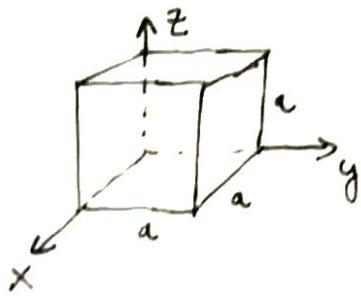


F1.



A lapok normálvektorai:

$(x; y)$  sík:  $\underline{n}_1 = -\hat{z}$ ; ezzel párhuzamos lap:  $-\underline{n}_1$   
 $(x; z)$  sík:  $\underline{n}_2 = -\hat{y}$ ;  $-||-$  :  $-\underline{n}_2$   
 $(y; z)$  sík:  $\underline{n}_3 = -\hat{x}$ ;  $-||-$  :  $-\underline{n}_3$

a)  $\underline{E} = E_0 \hat{z}$

alsó lap:  $\Psi_1 = \underline{E} \cdot \underline{n}_1 \cdot a^2 = -E_0 a^2$ ; felső lap:  $\Psi_1' = \underline{E} \cdot (-\underline{n}_1) a^2 = E_0 a^2$

az oldalsó lapok:  $\Psi = 0$ , mert pl.  $\Psi_{xz} = \underline{E} \cdot \underline{n}_2 \cdot a^2 = -E_0 a^2 \cdot \underbrace{\hat{z} \cdot \hat{y}}_{=0}$

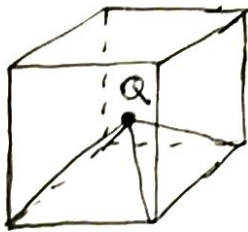
b)  $\underline{E} = \frac{E_0}{\sqrt{3}} (\hat{x} + \hat{y} + \hat{z})$

alsó lap:  $\Psi_1 = \underline{E} \cdot \underline{n}_1 a^2 = -\frac{E_0}{\sqrt{3}} a^2$ ; felső lap:  $\Psi_1' = \underline{E} \cdot (-\underline{n}_1) a^2 = \frac{E_0}{\sqrt{3}} a^2$

oldalsó lapok:  $\Psi_2 = \underline{E} \cdot \underline{n}_2 a^2 = -\frac{E_0}{\sqrt{3}} a^2$ ;  $\Psi_2' = \underline{E} \cdot (-\underline{n}_2) a^2 = \frac{E_0}{\sqrt{3}} a^2$

$\Psi_3 = \underline{E} \cdot \underline{n}_3 a^2 = -\frac{E_0}{\sqrt{3}} a^2$ ;  $\Psi_3' = \underline{E} \cdot (-\underline{n}_3) a^2 = \frac{E_0}{\sqrt{3}} a^2$

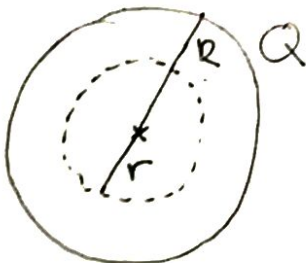
G



A szimmetria miatt egy lapon átmenő fluxus értéke a  $Q$  körüli  $r = \frac{\sqrt{3}}{2} a$  (testátló fele) sugarú gömbfelületen áthaladó fluxus  $\frac{1}{6}$ -ával megegyezik:

$\Psi_{gömb} = \frac{Q}{\epsilon_0}$  (Gauss-törvény)  $\rightarrow \Psi_{lap} = \frac{1}{6} \Psi_{gömb} = \frac{Q}{6\epsilon_0}$

F2.



a) ha  $0 \leq r \leq R$ :

Gauss-törvény:  $E \cdot 4r^2 \pi = \frac{1}{\epsilon_0} \cdot Q(r)$

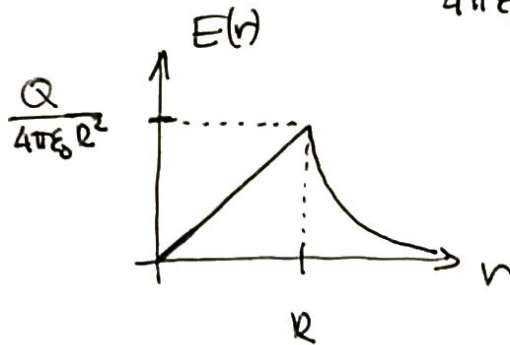
$$E \cdot 4r^2\pi = \frac{1}{\epsilon_0} \cdot Q \cdot \frac{\frac{4}{3}r^3\pi}{\frac{4}{3}R^3\pi}$$

$$E(r) = \frac{Q}{4\pi\epsilon_0} \cdot \frac{r}{R^3} = \frac{kQ}{R^3} \cdot r$$

ha  $R \leq r$ :

Gauss-törvény:  $E \cdot 4r^2\pi = \frac{1}{\epsilon_0} Q$

$$E(r) = \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2} = \frac{kQ}{r^2}$$



b)  $\varphi(r \rightarrow \infty) = 0$

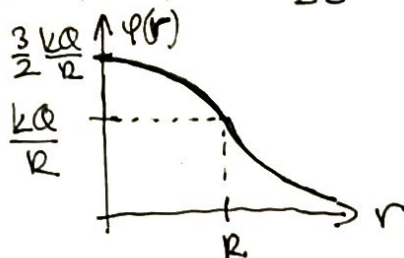
ha  $R \leq r$ :  $\varphi(r) = \frac{kQ}{r}$  (ponttöltés)

ha  $0 \leq r \leq R$ :  $\varphi(r) = \frac{kQ}{R} + \frac{E(R) + E(r)}{2} (R - r) =$

munka végzés  
∞-ből a gömbfelületig
gömbön belül  
lineáris a térerősség

$$= \frac{kQ}{R} + \frac{1}{2} \left( \frac{kQ}{R^2} + \frac{kQ}{R^3} r \right) (R - r) = \frac{kQ}{R} + \frac{1}{2} \frac{kQ}{R^3} (R + r)(R - r) =$$

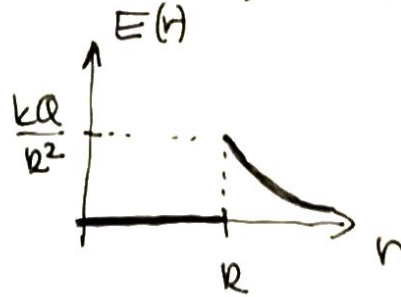
$$= \frac{kQ}{R} \left( 1 + \frac{1}{2} \frac{R^2 - r^2}{R^2} \right) = \frac{kQ}{2R^3} (2R^2 + R^2 - r^2) = \frac{kQ}{2R^3} (3R^2 - r^2)$$



g) A kömgömbön belül nincs elektrosztatikus t6r (lemp6kal6s)

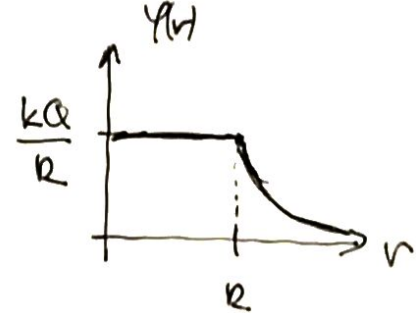
ha  $0 \leq r \leq R$ :  $E = 0$

ha  $R \leq r$ :  $E = \frac{kQ}{r^2}$



ha  $R \leq r$ :  $\varphi(r) = \frac{kQ}{r}$

ha  $0 \leq r \leq R$ :  $\varphi(r) = \text{all.} = \frac{kQ}{R}$



F3.

$R = 2\text{cm}$

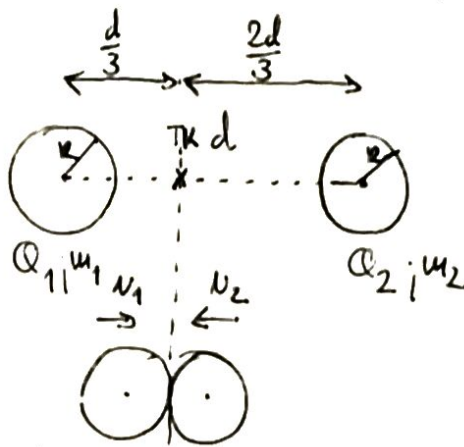
$Q_1 = -10\text{nC}$

$Q_2 = 20\text{nC}$

$m_1 = 10\text{g}$

$m_2 = 5\text{g}$

$d = 12\text{cm}$



A homog6n t6lt6sel-  
onl6s, szigetel6gy6mb-  
6k miatt  
pontt6lt6seknek  
tekinthet6nek fel.

Lend6letmegmarad6s:  $0 = m_1 v_1 - m_2 v_2 \rightarrow$

$\rightarrow v_2 = v_1 \cdot \frac{m_1}{m_2}$

Energiamegmarad6s:

$$-\frac{kQ_1 Q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{kQ_1 Q_2}{2R}$$

$$-\frac{kQ_1 Q_2}{d} + \frac{kQ_1 Q_2}{2R} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_1^2 \frac{m_1^2}{m_2^2}$$

$$2kQ_1 Q_2 \left( \frac{1}{2R} - \frac{1}{d} \right) = v_1^2 \left( 1 + \frac{m_1}{m_2} \right) m_1 \rightarrow v_1 = \sqrt{\frac{2kQ_1 Q_2}{m_1 \left( 1 + \frac{m_1}{m_2} \right)} \left( \frac{1}{2R} - \frac{1}{d} \right)}$$

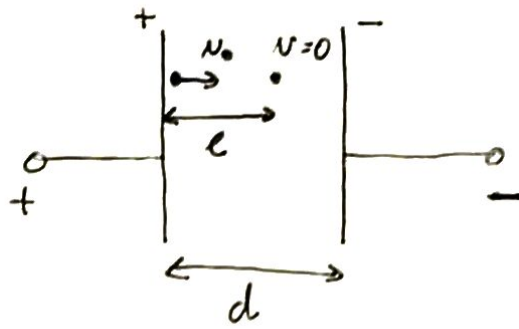
$= 0,045 \frac{\text{m}}{\text{s}} ; v_2 = 2v_1 = 0,09 \frac{\text{m}}{\text{s}}$

F4.

$$d = 10 \text{ mm}$$

$$U = 12 \text{ V}$$

$$E_0 = 9 \text{ eV}$$



$$q = 1.6 \cdot 10^{-19} \text{ C}$$

a) Munkatétel:  $E_0 = \frac{1}{2} m v_0^2 = \frac{e}{d} U \cdot q \rightarrow l = \frac{E_0 \cdot d}{q U} = 7,5 \text{ mm}$

b)  $v_0 = \sqrt{\frac{2 E_0}{m}} = 1,8 \cdot 10^6 \frac{\text{m}}{\text{s}}$

c) A térerősség:  $\frac{U}{d}$ , tehát  $a = \frac{U \cdot q}{m d} = 2,1 \cdot 10^{14} \frac{\text{m}}{\text{s}^2}$

F5.

$$\varphi(x, y, z) = 3 \frac{\text{V}}{\text{m}^2} \cdot x^2 + 7 \text{ V} = \alpha x^2 + \beta$$

$$x_0 = 1 \text{ m}; y_0 = 0; z_0 = 0; v_0 = 0$$

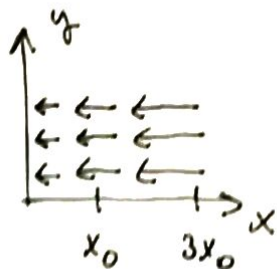
a) Mivel a potenciál a +x tengely irányában növekszik, ezért a térerősség -x irányú. Tehát az elektron +x irányba indul. A potenciál megváltozása:

$$\Delta \varphi = \varphi(3x_0) - \varphi(x_0) = \alpha \cdot 9x_0^2 - \alpha x_0^2 = 8\alpha x_0^2$$

munkatétel:  $q \cdot \Delta \varphi = \frac{1}{2} m v^2 = E_{\text{kin}} = 8\alpha q x_0^2 = 24 \text{ eV}$

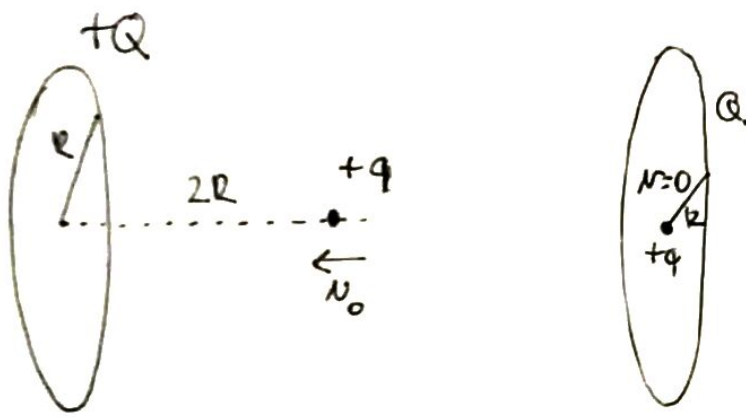
b)  $v = \sqrt{\frac{2 E_{\text{kin}}}{m}} = 2,9 \cdot 10^6 \frac{\text{m}}{\text{s}}$

c)  $E(x) = -2\alpha x = -6 \frac{\text{V}}{\text{m}^2} x$

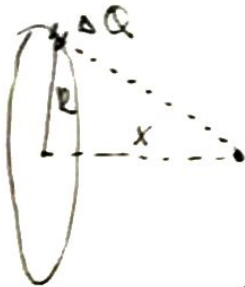


$$a_0 = \frac{q E(x_0)}{m} = 1,1 \cdot 10^{12} \frac{\text{m}}{\text{s}^2}$$

F6.



A potenciál  $x$  távolságra:



$$V(x) = \sum \frac{k \Delta Q}{\sqrt{R^2 + x^2}} = \frac{kQ}{\sqrt{R^2 + x^2}}$$

Energiamegmaradás:

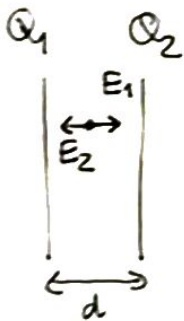
$$\frac{1}{2} m v_0^2 + U(x) \cdot q = 0 + U(R) \cdot q$$

$$\frac{1}{2} m v_0^2 + \frac{kqQ}{\sqrt{5}R} = \frac{kqQ}{R}$$

$$\frac{1}{2} m v_0^2 = \frac{kqQ}{R} \frac{5 - \sqrt{5}}{5}$$

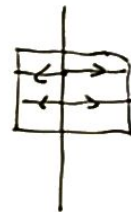
$$v_0 = \sqrt{\frac{2(5 - \sqrt{5})}{5} \cdot \frac{kqQ}{mR}}$$

F7.



$$C = \frac{\epsilon_0 A}{d}$$

egy lemez:



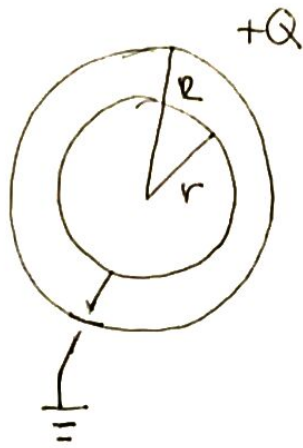
$$E \cdot 2A = \frac{1}{\epsilon_0} \cdot Q$$

$$E = \frac{Q}{2\epsilon_0 A}$$

Az eredő térerősség: 
$$E = \frac{Q_1 - Q_2}{2\epsilon_0 A}$$

$$U = E \cdot d = \frac{Q_1 - Q_2}{2} \frac{d}{\epsilon_0 A} = \frac{1}{2} \frac{Q_1 - Q_2}{C}$$

(F8.)



A kisebb gömb potenciálja nulla (földelt). Ahhoz, hogy ez teljesüljön, negatív töltés kell a kisebb gömbre

Szuperpozíciós elv:

$$0 = \frac{kQ}{R} + \frac{kq}{r}$$

← külső gömb potenciálja az  $r$  helyen

→  $q$  töltésű gömb potenciálja az  $r$  helyen

teljes:

$$q = -Q \cdot \frac{r}{R}$$