k+(-17{,}8)\cdot\delta[k]$$

$$u[k]=\varepsilon[k-3]\cdot(-0{,}4)^k$$

$$y[k]=?$$

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A válasz, ha $u[k]=\varepsilon[k-3]\cdot(-0{,}4)^{k-3} \overset{LTI}{=} \varepsilon[k]\cdot(-0{,}4)^k$

$$\Rightarrow u[k]=\varepsilon[k-3]\cdot(-0{,}4)^{k-3}\cdot(-0{,}4)^3=\varepsilon[k]\cdot(-0{,}4)^k\cdot(-0{,}4)^3$$

$$y[k]=\sum_{i=-\infty}^{\infty}u[i]\cdot h[k-i]=\underbrace{\sum_{i=-\infty}^{\infty}u[k-i]\cdot h[i]}_{\text{ezt használom}}$$

$$\rightarrow\sum_{i=-\infty}^{\infty}\left(\varepsilon[k-i]\cdot(-0{,}4)^{k-i}\cdot(-0{,}4)^3\right)\cdot\left(\varepsilon[i]\cdot 15{,}5\cdot 0{,}2^i+(-17{,}8)\cdot\delta[i]\right)=$$

$$=\sum_{i=-\infty}^{\infty}\varepsilon[k-i]\cdot(-0{,}4)^{k-i}\cdot(-0{,}4)^3\cdot\varepsilon[i]\cdot 15{,}5\cdot 0{,}2^i+\underbrace{\sum_{i=-\infty}^{\infty}\varepsilon[k-i]\cdot(-0{,}4)^{k-i}\cdot(-0{,}4)^3\cdot(-17{,}8)\cdot\delta[i]}_{\boxed{(-0{,}4)^k\cdot(-0{,}4)^3\cdot(-17{,}8)\cdot\varepsilon[k]=1{,}139\cdot(-0{,}4)^k}}$$

$$\rightarrow\sum_{i=0}^{k}(-0{,}4)^{k-i}\cdot(-0{,}4)^3\cdot 15{,}5\cdot 0{,}2^i=15{,}5\cdot(-0{,}4)^3\cdot(-0{,}4)^k\cdot\sum_{i=0}^{k}\left(\frac{0{,}2}{-0{,}4}\right)^i=$$

$$=15{,}5\cdot(-0{,}4)^3\cdot(-0{,}4)^k\cdot\frac{1-\left(\frac{0{,}2}{-0{,}4}\right)^{k+1}}{1-(-0{,}5)}=\frac{15{,}5}{1{,}5}\cdot(-0{,}4)^3\cdot(-0{,}4)^k\cdot\left(1-\frac{(0{,}2)^k}{(-0{,}4)^k}\cdot\left(\frac{0{,}2}{-0{,}4}\right)\right)=$$

$$=\frac{15{,}5}{1{,}5}\cdot(-0{,}4)^3\cdot(-0{,}4)^k-\frac{15{,}5}{1{,}5}\cdot\frac{(-0{,}4)^{\cancel{3}\,2}\cdot\cancel{(-0{,}4)^k}\cdot(0{,}2)^k\cdot(0{,}2)}{\cancel{(-0{,}4)^k}\cdot\cancel{(-0{,}4)}}=$$

$$=\frac{15{,}5}{1{,}5}\cdot(-0{,}4)^3\cdot(-0{,}4)^k-\frac{15{,}5}{1{,}5}\cdot(-0{,}4)^2\cdot(+0{,}2)\cdot(0{,}2)^k=\left(-0{,}661\cdot(-0{,}4)^k-0{,}331\cdot(0{,}2)^k\right)\varepsilon[k]$$

$$\Rightarrow\boxed{y[k]=\varepsilon[k]\left(0{,}478\cdot(-0{,}4)^k-0{,}331\cdot(0{,}2)^k\right)}$$

4.

$$R(t) = \varepsilon(t)\cdot 9{,}2\cdot e^{-3{,}7t}\cdot\cos(9{,}9t-6{,}8)$$

$$u(t)=1{,}5$$

$$y(t)=?$$

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$$y(t)=\int_{-\infty}^{\infty} u(\tau)\cdot R(t-\tau)\,d\tau = \int_{-\infty}^{\infty} u(t-\tau)\cdot R(\tau)\,d\tau$$

↑ ezt használom

$$\Rightarrow y(t)=\int_{-\infty}^{\infty} 1{,}5\cdot\varepsilon(\tau)\cdot 9{,}2\cdot e^{-3{,}7\tau}\cdot\cos(9{,}9\tau-6{,}8) \overset{d\tau}{=} \int_{0}^{\infty} 1{,}5\cdot 9{,}2\cdot e^{-3{,}7\tau}\cdot\cos(9{,}9\tau-6{,}8)\,d\tau =$$

$$= 13{,}8\int_0^\infty \underset{u}{e^{-3{,}7\tau}}\cdot\underset{v'}{\cos(9{,}9\tau-6{,}8)}\,d\tau \qquad u'=-3{,}7e^{-3{,}7\tau} \quad v=\frac{\sin(9{,}9\tau-6{,}8)}{9{,}9}$$

$$= e^{-3{,}7\tau}\cdot\frac{\sin(9{,}9\tau-6{,}8)}{9{,}9}-\int_0^\infty -3{,}7e^{-3{,}7\tau}\cdot\frac{\sin(9{,}9\tau-6{,}8)}{9{,}9}\,d\tau=$$

$$= e^{-3{,}7\tau}\cdot\frac{\sin(9{,}9\tau-6{,}8)}{9{,}9}+\frac{3{,}7}{9{,}9}\int_0^\infty \underset{u}{e^{-3{,}7\tau}}\cdot\underset{v'}{\sin(9{,}9\tau-6{,}8)}\,d\tau = \qquad u'=-3{,}7e^{-3{,}7\tau} \quad v=\frac{-\cos(9{,}9\tau-6{,}8)}{9{,}9}$$

$$= e^{-3{,}7\tau}\cdot\frac{\sin(9{,}9\tau-6{,}8)}{9{,}9}+\frac{3{,}7}{9{,}9}\left(e^{-3{,}7\tau}\cdot\frac{-\cos(9{,}9\tau-6{,}8)}{9{,}9}-\int_0^\infty -3{,}7e^{-3{,}7\tau}\cdot\frac{-\cos(9{,}9\tau-6{,}8)}{9{,}9}\,d\tau\right)=$$

$$= e^{-3{,}7\tau}\cdot\frac{\sin(9{,}9\tau-6{,}8)}{9{,}9}-\frac{3{,}7}{9{,}9}\,e^{-3{,}7\tau}\cdot\frac{\cos(9{,}9\tau-6{,}8)}{9{,}9}-\frac{3{,}7}{9{,}9}\cdot\frac{3{,}7}{9{,}9}\int_0^\infty e^{-3{,}7\tau}\cos(9{,}9\tau-6{,}8)\,d\tau$$

$$\Rightarrow \int_0^\infty e^{-3{,}7\tau}\cdot\cos(9{,}9\tau-6{,}8)\,d\tau=\frac{e^{-3{,}7\tau}\cdot\sin(9{,}9\tau-6{,}8)}{9{,}9}-\frac{3{,}7e^{-3{,}7\tau}\cdot\cos(9{,}9\tau-6{,}8)}{(9{,}9)^2}-\left(\frac{3{,}7}{9{,}9}\right)^2\int_0^\infty e^{-3{,}7\tau}\cdot\cos(9{,}9\tau-6{,}8)\,d\tau$$

átviszük erre az oldalra

$$\left(1+\left(\frac{3{,}7}{9{,}9}\right)^2\right)\int_0^\infty e^{-3{,}7\tau}\cdot\cos(9{,}9\tau-6{,}8)\,d\tau=\frac{9{,}9e^{-3{,}7\tau}\cdot\sin(9{,}9\tau-6{,}8)-3{,}7e^{-3{,}7\tau}\cdot\cos(9{,}9\tau-6{,}8)}{(9{,}9)^2} \quad \Big/\cdot\frac{1}{1+\left(\frac{3{,}7}{9{,}9}\right)^2}$$

$$13{,}8\cdot\int_0^\infty e^{-3{,}7\tau}\cdot\cos(9{,}9\tau-6{,}8)\,d\tau=\left[\frac{9{,}9\overset{\infty\text{-ben } \emptyset}{e^{-3{,}7\tau}}\cdot\sin(9{,}9\tau-6{,}8)-3{,}7\overset{\infty\text{-ben } \emptyset}{e^{-3{,}7\tau}}\cdot\cos(9{,}9\tau-6{,}8)}{(9{,}9)^2\cdot\left(1+\left(\frac{3{,}7}{9{,}9}\right)^2\right)}\,13{,}8\right]_0^\infty=$$

$$=0-\frac{\overbrace{9{,}9\cdot\sin(-6{,}8)}^{-4{,}892}-\overbrace{3{,}7\cos(-6{,}8)}^{3{,}217}}{\underbrace{(9{,}9)^2\cdot\left(1+\left(\frac{3{,}7}{9{,}9}\right)^2\right)}_{(9{,}9)^2+(3{,}7)^2}}\cdot 13{,}8=\frac{8{,}109\cdot 13{,}8}{98{,}01+13{,}69}=\boxed{1{,}002}$$