Kalk vizsga 1 — 2022. dec. 22

1.) (15p)

Gauss elim: det A =

$$= \begin{vmatrix} 4 & 0 & -1 \\ 2 & -1 & 0 \\ 10 & 8 & -1 \end{vmatrix} \overset{(2)}{=} 2\cdot \begin{vmatrix} 2 & 0 & -1/2 \\ 2 & -1 & 0 \\ 10 & 8 & -1 \end{vmatrix} = 2\cdot \begin{vmatrix} 2 & 0 & -1/2 \\ 0 & -1 & 1/2 \\ 0 & 8 & 3/2 \end{vmatrix} = 2\cdot \begin{vmatrix} 2 & 0 & -1/2 \\ 0 & -1 & 1/2 \\ 0 & 0 & \frac{11}{2} \end{vmatrix} =$$

II − I (2)
III − 5I (2)
III + 8II (2)

(2,5) $\det A = \underbrace{2\cdot 2\cdot(-1)\cdot\frac{11}{2}}_{\text{g.e. után főátlóbeli elemek szorzata}} = \underline{\underline{-22}}$

(2,5) rang: $r(A) = \underbrace{3}_{\text{g.e. után nem 0 sorok száma}}$

Mivel $\det A \neq 0 \Rightarrow \exists A^{-1}$. (2)

det A számolható sor/oszlop szerinti kifejtéssel is!

2.) (15p)

$$\lim_{n\to+\infty} \underbrace{\sqrt{n^4-3n+7}}_{A} - \underbrace{(n^2+5)}_{B} =$$

$$A - B \cdot \frac{A+B}{A+B} = \frac{A^2-B^2}{A+B} \quad (2)$$

$$(n^2+5)^2 = n^4+10n^2+25$$

$$= \lim_{n\to+\infty} \frac{n^4-3n+7-(n^2+5)^2}{\sqrt{n^4-3n+7}+(n^2+5)} = \lim_{n\to+\infty} \frac{n^4-3n+7-n^4-10n^2-25}{\sqrt{n^4-3n+7}+(n^2+5)} = \quad (2)\ (2)$$

$$= \lim_{n\to+\infty} \frac{-10n^2-3n-18}{\sqrt{n^4-3n+7}+n^2+5} = \lim \boxed{\frac{\frac{1}{n^2}}{\frac{1}{n^2}}} \cdot \frac{-10-\frac{3}{n}-\frac{18}{n^2}}{\sqrt{1-\frac{3}{n^3}+\frac{7}{n^4}}+1+\frac{5}{n^2}} = \quad (2)\ (3)$$

dom (1), dom, n^2 dom (1)

$$\frac{3}{n}\to 0,\quad \frac{18}{n^2}\to 0,\quad \frac{3}{n^3}\to 0,\quad \frac{7}{n^4}\to 0,\quad \frac{5}{n^2}\to 0$$

$$= \lim 1\cdot\frac{-10}{\sqrt{1}+1} = \underline{\underline{-5}} \quad (2)$$

3, (20p)

$z^2 - 2\cdot\bar{z} = -1 = 0\cdot i - 1$ $\quad z = a + ib$

$\bar{z} = a - ib$ 2 $\quad i^2 = -1$

$z^2 = a^2 + 2abi + i^2b^2 = a^2 - b^2 + 2abi$ 2

$a^2 - b^2 + 2abi - 2a + 2bi = 0\cdot i - 1$

$a^2 - b^2 - 2a + i(2b + 2ab) = 0\cdot i - 1$ 2

$\Updownarrow$ képzetes részek és valós részek egyenlőek } 1

$2b + 2ab = 0$ és $a^2 - b^2 - 2a = -1$ 2

$2b\cdot(1+a) = 0$

2

$\boxed{b = 0}$ $\qquad a = -1$

ha $b = 0$

$a^2 - 2a = -1$

$a^2 - 2a + 1 = 0$

$(a-1)^2 = 0$

$\boxed{a = 1}$ 3

Mo: $z_1 = 1 + 0\cdot i = 1$ 1

ha $a = -1$

$(-1)^2 - b^2 - 2(-1) = -1$

$1 - b^2 + 2 = -1$

$-b^2 = -4$

$b^2 = 4 \rightarrow b_{1,2} = \pm 2$ 3

Mo: $z_2 = -1 + 2i$ 1

$z_3 = -1 - 2i$

4.) (11p)

$f(x) = \cos(3x) - x^2$ $\quad f(x_0) = f(0) = \cos 0 - 0^2 = \boxed{1}$ 1

1 $f'(x) = -3\sin(3x) - 2x$ $\quad f'(0) = -3\cdot\sin 0 - 2\cdot 0 = \boxed{0}$ 1

1 $f''(x) = -9\cos(3x) - 2$ $\quad f''(0) = -9\cdot\cos 0 - 2 = \boxed{-11}$ 1

1 $f^{(3)}(x) = 27\sin(3x)$ $\quad f^{(3)}(0) = 27\cdot\sin 0 = \boxed{0}$ 1

1 $f^{(4)}(x) = 81\cdot\cos(3x)$ $\quad f^{(4)}(0) = 81\cdot\cos 0 = \boxed{81}$ 1

$T_4(x) = f(x_0) + f'(x_0)\cdot(x - x_0) + \frac{f''(x_0)}{2}\cdot(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \frac{f^{(4)}(x_0)}{4!}(x-x_0)^4$

képlet 3

$= 1 + 0\cdot x + \frac{(-11)}{2}\cdot x^2 + \frac{0}{3!}\cdot x^3 + \frac{81}{4!}\cdot x^4 = 1 - \frac{11}{2}x^2 + \frac{81}{24}x^4$ → 3

5.)(15p) parc. int. $\int f' \cdot g = f \cdot g - \int f \cdot g'$ 2

$$\int \underbrace{sh\left(\tfrac{x}{2}\right)}_{f' \; 1} \cdot \underbrace{(2x-5)}_{g \; 1} dx = \underbrace{2ch\left(\tfrac{x}{2}\right) \cdot (2x-5)}_{f \cdot g} - \underbrace{\int 2ch\left(\tfrac{x}{2}\right) \cdot 2 \, dx}_{-\int f \cdot g'} \ominus$$ 3

$f(x) = 2 \cdot ch\left(\frac{x}{2}\right)$ 3 $\quad g'(x) = 2$ 1

$4 \int ch\left(\frac{x}{2}\right) dx$ 1

$4 \cdot 2 \cdot sh\left(\frac{x}{2}\right)$ 2

$$\ominus \; 2ch\left(\tfrac{x}{2}\right) \cdot (2x-5) - 8 \cdot sh\left(\tfrac{x}{2}\right) + C$$ 1

6.)(20p)

$$\int_0^{+\infty} \frac{1}{2+3x+x^2} dx \overset{*}{=} \int_0^{+\infty} \overbrace{\frac{1}{x+1} - \frac{1}{x+2}}^{1} = \overbrace{\lim_{A \to +\infty} \int_0^{A} \frac{1}{x+1} - \frac{1}{x+2} dx}^{2} \ominus$$

$x^2+3x+2=0$

$x_{1,2} = \dfrac{-3 \pm \sqrt{9-8}}{2}$ $\begin{matrix} -1 \\ -2 \end{matrix}$

$$\ominus \lim_{A \to +\infty} \left[\ln|x+1| - \ln|x+2| \right]_{x=0}^{A} \ominus$$ 2

$$= \lim_{A \to +\infty} \left[\ln \left| \frac{x+1}{x+2} \right| \right]_{x=0}^{A} \ominus$$ 2

2 *: parc. törtekre bontás:

$$\frac{1}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)}$$ 6

$1 = A(x+2) + B(x+1)$

$x=-1: \; 1 = A(-1+2) \Rightarrow A = 1$

$x=-2: \; 1 = B(-2+1) \Rightarrow B = -1$

$$\ominus \lim_{A \to +\infty} \left(\overbrace{\ln \left| \frac{A+1}{A+2} \right|}^{1} - \underbrace{\overbrace{\ln \left| \frac{0+1}{0+2} \right|}^{1}}_{\ln 1/2} \right) = \underbrace{\ln 1}_{0} - \ln 1/2 = -\ln \frac{1}{2}$$ 1 1

$$\frac{A+1}{A+2} = \frac{\cancel{A}}{\cancel{A}} \cdot \frac{1+\frac{1}{A}}{1+\frac{2}{A}} \to 1$$ 1

f) (15p) $f(x)=\dfrac{x+1}{x+2}$ $D_f=\mathbb{R}\setminus\{-2\}$ 2

$$f'(x)=\frac{1\cdot(x+2)-(x+1)\cdot 1}{(x+2)^2}=\frac{x+2-x-1}{(x+2)^2}=\frac{1}{(x+2)^2}\geq 0$$ 5 2

Mon. táblázat ÉT: -2 2

	$(-\infty,-2)$	$(-2,+\infty)$
f'	+	+
f	↗ mon. nő	↗ mon. nő

4