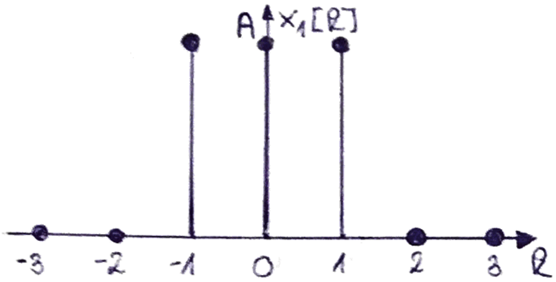
Rendszerelmélet 3.HF

2017.11.28.

1. feladat



$A = 6{,}1$

$X_1(e^{j\vartheta}) = ?$

$$X_1(e^{j\vartheta}) = \sum_{k=-1}^{1} x_1[k] e^{-j\vartheta k} = 6{,}1 e^{j\vartheta} + 6{,}1 + 6{,}1 e^{-j\vartheta} = 6{,}1 + 6{,}1(e^{j\vartheta} + e^{-j\vartheta}) =$$

$$= 6{,}1 + 12{,}2\left(\frac{e^{j\vartheta} + e^{-j\vartheta}}{2}\right) = 6{,}1 + 12{,}2\cos\vartheta$$

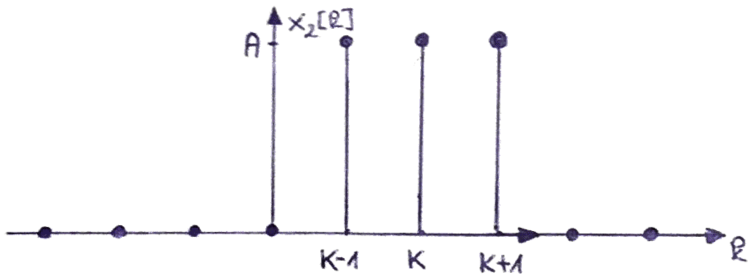
$\vartheta = 72$:

$$\mathrm{Re}\{X_1(e^{j72})\} = 6{,}1 + 12{,}2\cos 72 = -5{,}700457177$$

$$\mathrm{Im}\{X_1(e^{j72})\} = \emptyset$$ – a jel páros $\Rightarrow$ a spektrum tisztán valós (+valós)

$$E_1 = \sum_{-\infty}^{\infty} |x_1[k]|^2 = 6{,}1^2 + 6{,}2^2 + 6{,}2^2 = 3 \cdot 6{,}1^2 = 111{,}63$$

2. feladat



$A = 6{,}1$

$K = 14$

$X_2(e^{j\vartheta}) = ?$

$$x_2[k] = x_1[k-K] \Rightarrow \mathcal{F}\{x_2[k]\} = X_1(e^{j\vartheta}) \cdot e^{-jK\vartheta} = (6{,}1 + 12{,}2\cos\vartheta) e^{-j14\vartheta}$$

$$\mathrm{Re}\{X_2(e^{j\vartheta})\} = 6{,}1\cos(14\vartheta) + 12{,}2\cos\vartheta\cos(14\vartheta)$$

$$\mathrm{Im}\{X_2(e^{j\vartheta})\} = -6{,}1\sin(14\vartheta) - 12{,}2\cos\vartheta\sin(14\vartheta)$$

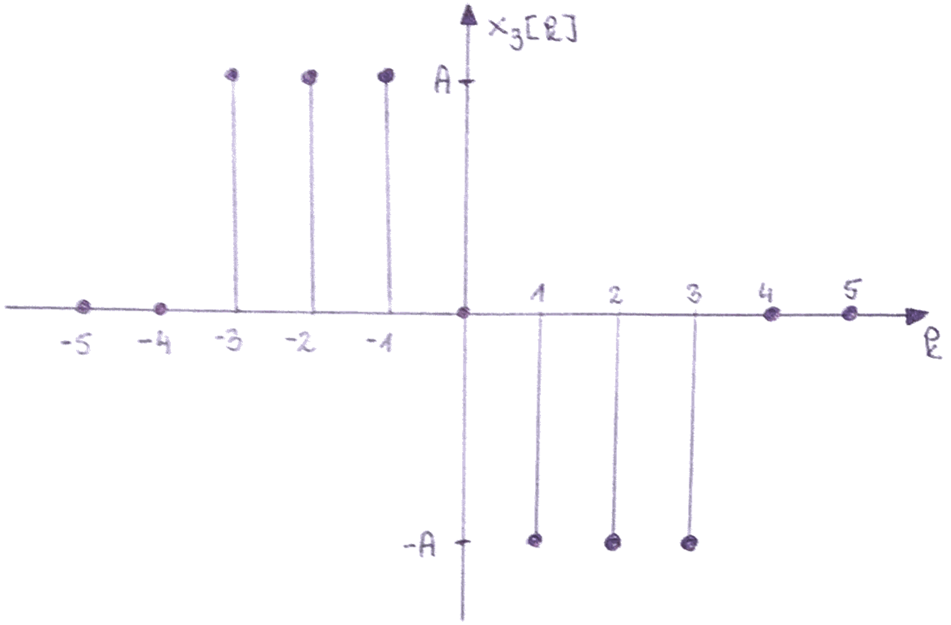
$\vartheta = 101$:

$$\mathrm{Re}\{X_2(e^{j\vartheta})\} = 16{,}30548064$$

$$\mathrm{Im}\{X_2(e^{j\vartheta})\} = -4{,}747128464$$

$$E_2 = E_1 = 111{,}63$$

3. feladat



$A = 6{,}1$

$X_3(e^{j\vartheta}) = ?$

$$x_3[k] = x_1[k+2] - x_1[k-2] \Rightarrow (6{,}1 + 12{,}2\cos\vartheta)e^{-j\vartheta(-2)} - (A + 2A\cos\vartheta)e^{-j\vartheta 2} =$$

$$= (6{,}1 + 12{,}2\cos\vartheta)\left(\frac{e^{j\vartheta 2} - e^{-j\vartheta 2}}{2j}\right)\cdot 2j = (12{,}2j + 24{,}4j\cos\vartheta)\sin(2\vartheta) = X_3(e^{j\vartheta})$$

$Re\{X_3(e^{j\vartheta})\} = \varnothing$ — $x_3[k]$ páratlan

$Im\{X_3(e^{j\vartheta})\} = (12{,}2 + 24{,}4\cos\vartheta)\sin(2\vartheta)$

$\vartheta = 72$:

$Re\{X_3(e^{j72})\} = \varnothing$

$Im\{X_3(e^{j72})\} = 5{,}598095138$

$E_3 = 2\cdot E_1 = 6\cdot A^2 = 223{,}26$

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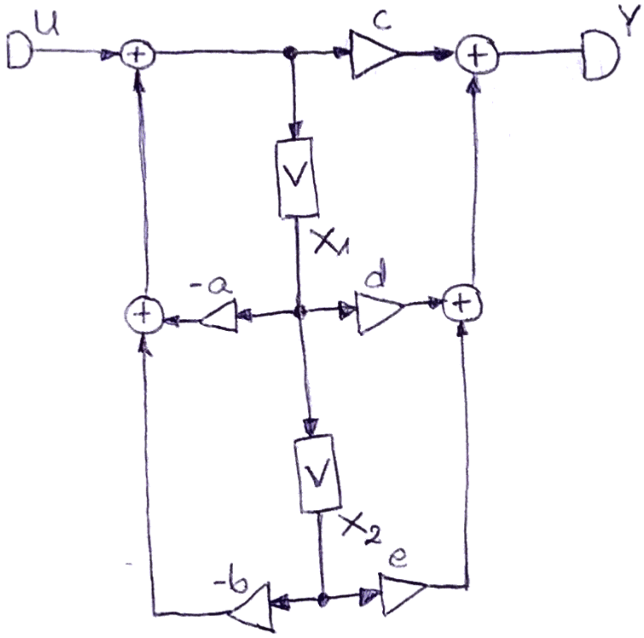
4. feladat



$a = -6{,}4$

$b = 123{,}88$

$c = -11{,}47$

$d = 18{,}94$

$e = 183$

$u(t) = 0{,}58 \cdot \varepsilon(t)$

1. $H(j\omega) = ?$

A rendszer kanonikus, ezért az átviteli karakterisztika felírható így:

$$H(j\omega) = \frac{c\cdot(j\omega)^2 + d\cdot(j\omega) + e}{1\cdot(j\omega)^2 + a(j\omega) + b} = \frac{-11{,}47(j\omega)^2 + 18{,}94(j\omega) + 183}{(j\omega)^2 + (-6{,}4)\cdot(j\omega) + 123{,}88}$$

2. A rendszer válasza $u(t)$-re?

$u(t)$ belépő jel, ezért érdemes inkább Laplace-transzformációval dolgozni:

$$U(s) = \mathcal{L}\{u(t)\} = \frac{0{,}58}{s} \qquad H(s) = H(j\omega)\Big|_{j\omega = s} = \frac{-11{,}47s^2 + 18{,}94s + 183}{s^2 - 6{,}4s + 123{,}88}$$

$$Y(s) = H(s)\cdot U(s) = \frac{-6{,}6526s^2 + 10{,}9852s + 106{,}14}{s(s^2 - 6{,}4s + 123{,}88)} = \frac{A}{s} + \frac{B}{s - p_1} + \frac{C}{s - p_2} = \dots$$

(p_1, p_2 a $H(s)$ nevezőjének a gyökei

$$\dots = Y(s) \longrightarrow y(t) = \varepsilon(t)\Big(0{,}856797 + (-3{,}7084 - 0{,}37629j)\cdot e^{(3{,}2 - 10{,}6602j)t} +$$

$$+ (-3{,}7084 + 0{,}37629j)\cdot e^{(3{,}2 + 10{,}6602j)t}\Big)$$

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5. feladat

$$s_{AM-DSB-SC}(t) = \underset{A_1}{1}\cos(2\pi \cdot \underset{f_{m_1}}{1057800}t)\left(\underset{A_2}{2,58}\cos(2\pi \cdot \underset{f_{m_2}}{1590}t) + \underset{A_3}{4,77}\cos(2\pi \cdot \underset{f_{m_3}}{9096}t)\right)$$

$$\sum_{n=1}^{8} = B_n \cdot \delta(f - f_n)$$

$f_1 = -f_{m_1} - f_{m_2} - f_{m_3}$ $\quad B_1 = \frac{A_3}{4}$

$f_2 = -f_{m_1} - f_{m_2}$ $\quad B_2 = \frac{A_2}{4}$

$f_3 = -f_{m_1} + f_{m_2}$ $\quad B_3 = \frac{A_2}{4}$

$f_4 = -f_{m_1} + f_{m_3}$ $\quad B_4 = \frac{A_3}{4}$

$f_5 = f_{m_1} - f_{m_3}$ $\quad B_5 = \frac{A_3}{4}$

$f_6 = f_{m_1} - f_{m_2}$ $\quad B_6 = \frac{A_2}{4}$

$f_7 = f_{m_1} + f_{m_2}$ $\quad B_7 = \frac{A_2}{4}$

$f_8 = f_{m_1} + f_{m_2} + f_{m_3}$. $\quad B_8 = \frac{A_3}{4}$

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6. feladat

$$s_{AM-DSB-NSC}(t) = \underset{A_1}{}\cos(2\pi \cdot \underset{f_{m_1}}{1072100}t)\left(\underset{A}{10} + \underset{A_2}{4,22}\cos(2\pi \cdot \underset{f_{m_2}}{4970}t) + \underset{A_3}{3,95}\cos(2\pi \cdot \underset{f_{m_3}}{8460}t)\right)$$

$$m = \frac{A_2 + A_3}{10} \cdot 100 \qquad A = \frac{4 \cdot \left(\frac{A_2}{4}\right)^2 + 4 \cdot \left(\frac{A_3}{4}\right)^2}{2 \cdot \left(\frac{A}{2}\right)^2} \cdot 100$$

$$\sum_{n=1}^{10} = B_n \cdot \delta(f - f_n)$$

$B_1 = \frac{A_3}{4}$ $\quad f_1 = -f_{m_1} - f_{m_3}$ $\qquad B_6 = \frac{A_3}{4}$ $\quad f_6 = f_{m_1} - f_{m_3}$

$B_2 = \frac{A_2}{4}$ $\quad f_2 = -f_{m_1} - f_{m_2}$ $\qquad B_7 = \frac{A_2}{4}$ $\quad f_7 = f_{m_1} - f_{m_2}$

$B_3 = A/2$ $\quad f_3 = -f_{m_1}$ $\qquad B_8 = A/2$ $\quad f_8 = f_{m_1}$

$B_4 = \frac{A_2}{4}$ $\quad f_4 = -f_{m_1} + f_{m_2}$ $\qquad B_9 = \frac{A_2}{4}$ $\quad f_9 = f_{m_1} + f_{m_2}$

$B_5 = \frac{A_3}{4}$ $\quad f_5 = -f_{m_1} + f_{m_3}$ $\qquad B_{10} = \frac{A_3}{4}$ $\quad f_{10} = f_{m_1} + f_{m_3}$

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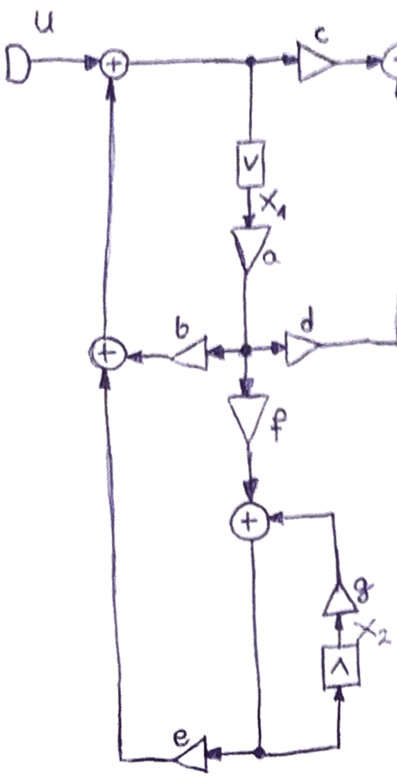
7. feladat



$a = -0,54$

$b = -0,6$

$c = 0,83$

$d = 0,5$

$e = 0,8$

$f = 0,1$

$g = -0,83$

$$\Rightarrow \underline{\underline{A}} = \begin{bmatrix} -0,2808 & -0,664 \\ 0,054 & -0,83 \end{bmatrix} \quad \underline{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\underline{C}^T = [0,036936 \quad -0,55112] \quad D = 0,83$$

$$H(z) = \underline{C}^T [z\underline{\underline{I}} - \underline{\underline{A}}]^{-1} \underline{B} + D \Rightarrow H(z) = \frac{0,83z^2 + 0,9589z + 0,2241}{z^2 + 1,1108z + 0,26892}$$

zérusok: $0,83z^2 + 0,9589z + 0,2241 = 0$

$$\Rightarrow z_1 = -0,3253012048$$
$$z_2 = -0,83$$

pólusok: $z^2 + 1,1108z + 0,26892 = 0$

$$\Rightarrow p_1 = -0,3565302939$$
$$p_2 = -0,7542697061$$

$$H(z) = \frac{0,83z^2 + 0,9589z + 0,2241}{z^2 + 1,1108z + 0,26892} = \frac{0,83(z^2 + 1,1108z + 0,26892) + 0,036936z + 0,0008964}{z^2 + 1,1108z + 0,26892} =$$

$$= 0,83 + \frac{0,036936z + 0,0008964}{(z - p_1)(z - p_2)} = 0,83 + \frac{A}{z - p_1} + \frac{B}{z - p_2}$$

...

$$A = -0,0371751634 \quad B = 0,0816763682$$

$$\Rightarrow H(z) = 0,83 + \left(\frac{-0,0371751634z}{z + 0,3565302939} + \frac{0,0816763682z}{z + 0,7542697061} \right) z^{-1}$$

$$h[k] = Z^{-1}\{H(z)\} = 0,83\delta[k] + \varepsilon[k-1]\left(A \cdot p_1^{k-1} + B \cdot p_2^{k-1}\right)$$