Jelek és rendszerek 1. gyakorlat

II. 18.
2. hét
1/4

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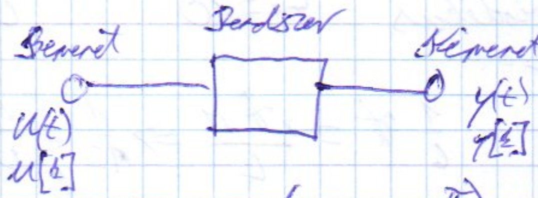


$f = 1\,Hz$

$y(t) = 3 \cdot \cos(2\pi f t + \pi/2)$

$f = 10\,Hz$

$y(t) = 6 \cos(2\pi f t + \pi/2)$

$\vdots$

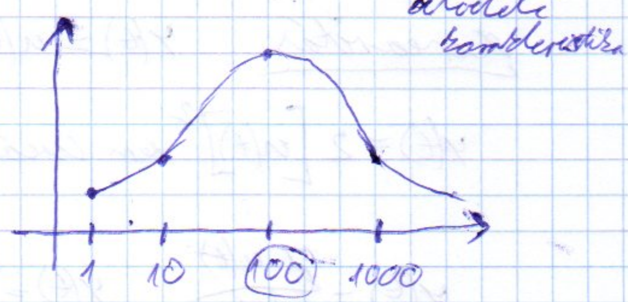
$u(t) = 3 \cos\left(2\pi f t + \frac{\pi}{2}\right)$

$f_1 = 1\,Hz$

$f_2 = 10\,Hz$

$f_3 = 100\,Hz$

$f_4 = 1\,kHz$



mélynyomó szűrője

Jelek

Periodicitás

$u(t) = A \cos(\overbrace{2\pi f}^{\omega} t + \varphi)$

$u[k] = A \cos(\underbrace{2\pi \nu}_{\Theta} k + \varphi)$

$\omega = 2\pi \cdot f = \frac{2\pi}{T}$

$\Theta = 2\pi \frac{M}{L}$

$u(t) = 4 \cos\left(\pi t + \frac{\pi}{2}\right)$

$\pi = \omega = \frac{2\pi}{T}$

$T = 2\,s$

$u[k] = 2 \cos(\underbrace{0{,}2}_{\Theta} k + \pi/3)$

$\Theta = 0{,}2 = 2\pi \cdot \frac{M}{L}$ $\quad M, L \in \mathbb{Z}$

$\Downarrow$

nem periodikus

$u[k] = \sqrt{2} \cdot \cos(\boxed{0{,}2\pi} k + \sqrt{3})$

Periodikus?
$L = ?$

$0{,}2\pi = \frac{2\pi M}{L}$ $\quad 0{,}1^* = \frac{M}{L} \Rightarrow L = 10$

$* \ \frac{3}{10} = \frac{M}{L} \Rightarrow L = 10$

$\frac{1{,}5}{10} = \frac{3}{20} \Rightarrow L = 20$

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$u[k] = 1 \cdot \cos(0{,}2\pi k) + 2\sin\left(\frac{11}{6}\pi k\right)$

Periodikus $L = ?$ $\quad L_1 = 10 \quad L_2 = 12$ Ha azonos frek. 2 két verzió akkor lehet összevonni

$\frac{1}{6}\pi = \frac{2\pi M}{L} \Rightarrow \frac{1}{12} = \frac{M}{L} \Downarrow L = 12$

$L = \text{lkkt.}(L_1, L_2)$

$L = 60$

Rendszer $\quad y(t) = u(t)$

$y(t) = 2u(t)$

Linearitás $\quad y(t) = u(t) + 2 \cdot u(t)$

$y(t) = 2[u(t)]^2$ nem lineáris

$y(t) = \frac{d\,u(t)}{dt} \qquad y(t) = \int u(t)\,dt$

Invariáns

- mindig ugyanúgy működik

Kauzalitás

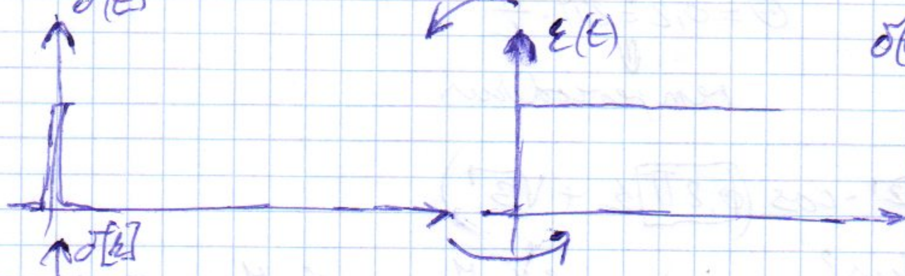
- a jövő nem befolyásolja a jelent

$\delta(t)$

$\varepsilon(t)$

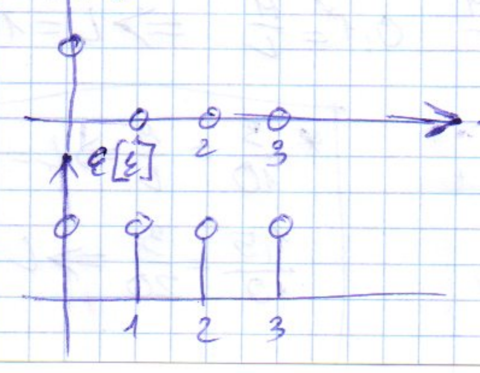
$\delta[k]$

$\varepsilon[k]$

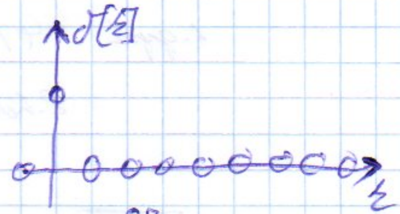


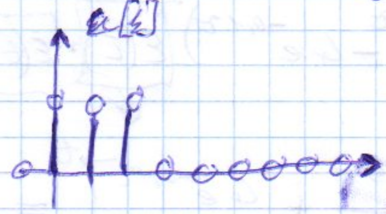
$\delta(t) = \frac{d\,\varepsilon(t)}{dt}$

$\varepsilon(t) = \int \delta(t)\,dt$



J. csk. II. 18.
1. gyak 3/4
2. hét



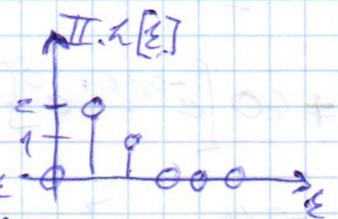


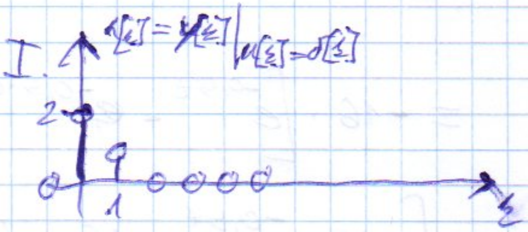
I. II. III.

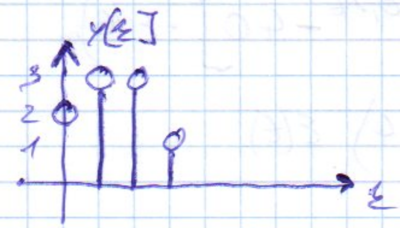
$$u[k] = \delta[k] + \delta[k-1] + \delta[k-2]$$

$$u[k] = \delta[k]$$

$$y[k] = \ldots = h[k] = 2 \cdot \delta[k] + \delta[k-1]$$







Konvolúció

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot u(t-\tau) \, d\tau = \int_{-\infty}^{\infty} h(t-\tau) \cdot u(\tau) \, d\tau$$

$$y[k] = \sum_{i=-\infty}^{\infty} h[i] \cdot u[k-i] = \sum_{i=-\infty}^{\infty} h[k-i] \, u[i]$$

Fejes R. II.18.
1. gyak 4/4
2 hét

$h(t) = \left(8\, e^{-0{,}5t} - 4 \cdot e^{-0{,}1t}\right)\, \varepsilon(t)$

$u(t) = \varepsilon(t)$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot u(t-\tau)\, d\tau = \int_{\cancel{-\infty}\ 0}^{\cancel{\infty}\ t} \left(8\, e^{-0{,}5\tau} - 4 e^{-0{,}1\tau}\right) \boxed{\varepsilon(\tau)} \cdot \boxed{\varepsilon(t-\tau)}\, d\tau$$

$$= \int_0^t 8 \cdot e^{-0{,}5\tau} - 4\, e^{-0{,}1\tau}\, d\tau = \left[8 \cdot \frac{e^{-0{,}5\tau}}{-0{,}5} + \frac{4 e^{-0{,}1\tau}}{0{,}1} \right]_0^t =$$

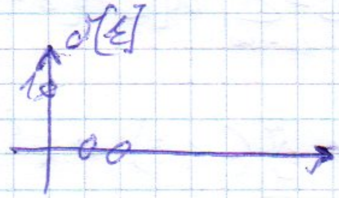
$$= -16 \cdot \left[e^{-0{,}5t} - e^{-0{,}5 \cdot 0} \right] + 40 \left[e^{-0{,}1t} - e^{0} \right] =$$

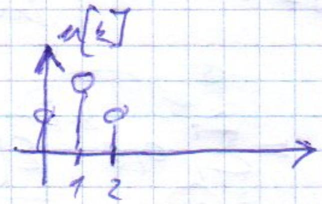
$$= \left[-16\, e^{-0{,}5t} + 16 + 40\, e^{-0{,}1t} - 40 \right] \cdot \varepsilon(t)$$

$$= \left(-16 \cdot e^{-0{,}5t} + 40 e^{-0{,}1t} - 24 \right) \varepsilon(t)$$

$h(t) = y(t) \Big|_{u(t)=\delta(t)}$

$h[k] = y[k] \Big|_{u[k]=\delta[k]}$





$$y(t) = \int_{-\infty}^{\infty} h(\tau)\cdot u(t-\tau)\,d\tau = \int_{-\infty}^{\infty} h(t-\tau)\cdot u(\tau)\,d\tau$$

$$y[k] = \sum_{i=-\infty}^{\infty} h[i]\cdot u[k-i] = \sum_{i=-\infty}^{\infty} h[k-i]\,u[i]$$

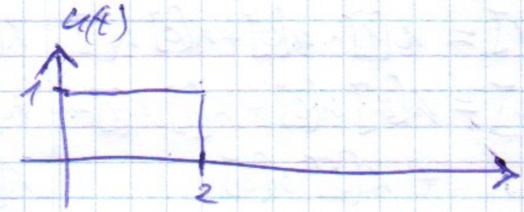
1. Példa

$h(t) = \left(8\,e^{-0,5t} - 4\cdot e^{-0,1t}\right)\varepsilon(t) \qquad u_a(t) = \varepsilon(t) \Rightarrow y_a(t) = ?$

$y(t) = \left(-16\cdot e^{-0,5t} + 40\,e^{-0,1t} - 24\right)\cdot\varepsilon(t)$

b) $u(t) = \boxed{\varepsilon(t)} - \boxed{\varepsilon(t-2)}$

Linearitás
Invariáns



$u_b(t) = u_a(t) - u_a(t-2)$

$y_b(t) = y_a(t) - y_a(t-2) = \left(-16\,e^{-0,5t} + 40\,e^{-0,1t} - 24\right)\varepsilon(t) -$
$- \left(-16\,e^{-0,5(t-2)} + 40\cdot e^{-0,1(t-2)} - 24\right)\cdot\varepsilon(t-2)$

c) $u_c(t) = \left[1 - \varepsilon(t)\right] + 2\,\varepsilon(t) \qquad u_{c1}(t) = 1$

$1 + \varepsilon(t) \qquad \lim_{t\to\infty} y_a(t) = -24$

$y_{ct} = y_a(t) - 24$

2.Fel.

$h[k] = -1{,}5 \cdot 0{,}8^k \cdot \varepsilon[k]$

$u_a[k] = 0{,}6^k \, \varepsilon[k]$

$y_a[k] = ?$

$$y_a[k] = \sum_{i=-\infty}^{\infty} u[i]\, h[k-i] = \sum_{i=-\infty}^{\infty} u[k-i] \cdot h[i] =$$

$$= \sum_{i=-\infty}^{\infty} 0{,}6^{k-i} \boxed{\varepsilon[k-i]} (-1{,}5)\, 0{,}8^i \boxed{\varepsilon[i]} = \sum_{i=0}^{k}$$

$$= 0{,}6^k \cdot (-1{,}5) \sum_{i=0}^{k} \left(\frac{4}{3}\right)^i \cdot \varepsilon[i] \cdot \varepsilon[k-i] =$$

$$= -1{,}5 \cdot 0{,}6^k \cdot \frac{1 - \left(\frac{4}{3}\right)^k \cdot \left(\frac{4}{3}\right)}{1 - \frac{4}{3}} = -1{,}5 \cdot (-3) \cdot$$

$$\cdot\, 0{,}6^k \cdot \left[1 - \left(\frac{4}{3}\right)^k \cdot \frac{4}{3}\right] = \underline{4{,}5 \cdot 0{,}6^k - 6 \cdot 0{,}8^k \ \varepsilon[k]}$$

$$S_n = \frac{1-q^{n+1}}{1-q} = \frac{q^{n+1}-1}{q-1}$$

b) $y[k] = ?$ $\quad k = 0, 1, 2$

$y[0] = h[0] \cdot u[0]$

$y[1] = h[1] \cdot u[0] + h[0]\, u[1]$

$y[2] = h[2]\, u[0] + h[1]\, u[1] + h[0]\, u[2]$

Ha $h[k]$ és $u[k]$ is belép...

$h[0] = -1{,}5$

$h[1] = -1{,}5 \cdot 0{,}8$

$h[2] = -1{,}5 \cdot 0{,}64$

$u[0] = 0{,}6^0 = 1$

$u[1] = 0{,}6$

$u[2] = 0{,}36$

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2. gyak 4. hét

c)

$u_c[k] = 3$

$y_c[k] = ?$

$$y_a[k] = \cancel{-0,6}\left(4,5 \cdot 0,6^k - 6 \cdot 0,8^k\right) \varepsilon[k]$$

$$y_c[k] = -22,5 \quad \Big\| \quad y_c[k] = \sum_{i=-\infty}^{\infty} h[k]\, u[k-i] = \ldots$$

d) $u_d[k] = 0,6^{k-5} \cdot \varepsilon[k-5]$

$y_d[k] = ?$

$$y_d[k] = y_a[k-5]$$

e) $u_e[k] = 0,6^{k-2}\, \varepsilon(k) = 0,6^{-2} \cdot 0,6^k \cdot \varepsilon(k) =$

$y_e[k] = ?$ $\quad y_e[k] = y_a[k] \cdot \dfrac{1}{0,36}$

$$u_f[k] = 0,6^k\, \varepsilon[k-2] = \boxed{(0,6)^{k-2} \cdot \varepsilon[k-2]} \cdot (0,6)^2$$

III.4.

f. és R.
2. gyak

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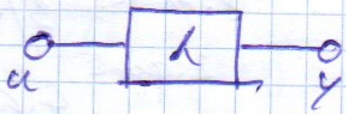
4. hét



$y = h * u$

$\boxed{FI}$ $\dot{x}(t) = \underline{A}\,\underline{x}(t) + \underline{B}\cdot u(t)$ $\boxed{DI}$ $\underline{x}[k+1] = \underline{A}\,\underline{x}[k] + \underline{B}\,u[k]$

$y(t) = \underline{C}^T \underline{x}(t) + D\cdot u(t)$ $\quad y[k] = \underline{c}^T \underline{x}[k] + D\cdot u[k]$

$\boxed{DI}$

$$x_1[k+1] = 0\,x_1[k] + (-0{,}24)\,x_2[k] + (-0{,}24)\,u[k]$$
$$x_2[k+1] = 1\,x_1[k] + x_2[k] + 1{,}5\,u[k]$$
$$y[k] = 0\cdot x_1[k] + 1\cdot x_2[k] + 1\,u[k]$$

$$\underline{A}\begin{pmatrix}0 & -0{,}24\\ 1 & 1\end{pmatrix} \quad B\begin{pmatrix}-0{,}24\\ 1{,}5\end{pmatrix} \quad c^T(0\;1) \quad D = 1$$

$\boxed{FI}$

$$\dot{x}_1(t) = A_{11}\,x_1(t) + A_{12}\,x_2(t) + B_1\,u(t)$$
$$\dot{x}_2(t) = A_{21}\,x_1(t) + A_{22}\,x_2(t) + B_2\,u(t)$$
$$y(t) = c_1^T\,x_1(t) + c_2^T\,x_2(t) + D\cdot u(t)$$

$$\underline{A}\begin{pmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix} \quad B\begin{pmatrix}B_1\\ B_2\end{pmatrix} \quad \underline{c}(c_1\;c_2) \quad D = D$$

$\boxed{DI}$

$u[k] = 0{,}5^k\, \varepsilon[k]$

$y[k] = ?$

1, $\lambda_1, \lambda_2 = ?$
2, $L_1, L_2 = ?$
3, $A^i = ?$
4, $y[k] = \ldots$

$$y[k] = D \cdot u[k] + \underline{c}^T \sum_{i=0}^{k-1} \underline{A}^{k-1-i}\, \underline{B}\, u[i]$$

4. Lat

$$c^T \cdot A^i \cdot B = c^T\left(\lambda_1^i \cdot \underline{\underline{L}}_1 + \lambda_2^i \underline{\underline{L}}_2\right) \cdot B$$

1, $|\lambda \underline{\underline{E}} - A| = 0$

$$\det\begin{pmatrix} \lambda & +0{,}24 \\ -1 & \lambda - 1 \end{pmatrix} = \lambda(\lambda-1) + 0{,}24 = \lambda^2 - \lambda + 0{,}24 = 0$$

$$\lambda(\lambda-1) = -0{,}24$$

$$\lambda_{12} = \frac{1 \pm \sqrt{1 - 4 \cdot 0{,}24}}{2} \begin{cases} 0{,}6 & \lambda_1 \\ 0{,}4 & \lambda_2 \end{cases}$$

2, $$L_1 = \frac{1}{\lambda_1 - \lambda_2} \cdot \left[\underline{\underline{A}} - \lambda_2 \underline{\underline{E}}\right] = \frac{1}{0{,}2}\begin{pmatrix} \overset{-0{,}4}{\cancel{-\lambda}} & -0{,}24 \\ 1 & \underset{0{,}6}{\cancel{\lambda - \lambda}} \end{pmatrix} =$$

$$\sum_{\forall i} L_i = \underline{\underline{E}}$$

$$= \cancel{-5\lambda_2}\begin{bmatrix} -2 & -1{,}2 \\ 5 & 3 \end{bmatrix}$$

$$L_2 = \frac{1}{-0{,}2} \cdot \left(\begin{bmatrix} 0 & -0{,}24 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0{,}6 & 0 \\ 0 & 0{,}6 \end{bmatrix}\right) =$$

$$= -5\begin{pmatrix} -0{,}6 & -0{,}24 \\ 1 & 0{,}4 \end{pmatrix} = \begin{bmatrix} 3 & 1{,}2 \\ -5 & -2 \end{bmatrix}$$

3) $$A^i = 0{,}6^i \begin{pmatrix} -2 & -1{,}2 \\ 5 & 3 \end{pmatrix} + 0{,}4^i \begin{pmatrix} 3 & 1{,}2 \\ -5 & -2 \end{pmatrix}$$

$$[0\ \ 1]\begin{bmatrix} -2 & -1{,}2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1{,}2 \\ -5 & -2 \end{bmatrix} - \underline{3{,}3} = \underline{c}^T \underline{L}_1 \underline{B}$$

$$c^T \underline{L}_2 \underline{B} = -1{,}8$$

$$c^T A^i B = 0{,}6^i \cdot 3{,}3 + 0{,}4^i(-1{,}8)$$

J. és R.
2,5. gyak 6/8
(Blasz gyak)
ö. Lt

$\boxed{DI}$

$$x_1[k+1] = A_{11}\, x_1[k] + A_{21}\, x_2[k] + B_1\, u[k]$$

$$x_2[k+1] = A_{21}\, x_1[k] + A_{22}\, x_2[k] + B_2\, u[k]$$

$$y[k] = c_1\, x_1[k] = c_2\, x_2[k] + D \cdot u[k]$$

$$u[k] = 0{,}5^k \cdot \varepsilon[k]$$

$$\underline{c}^T \underline{A}^i \cdot \underline{B} = 0{,}6^i \cdot 3{,}3 + 0{,}4^i (-1{,}8)$$

$$y[k] = D \cdot u[k] + \underline{c}^T \sum_{i=0}^{k-1} \underline{\underline{A}}^{k-1-i} \cdot \underline{B} \cdot u[i]$$

4, $y[k] = 0{,}5^k \cdot \varepsilon[k] + \left\{ \sum_{i=0}^{k-1} \left[3{,}3 \cdot 0{,}6^{k-1-i} + (-1{,}8) \cdot 0{,}4^{k-1-i} \right] \cdot u[i] \right\} = \dots =$

$$= 0{,}5^k \cdot \varepsilon[k] + \left[33 \cdot 0{,}6^{k-1} + 18 \cdot 0{,}4^{k-1} \right] \varepsilon[k] \quad \checkmark$$

$\boxed{FI}$

$$x_1'(t) = 0\; x_1(t) + 1 \cdot x_2(t) + 0 \cdot u(t)$$

$$x_2'(t) = (-3)\, x_1(t) + (-4)\, x_2(t) + 1 \cdot u(t)$$

$$y(t) = 1 \cdot x_1(t) + 5\, x_2(t) + \emptyset\, u(t)$$

$$A \begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix} \quad B \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad c^T [1, 5] \quad D = 0$$

1, λ_1, λ_2

2, L_1, L_2

3, $c^T e^{\underline{A} t} \cdot B = c^T (\lambda_1^k \cdot L_1 + \lambda_2^k \cdot L_2) B$

4, $y(t)$

1, $\det(\underline{\underline{A}} - \lambda \underline{\underline{E}}) = \left|\begin{pmatrix}\lambda & 0\\ 0 & \lambda\end{pmatrix} - \begin{pmatrix}0 & 1\\ -3 & -4\end{pmatrix}\right| = \left|\begin{matrix}\lambda & -1\\ 3 & \lambda+4\end{matrix}\right| = \lambda(\lambda-4)+3=0$

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$\Downarrow$

$\lambda^2+4\lambda+3=0$

$\lambda_1 = -1$

$\lambda_2 = -3$

2, $L_1 = \frac{1}{\lambda_1-\lambda_2}\left[\underline{\underline{A}} - \lambda_2 \underline{\underline{E}}\right] =$

$= \ldots = \cancel{\begin{bmatrix}-2 & -1,2\\ 5 & 3\end{bmatrix}} = \begin{bmatrix}1,5 & 0,5\\ -1,5 & -0,5\end{bmatrix}$

$L_2 = \frac{1}{\lambda_2-\lambda_1}\left[\underline{\underline{A}} - \lambda_1 \underline{\underline{E}}\right] = \cancel{\begin{bmatrix}3 & 1,2\\ -5 & -2\end{bmatrix}} = \begin{bmatrix}-0,5 & -0,5\\ 1,5 & 1,5\end{bmatrix}$ $\quad L_1 + L_2 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

3, $c^T e^{At} \cdot B = \begin{bmatrix}1 & 5\end{bmatrix} \cancel{(-1)} \left(e^{\lambda_1 t}\cdot L_1 + e^{\lambda_2 t}\cdot L_2\right)\begin{bmatrix}0\\ 1\end{bmatrix} =$

$= \begin{bmatrix}1 & 5\end{bmatrix}\cancel{\begin{bmatrix}-2 & -1,2\\ 5 & 3\end{bmatrix}}\begin{bmatrix}0\\ 1\end{bmatrix}e^{-t} + \begin{bmatrix}1 & 5\end{bmatrix}\cancel{\begin{bmatrix}3 & 1,2\\ -5 & -2\end{bmatrix}}\begin{bmatrix}0\\ 1\end{bmatrix}e^{-3t} =$

$\left(= \begin{bmatrix}\underbrace{-2+25}_{23} & \underbrace{-4,4+15}_{12,6}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}e^{-1t} + \ldots + e^{-3t} = \right)$

$= \cancel{12,6\, e^{-t} + 7 \cdot e^{-3t}} = -2e^{-t} + 7e^{-3t}$

4, $y(t) = \underbrace{c\, e^{At}\cdot x(0)}_{\text{kezdő állapot 0 és belső gerj} = 0} + \underline{\underline{C}}\int_0^t e^{A(t-\tau)}\cdot \underline{\underline{B}}\, u(\tau)\, d\tau + D\cdot u(t)$

$y(t) = \int_{-0}^{t}\left[-2e^{-(t-\tau)} + 7e^{-3(t-\tau)}\right] u(\tau)\, d\tau =$

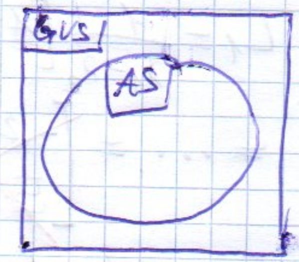
$= \ldots = \left[\frac{1}{3} + 2e^{-t} - \frac{7}{3}\cdot e^{-3t}\right]\varepsilon(t)$

J. és R. III. 11.
2,5 gyak. 8/8

FI

DI

5. lét



ASZ: A, vagy λ_1, λ_2

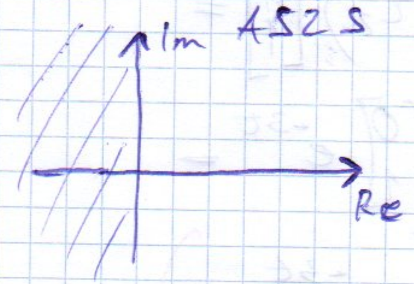
GVS: A, B, C, D vagy $h[k]$, $h(t)$

FI

$ASZ.S$ $\underbrace{\forall \lambda_i \quad Re\, \lambda_i < 0}$

$\Downarrow$

$ASZ\ S$



$\lambda^2 + a_1\lambda + a_2 = 0$

$\Downarrow$

$\left.\begin{matrix} a_1 > 0 \\ a_2 > 0 \end{matrix}\right\} \Rightarrow ASZ.\ S.$ Hurwitz krit

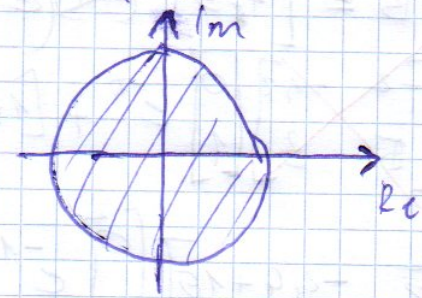
GVS_∞

$\int_{-\infty}^{\infty} h(t)\, dt < \infty \Rightarrow GVS$

DI

$ASZ.S.$

$\forall |\lambda_i| < 1$



$\lambda^2 + a_1\lambda + a_2 = 0$

$\left.\begin{matrix} 1 + a_1 + a_2 > 0 \\ 1 - a_1 + a_2 > 0 \\ |a_2| < 1 \end{matrix}\right\} \Rightarrow ASZ.S.$ Jury krit.

GVS

$\sum_{k=-\infty}^{\infty} h[k] < \infty \Rightarrow GVS$

$A \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}$ ha $c_1 = c_2 = 0 = GVS$

$B \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$C [c_1\ c_2]$

$D = 1$

Jelek és Rendszerek
3. gyakorlat

Eddig

$y(t) = h(t) * u(t)$

$y[k] = h[k] * u[k]$

$x_1[k+1] = \dots$

$x_2[k+1] = \dots$

$y[k] = \dots$

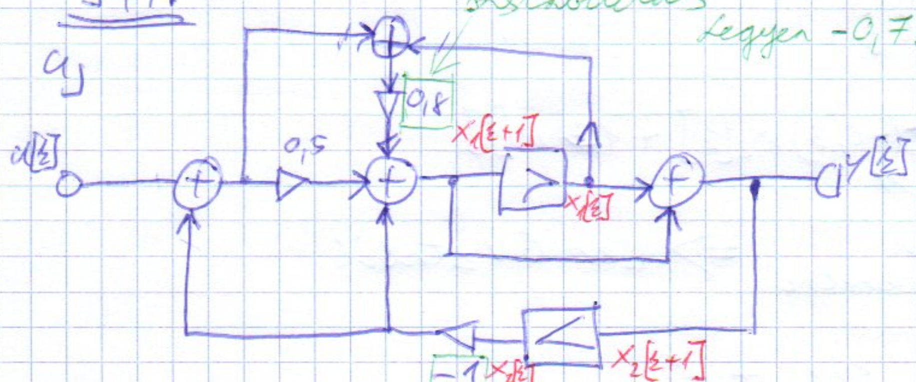
$x_1'(t) = \dots$

$x_2'(t) = \dots$

$y(t) = \dots$

17H

a)



$$x_1[k+1] = 0{,}8\left(x_1[k] + u[k] - x_2[k]\right) + 0{,}5\left(u[k] - x_2[k]\right) - x_2[k]$$
$$= 0{,}8\, x_1[k] - 2{,}3\, x_2[k] + 1{,}3\, u[k]$$

$$x_2[k+1] = x_1[k] + \underbrace{0{,}8\left(x_1[k] + u[k] - x_2[k]\right) + 0{,}5\left(u[k] - x_2[k]\right) - x_2[k]}_{x_1[k+1]}$$

$$= 1{,}8\, x_1[k] - 2{,}3\, x_2[k] + 1{,}3\, u[k]$$

$$y[k] = x_2[k+1] = 1{,}8\, x_1[k] - 2{,}3\, x_2[k] + 1{,}3\, u[k]$$

$$\underline{A} \begin{pmatrix} 0{,}8 & -2{,}3 \\ 1{,}8 & -2{,}3 \end{pmatrix} \quad B \begin{bmatrix} 1{,}3 \\ 1{,}3 \end{bmatrix} \quad C^T \begin{bmatrix} 1{,}8 , & -2{,}3 \end{bmatrix} \quad D = 1{,}3$$

b) $\det(\underline{\underline{A}} - \lambda \underline{\underline{E}}) = 0$

6. let

$$\begin{vmatrix} 0{,}8-\lambda & -2{,}3 \\ 1{,}8 & -2{,}3-\lambda \end{vmatrix} = (0{,}8-\lambda)(2{,}3+\lambda)\cdot(-1) - (-2{,}3)\cdot 1{,}8 =$$

$$= \dots = \begin{cases} \lambda_1 = \\ \lambda_2 = \end{cases} \lambda_{12} = -0{,}75 \pm 1{,}32\,j$$

A.S. $\forall\, i$-re $|\lambda_i| < 1$

$|\lambda| = \sqrt{a^2 + b^2}$ Nem A.S.

$\hookrightarrow \lambda = a + bj$

$y[k] = 1{,}8\, x_1[k] \mp (-2{,}3)\, x_2[k] + 1{,}3\, u[k]$

Mindkét á.v. kijut a kimenetre $\Rightarrow$ Nem G.V. stabil

Stabilitás megállapítás

⋮

$$x_1[k+1] = -0{,}7\, x_1[k] - 0{,}8\, x_2[k] - 0{,}2\, u[k]$$

$$x_2[k+1] = 0{,}3\, x_1[k] - 0{,}8\, x_2[k] - 0{,}2\, u[k]$$

$$y[k] = 0{,}3\, x_1[k] - 0{,}8\, x_2[k] - 0{,}2\, u[k]$$

A, B, C^T, D

$u[k] = \varepsilon[k]$

Lépésenkénti megoldás?

$y[k] = ?$

k	$x_1[k]$	$x_2[k]$	$u[k]$	$y[k]$
0	0	0	1	$-0{,}2$
1	$(-0{,}2)\cdot\cancel{(-0{,}2)}$ $\cancel{(-0{,}04)}$ $-0{,}2$	$-0{,}2$	1	$0{,}3\cdot(-0{,}2)-0{,}8\cdot(0{,}2)-0{,}2=\underline{\underline{-0{,}1}}$ $-\frac{6}{100}+\frac{16}{100}-\frac{20}{100}$
2	$0{,}1$	$-0{,}1$	1	$-0{,}09$
3	$-0{,}19$	$-0{,}09$	1	$-0{,}185$

$k=2$

$$x_1[k]=(-0{,}7)\cdot(-0{,}2)+(-0{,}8)(-0{,}2)-0{,}2=0{,}1$$

$$u(t)=3\cos\left(2\pi f t+\frac{\pi}{3}\right)\longrightarrow \overline{U}=3\,e^{j\frac{\pi}{3}}$$

$$u[k]=\sqrt{2}\cos\left(2\pi f k+\frac{\pi}{4}\right)\longrightarrow \overline{U}=\sqrt{2}\,e^{j\frac{\pi}{4}}$$

$$\overline{H}=A_H\cdot e^{j\varphi_H}=\sqrt{7}\cdot e^{j\frac{\pi}{8}}$$

$$y(t)=\underbrace{|\overline{Y}|}\cos\left(2\pi f t+\underbrace{}^{\mathrm{arc}(\overline{Y})}\right)\qquad \overline{Y}=\overline{H}\cdot\overline{U}=\sqrt{7}\cdot 3\cdot e^{j\left(\frac{\pi}{3}+\frac{\pi}{8}\right)}$$

$$y[k]=|\overline{Y}|\cos\left(2\pi f k+\mathrm{arc}(\overline{Y})\right)\qquad y(t)=3\cdot\sqrt{7}\cos\left(2\pi f t+\frac{\pi}{3}+\frac{\pi}{8}\right)$$

Átviteli karakterisztika

DI $\quad H(j\omega)\overset{\text{Pl.}}{=}\dfrac{1}{j\omega+3}$

$\downarrow\uparrow$

FI $\quad H(e^{j\vartheta})$

III. 18.

J. és R. 3. gyak

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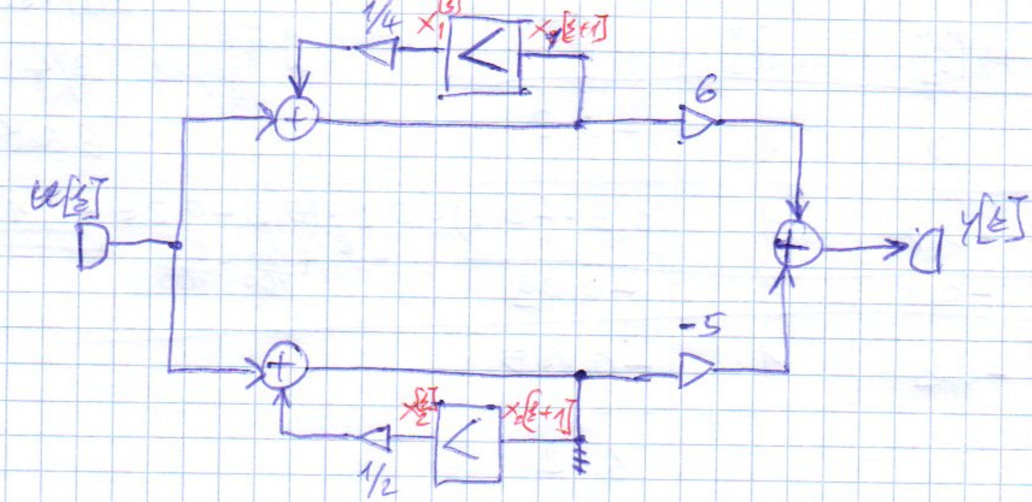
6. hét



$$x_1[k+1] = \frac{1}{4} x_1[k] + 0\, x_2[k] + 1\, u[k]$$

$$x_2[k+1] = 0\, x_1[k] + \frac{1}{2} x_2[k] + 1 \cdot u[k]$$

$$y[k] = 6\, x_1[k+1] - 5\, x_2[k+1] =$$

$$= \underset{1,5}{\frac{6}{4}} x_1[k] - \underset{2,5}{\frac{5}{2}} x_2[k] + u[k]$$

$x[k] = X(e^{j\omega})$

$x[k+1] = X(e^{j\omega}) \cdot e^{-j\nu}$

$y[k] = Y(e^{j\nu})$

(1) $X_1(e^{j\nu}) \cdot (e^{-j\nu}) = 0{,}25 \cdot X_1(e^{j\nu}) + 0 \cdot X_2(e^{j\nu}) + U(e^{j\nu})$

Késleltetés

$x[k+1] = X(e^{j\omega}) \cdot \boxed{e^{j\nu}}$

(2) $X_2(e^{j\nu})(e^{-j\nu}) = 0\, X_1(e^{j\nu}) + 0{,}5\, X_2(e^{j\nu}) + U(e^{j\nu})$

Deriválás

$\frac{dx}{dt} = x'(t) = X(j\omega)(j\omega)$

(3) $Y(e^{j\nu}) = 1{,}5\, X_1(e^{j\nu}) - 2{,}5\, X_2(e^{j\nu}) + U(e^{j\nu})$

$(1) \rightarrow X_1 \rightarrow (2) \rightarrow X_1, X_2 \rightarrow (3) \rightarrow H = \frac{Y}{U}$