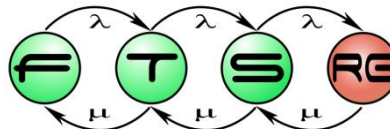
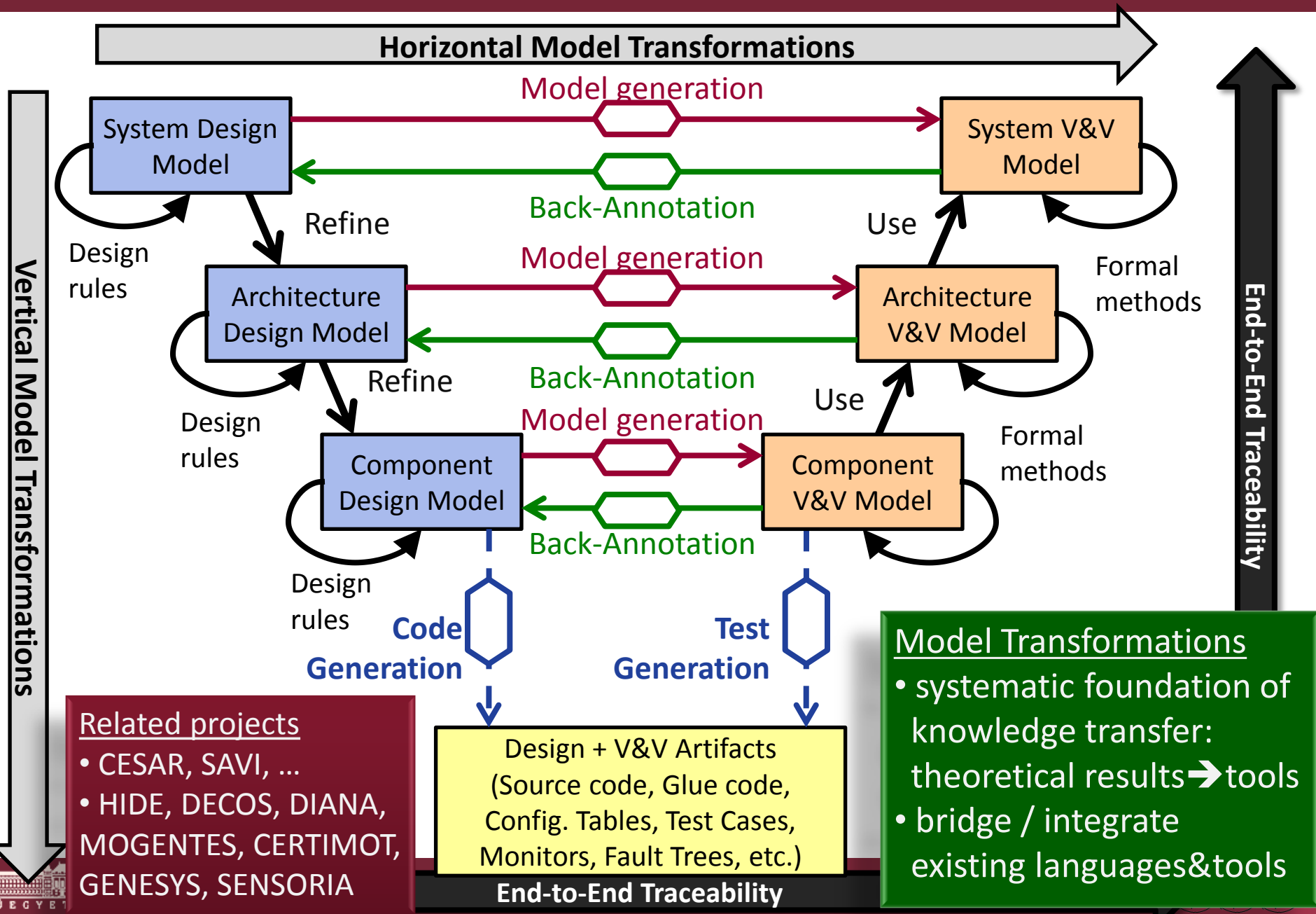


Foundations of Model Transformation

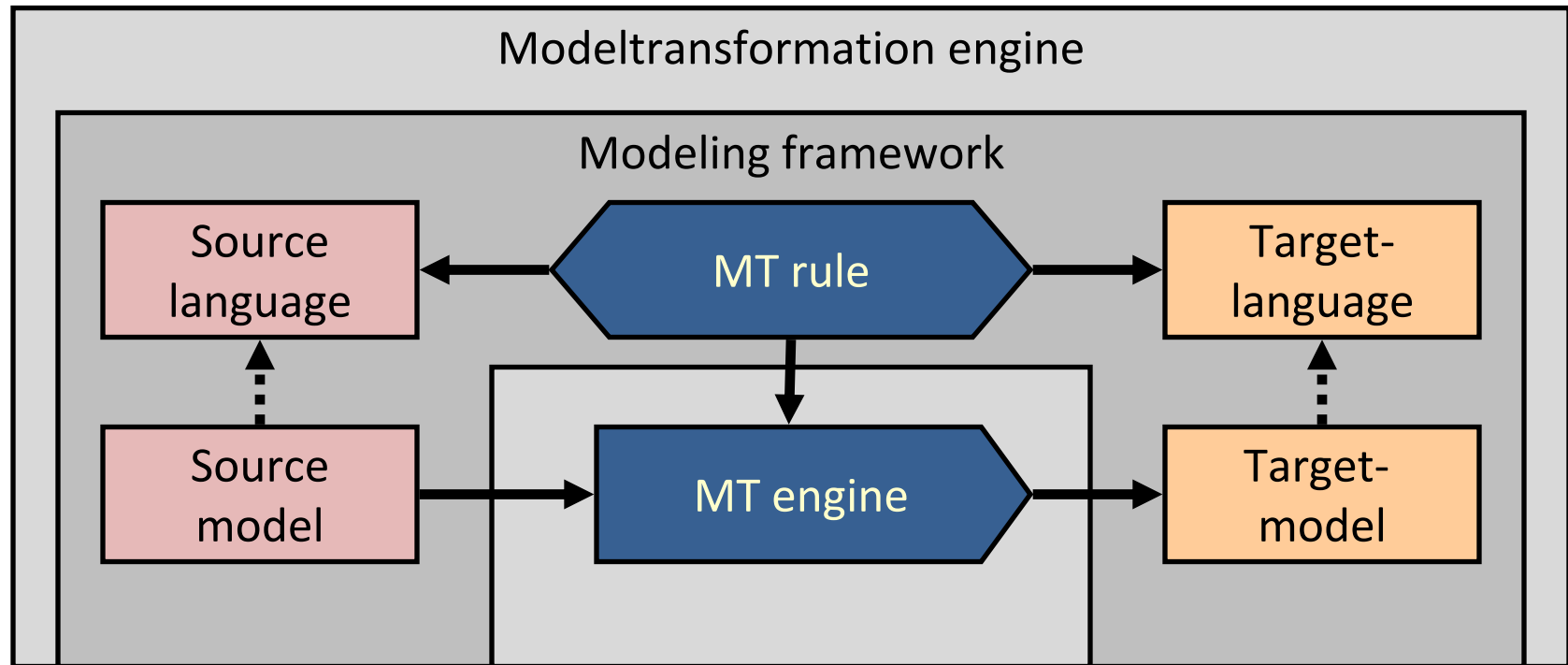
Model Driven Systems Development
Lecture 9-10



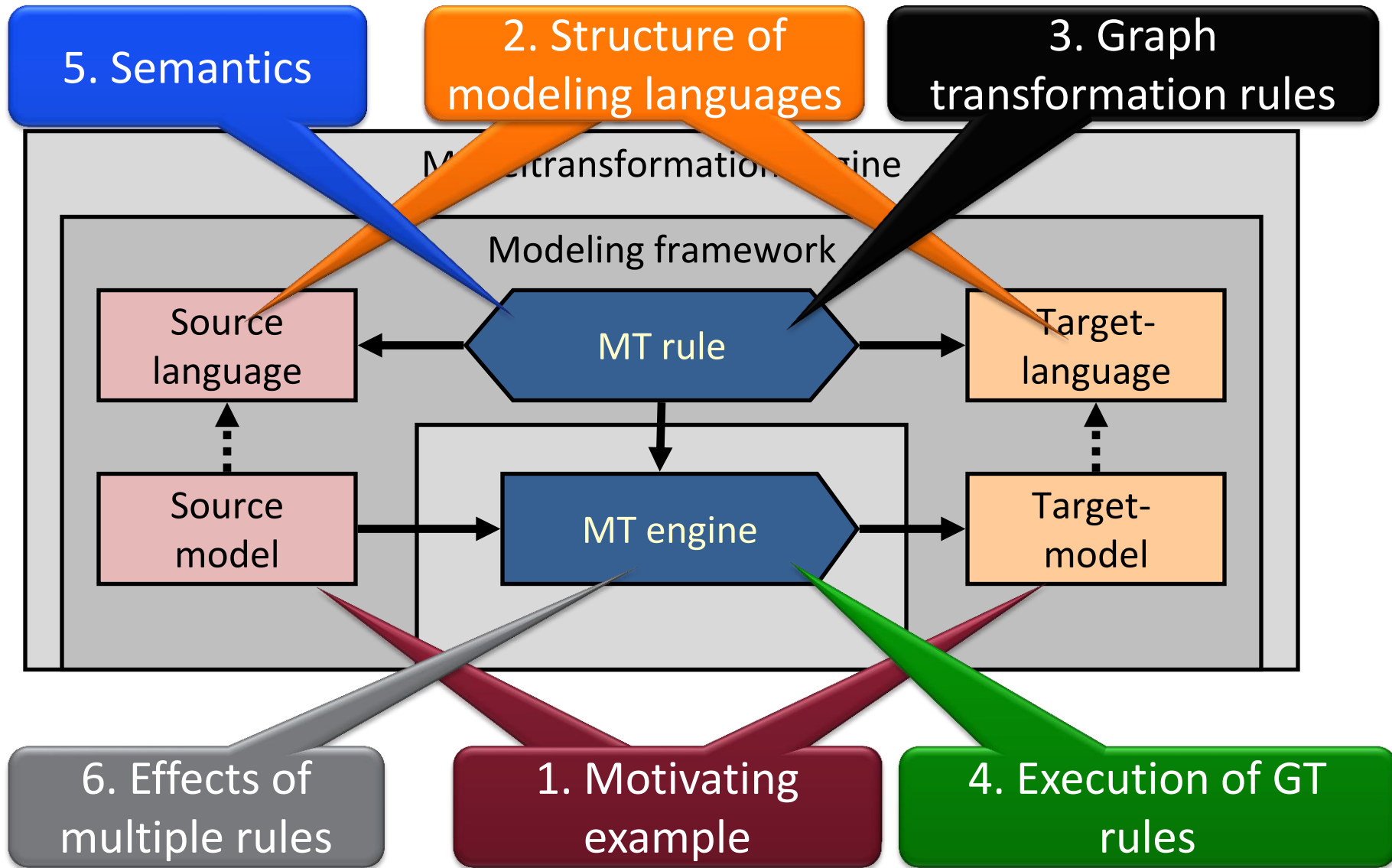
Models and Transformations in Critical Systems



Definition of Model Transformation



Overview



1. Motivating Example

Object Relational Schema mapping

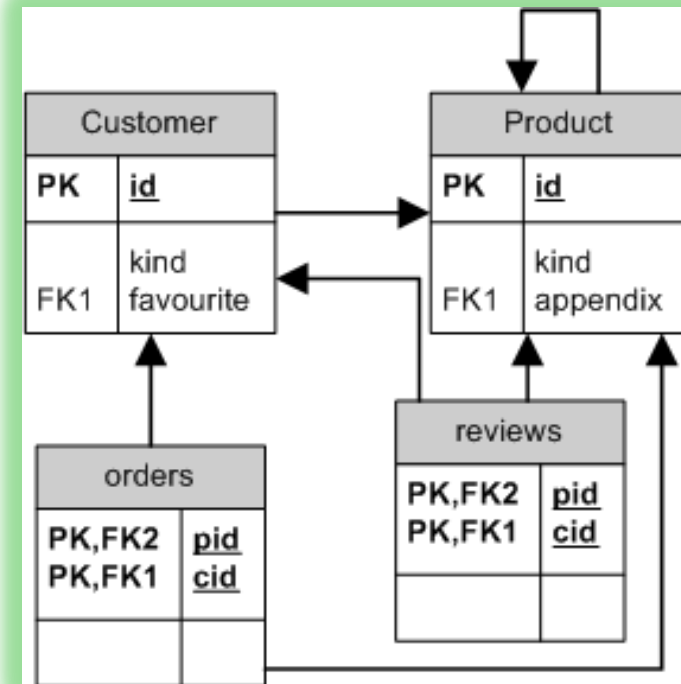
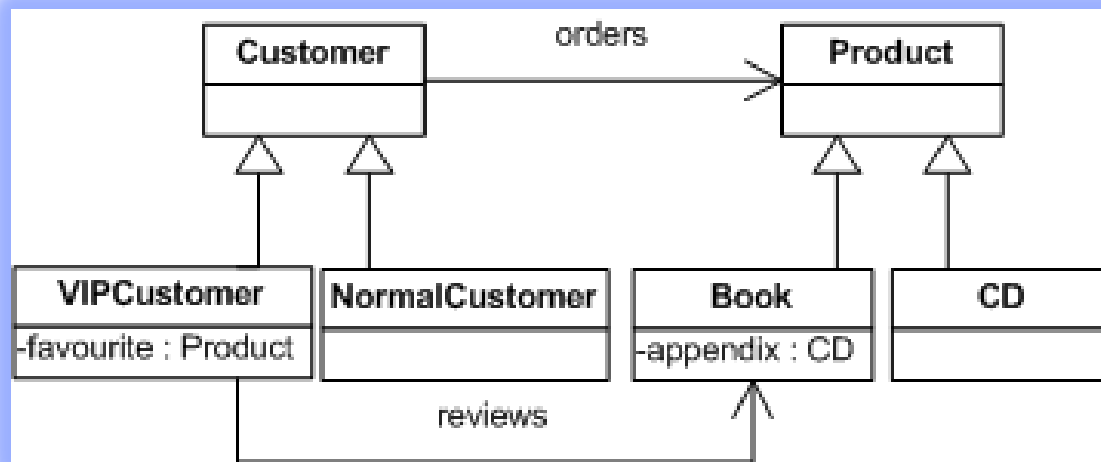
Example: Object-relational mapping

■ Important as:

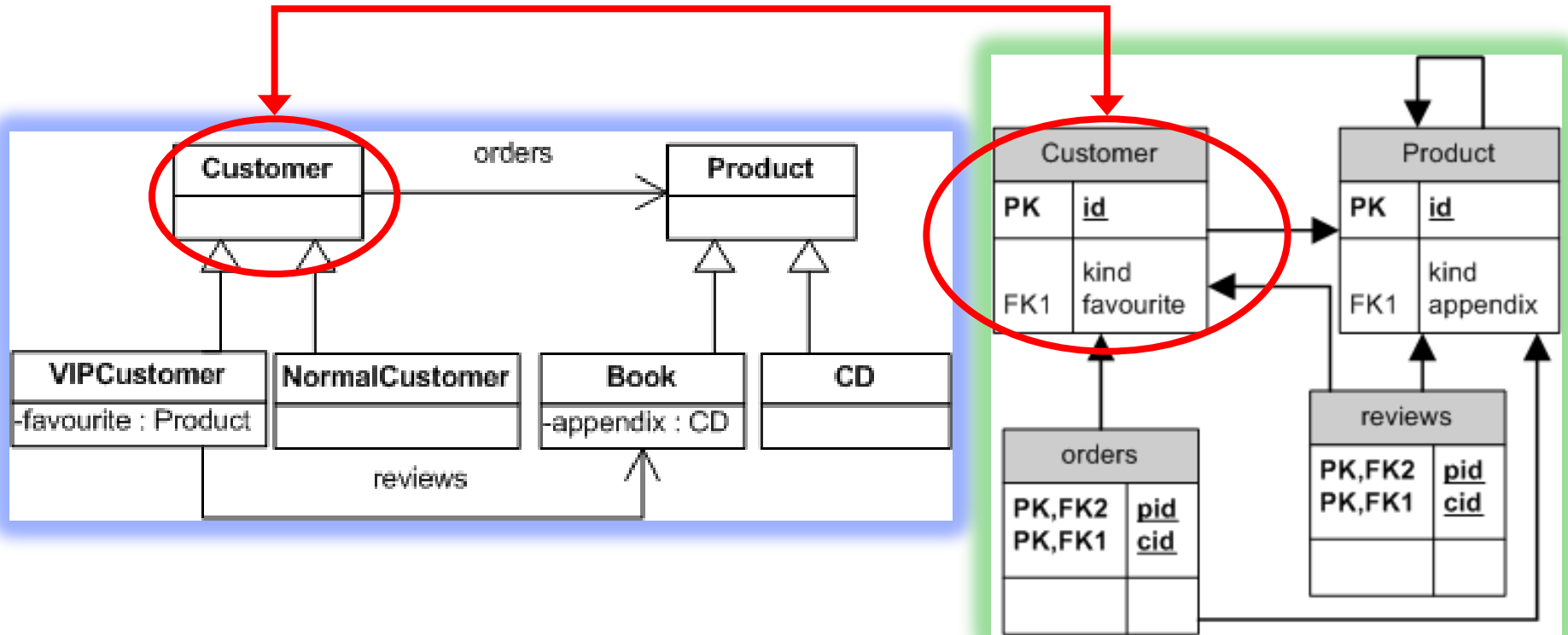
- Model transformation benchmark
- Most widely used industrial model transformation (pl. Hibernate, EJB, CDO)

■ Objective:

- **Input:**
UML class diagram
- **Output**
Relational database schema



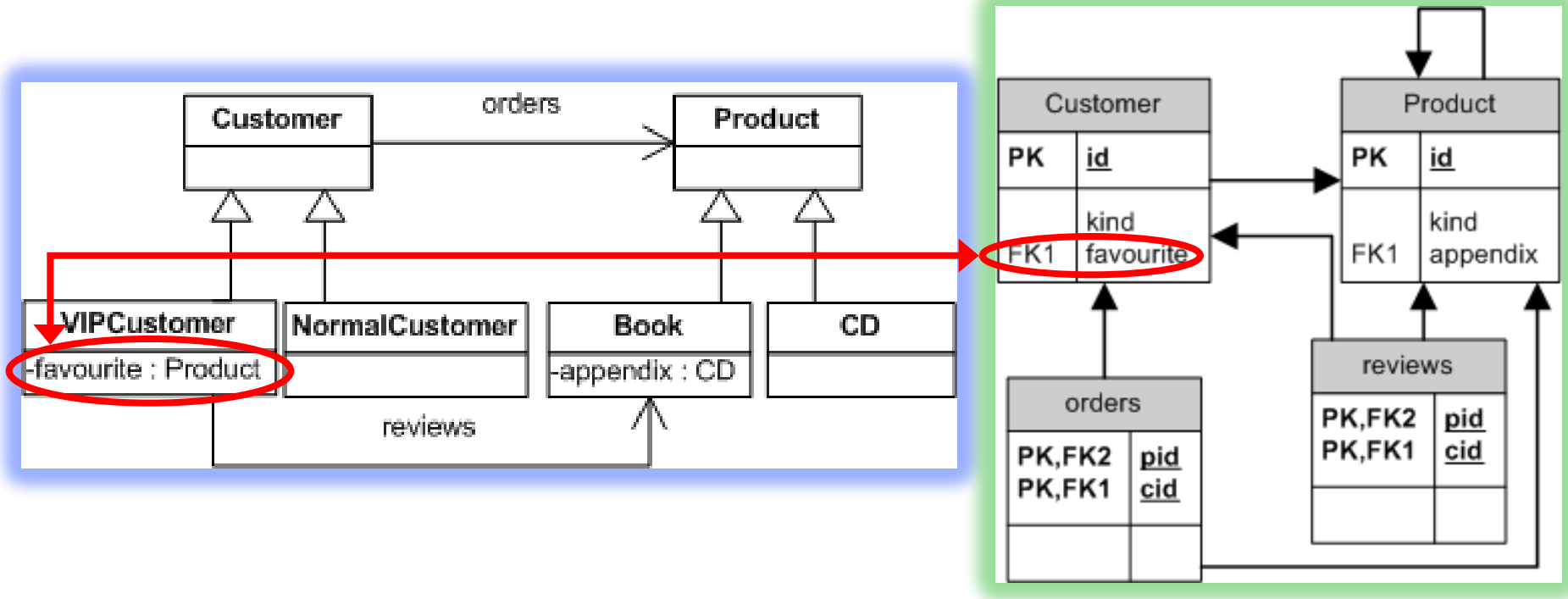
Informal definition of the MT rules of the mapping



Topmost (generalization) classes → Database table + 2 column:

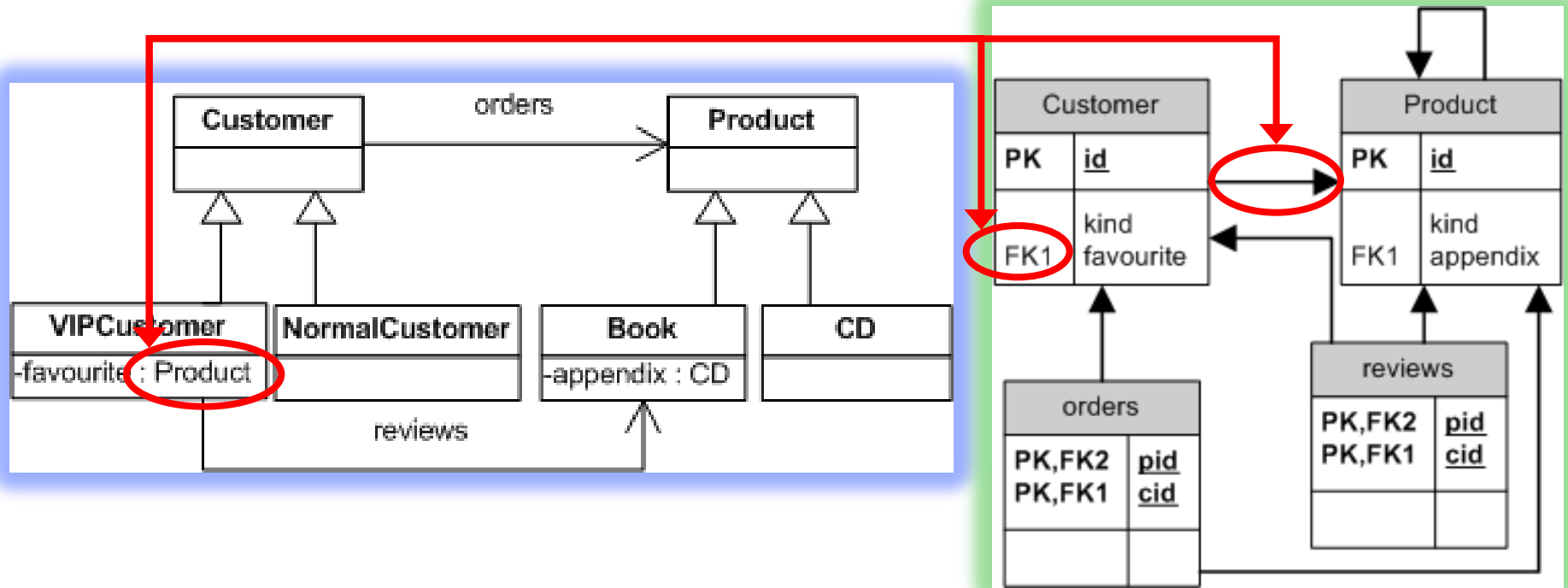
- Unique identifier (primary key),
- type definition

Informal definition of the MT rules of the mapping



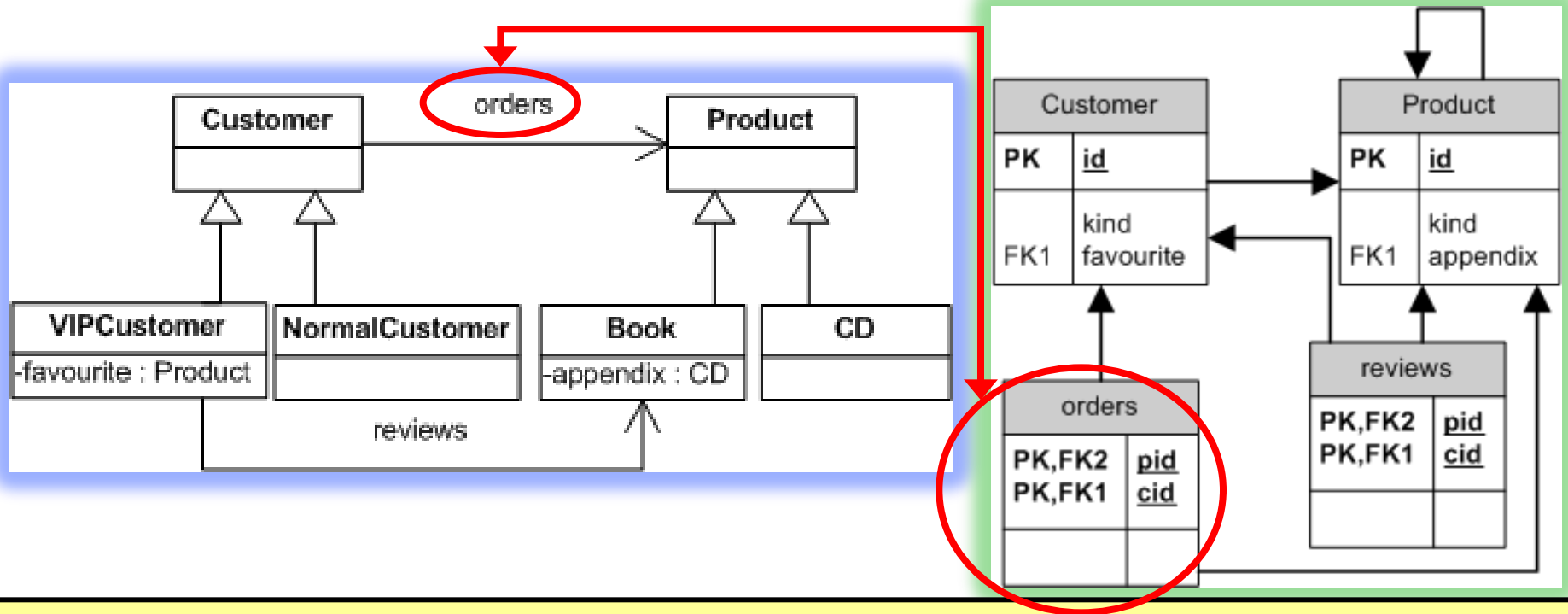
Class attributes → (contained by the topmost classes) Column of the table

Informal definition of the MT rules of the mapping



Type of the attributes → foreign key

Informal definition of the MT rules of the mapping



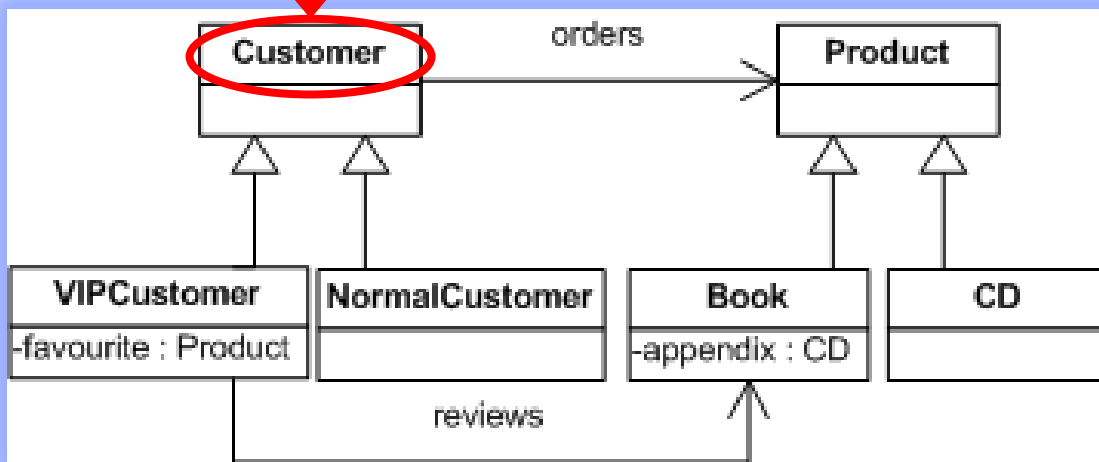
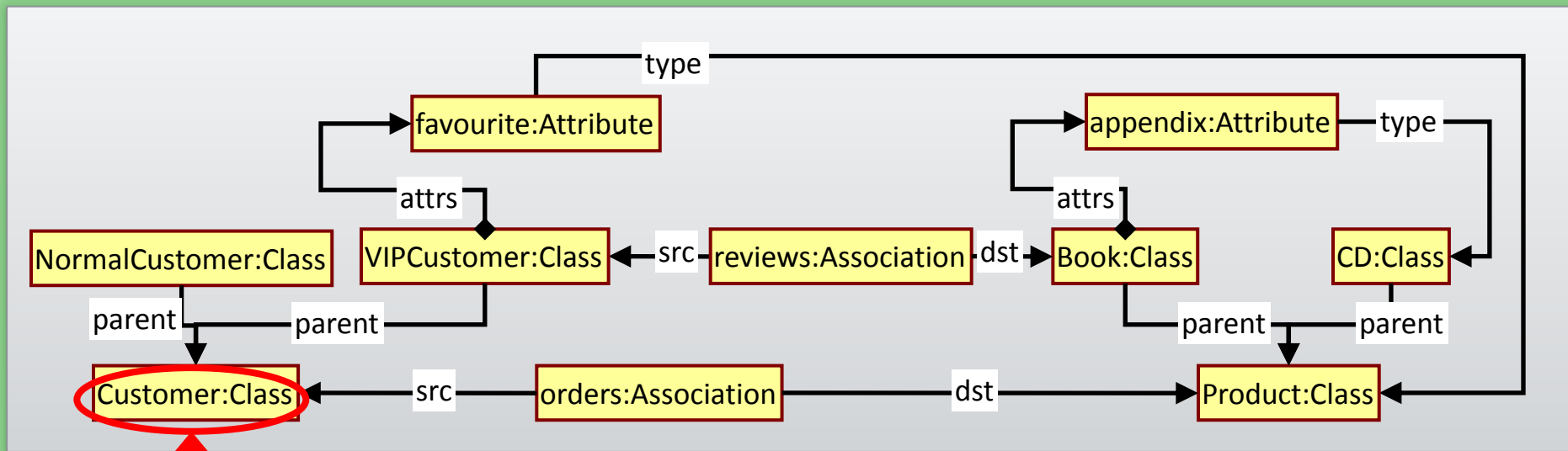
Association ➔ A table with two columns

- source and target identifiers
- foreign keys (for consistency)

2. Structure of Modeling Languages

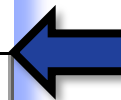
Revision

Structure of Modeling languages (UML)



Abstract syntax

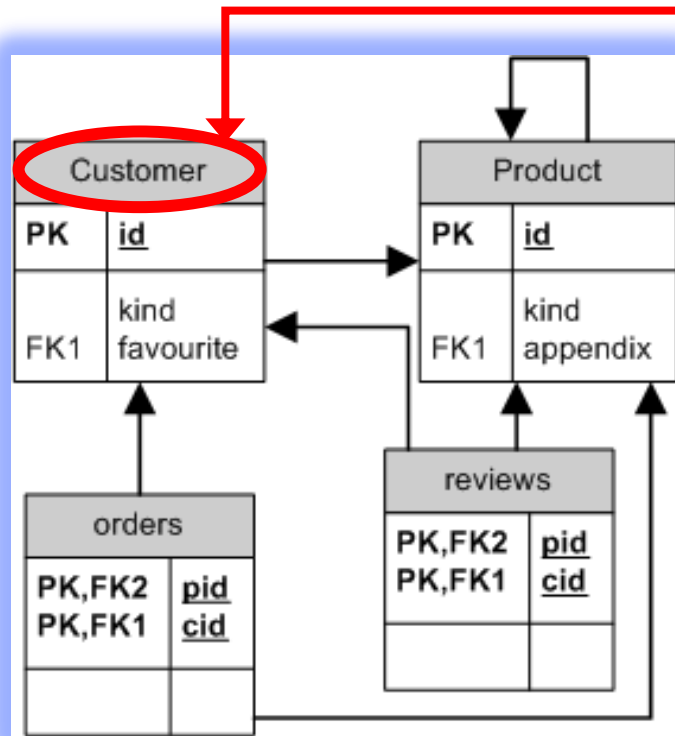
- Graph based model representation
- Machine readable



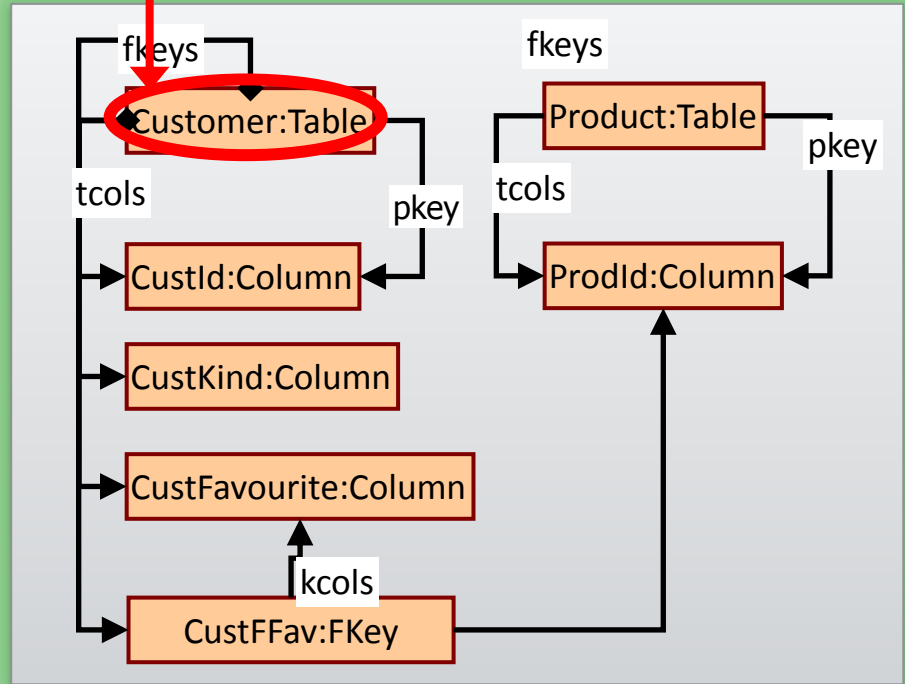
Concrete syntax

- Visual/textual representation
- Human readable

Structure of Modeling languages (RDBMS Schema)

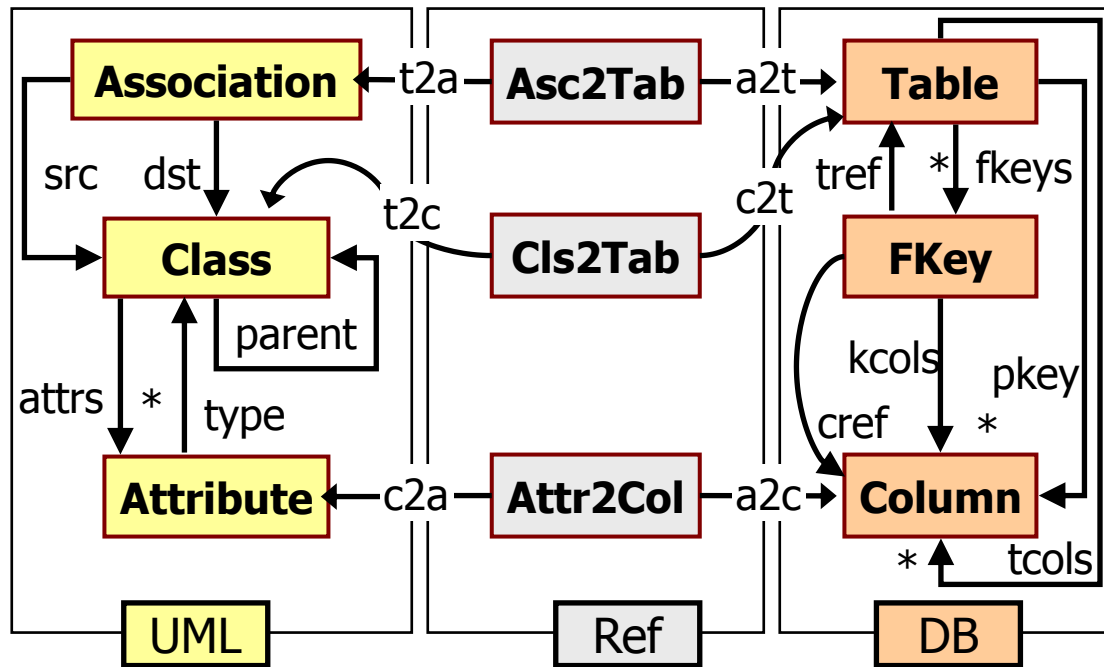


Concrete syntax

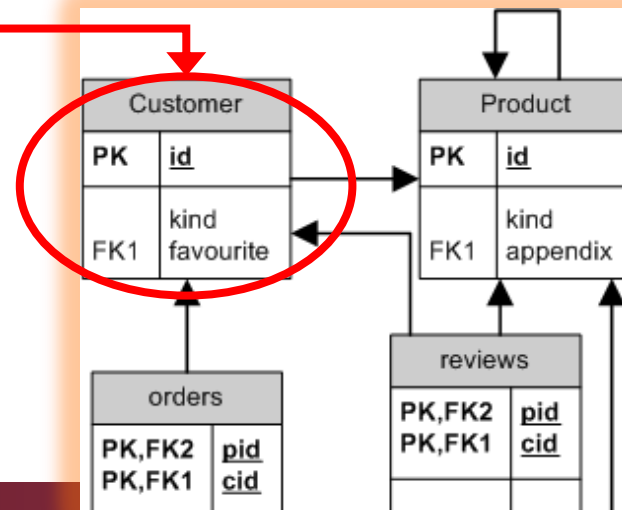
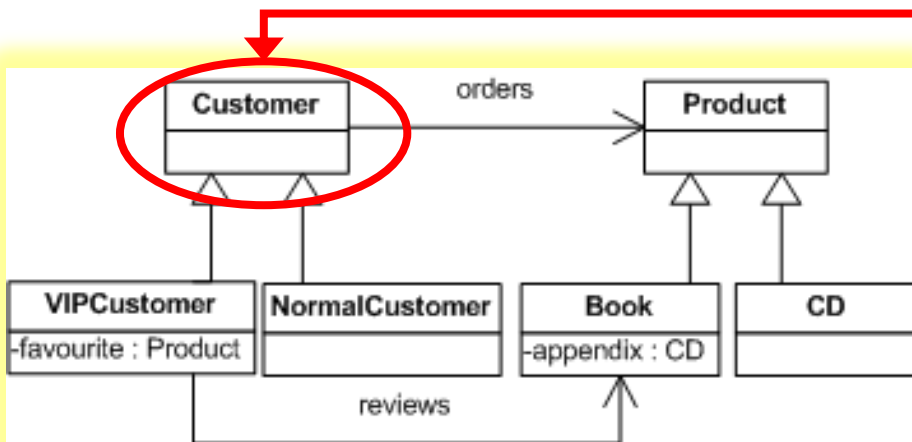


Abstract syntax

Metamodel of the O-R mapping

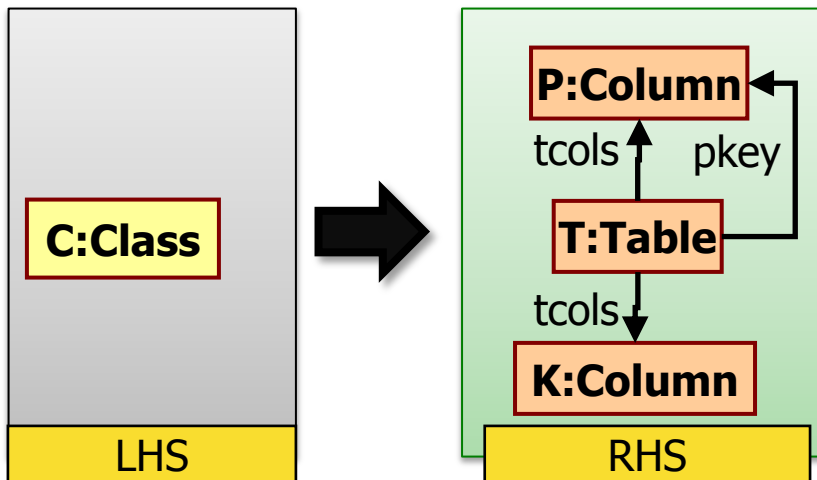


- Source + Target metamodel
- Traceability metamodel:
 - For saving the relations between the source and the target languages
- Motivation: critical embedded systems
 - Traceability
 - Requirement → Source code



3. Graph Transformation Rules

Structure of a GT rule



■ Graph Transformation Rules

○ Left hand side - LHS

- Graph pattern
- Precondition for the rule application

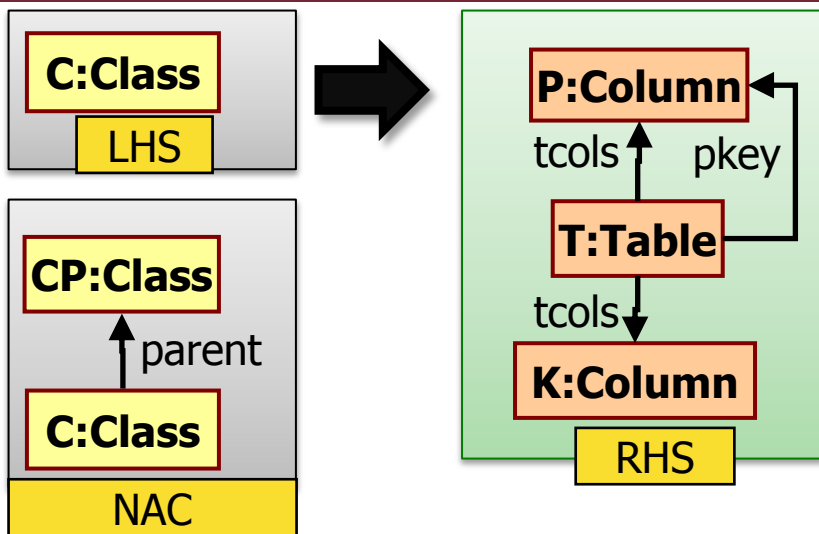
○ Right hand side - RHS:

- Graph pattern + LHS mapping
- Declarative definition of the rule application
 - What we get (and not how we get it)

■ Graph Transformation (GT):

- Declarative and formal paradigm
- Rule base transformation
- Match of the LHS → match of the RHS
- Generalization of Chomsky grammars (hierarchy) (text → graph)

Structure of a GT rule



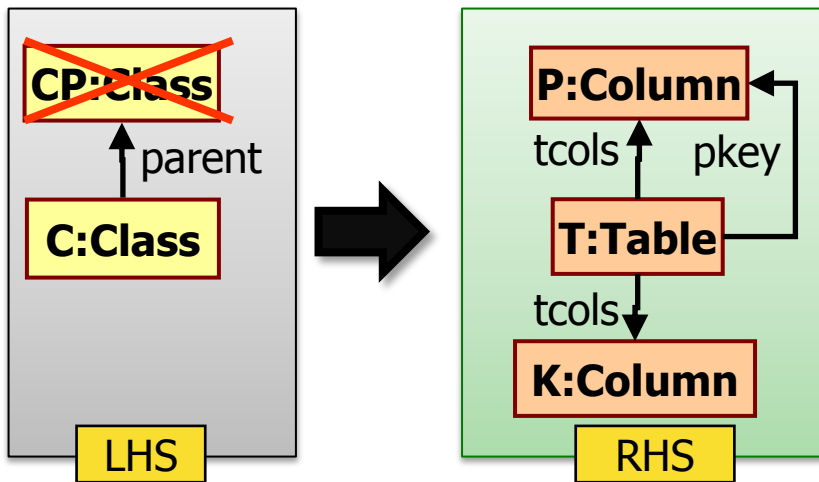
■ Graph Transformation Rules

- **Left hand side - LHS**
 - Graph pattern
 - Precondition for the rule application
- **Right hand side - RHS:**
 - Graph pattern + LHS mapping
 - Declarative definition of the rule application
 - What we get (and not how we get it)
- **Negative Application Condition(NAC):**
 - Graph pattern + LHS mapping
 - Negative precondition of the rule application
 - If it can be made true → the rule cannot be applied
 - Multiple NACs → only one is true → rule cannot be applied

■ Graph Transformation (GT):

- Declarative and formal paradigm
- Rule base transformation
- Match of the LHS → Image of the RHS
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Structure of a GT rule



■ Graph Transformation Rules

- **Left hand side - LHS**
 - Graph pattern
 - Precondition for the rule application
- **Right hand side - RHS:**
 - Graph pattern + LHS mapping
 - Declarative definition of the rule application
 - What we get (and not how we get it)
- **Negative Application Condition(NAC):**
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■ Graph Transformation (GT):

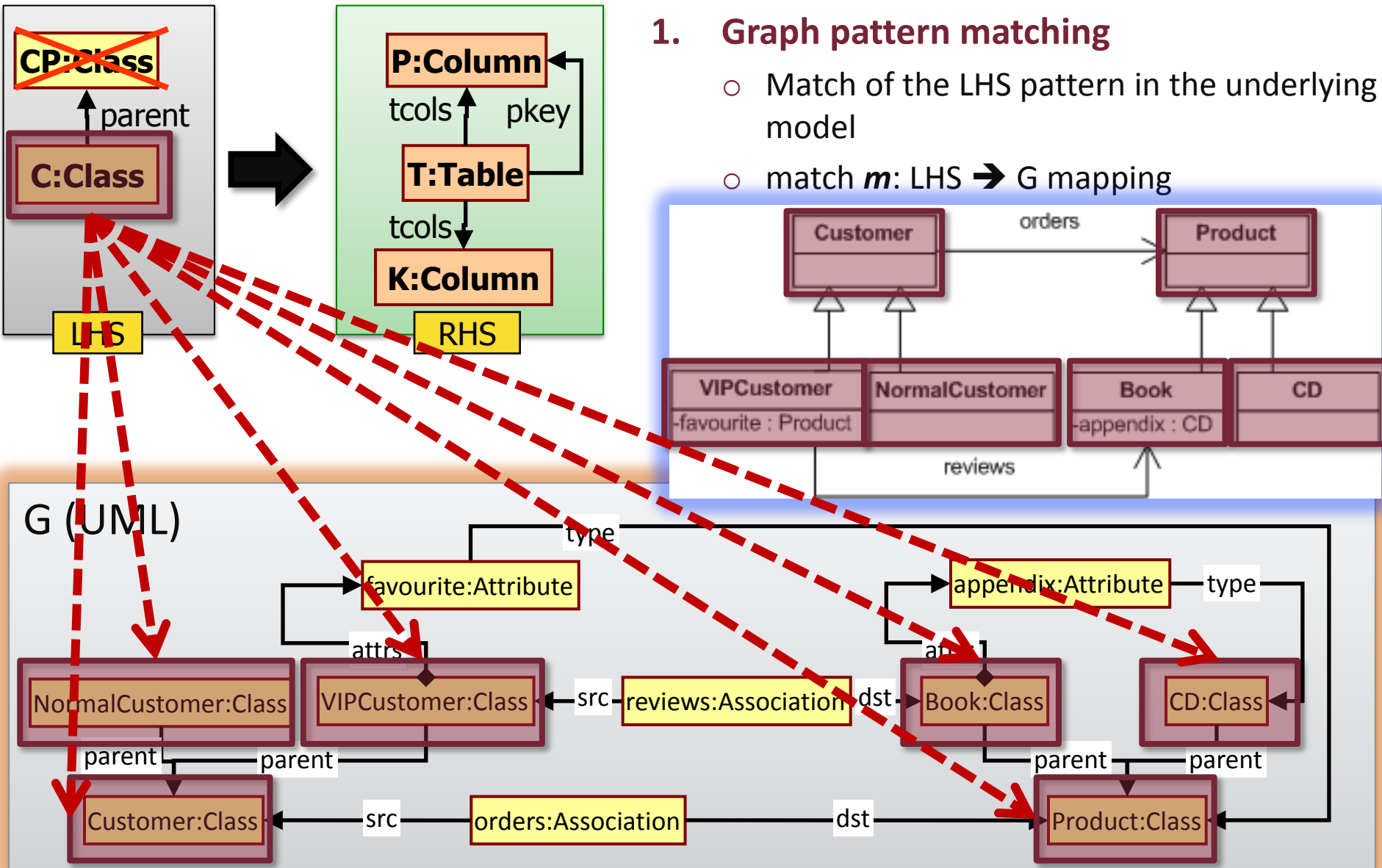
- Declarative and formal paradigm
- Rule base transformation
- Match of the LHS → Image of the RHS
- Generalization of Chomsky grammars (hierarchy) (text → graph)

4. Application of Graph Transformation Rules

Application of GT rules

1. Graph pattern matching

- Match of the LHS pattern in the underlying model
- match *m*: LHS → G mapping

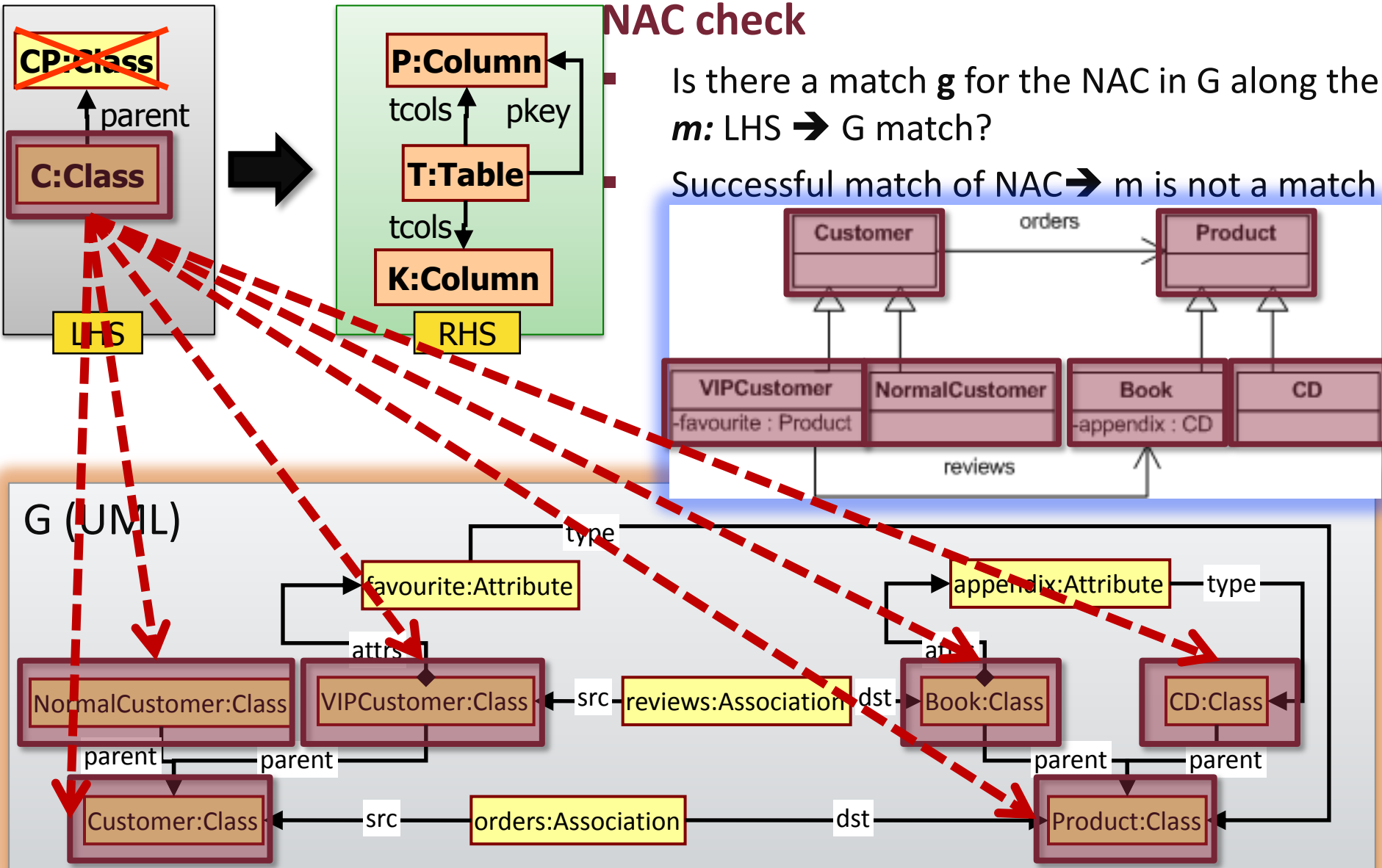


Application of GT rules

NAC check

Is there a match g for the NAC in G along the m : LHS \rightarrow G match?

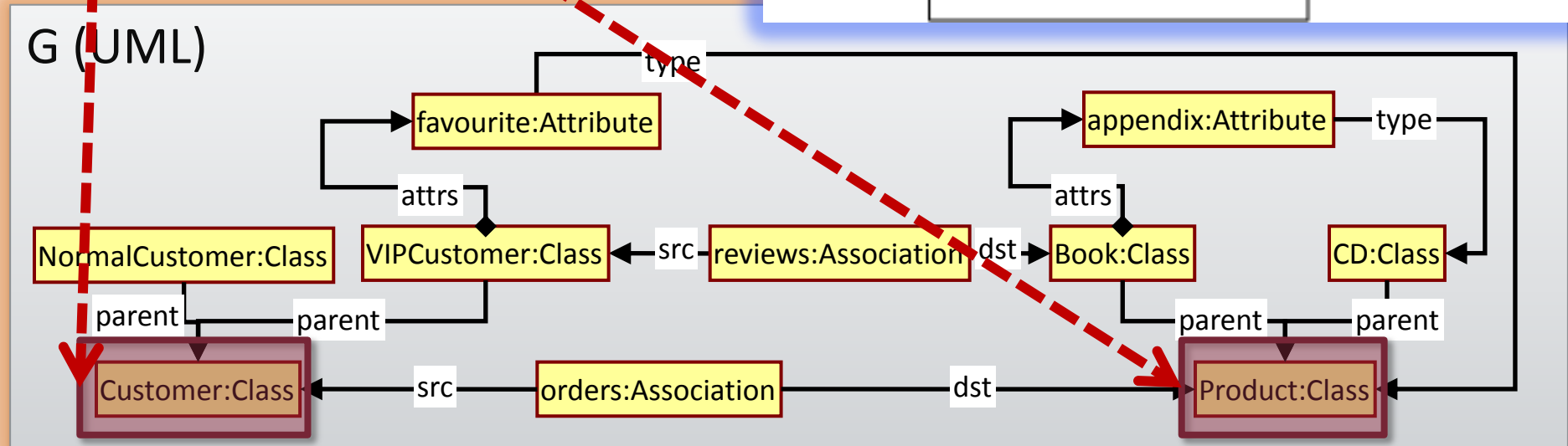
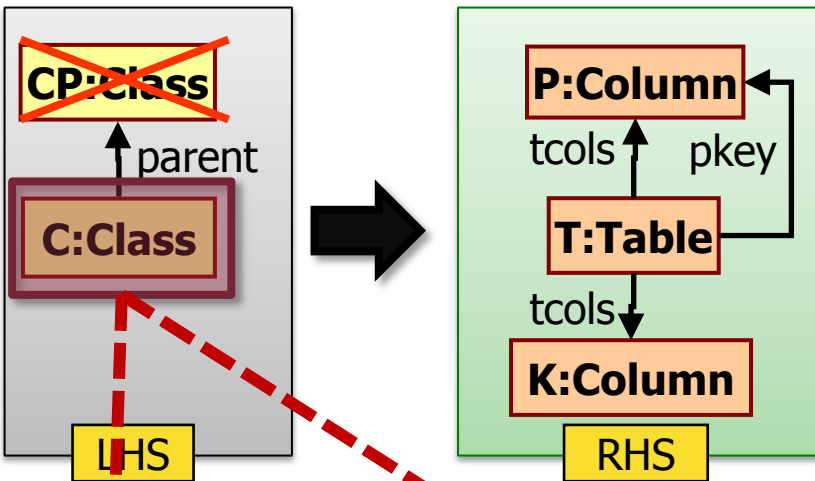
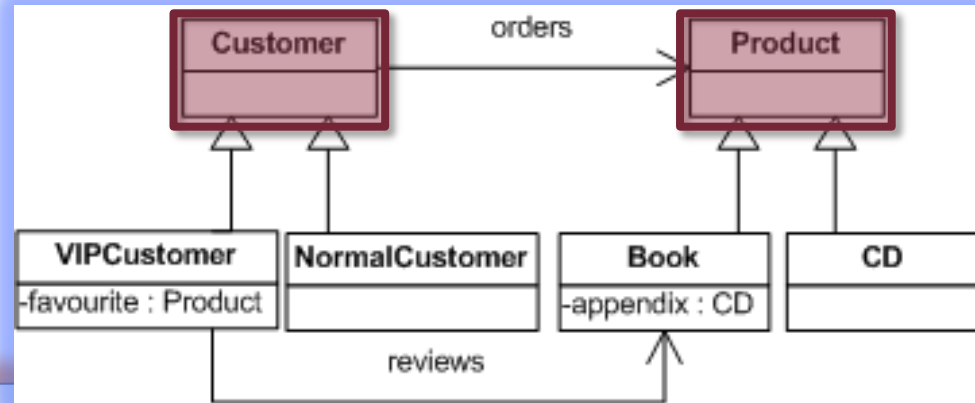
Successful match of NAC \rightarrow m is not a match



Application of GT rules

3. Non-deterministic selection

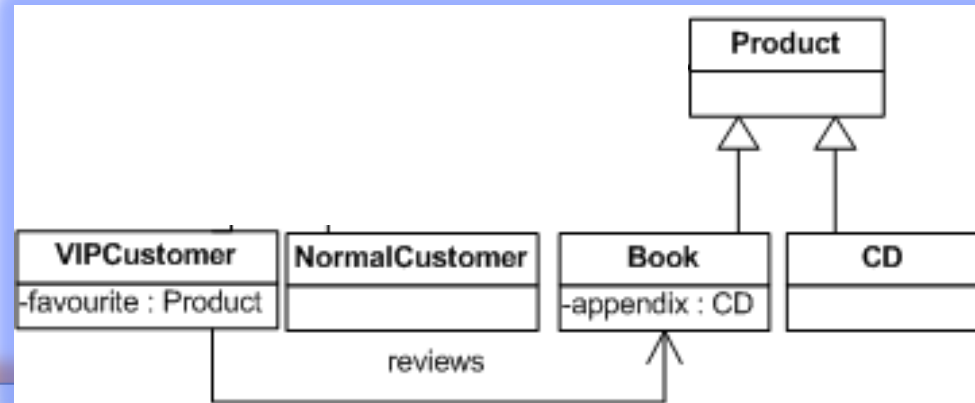
- Random selection of a match (if more than one)
- No match → rule fails



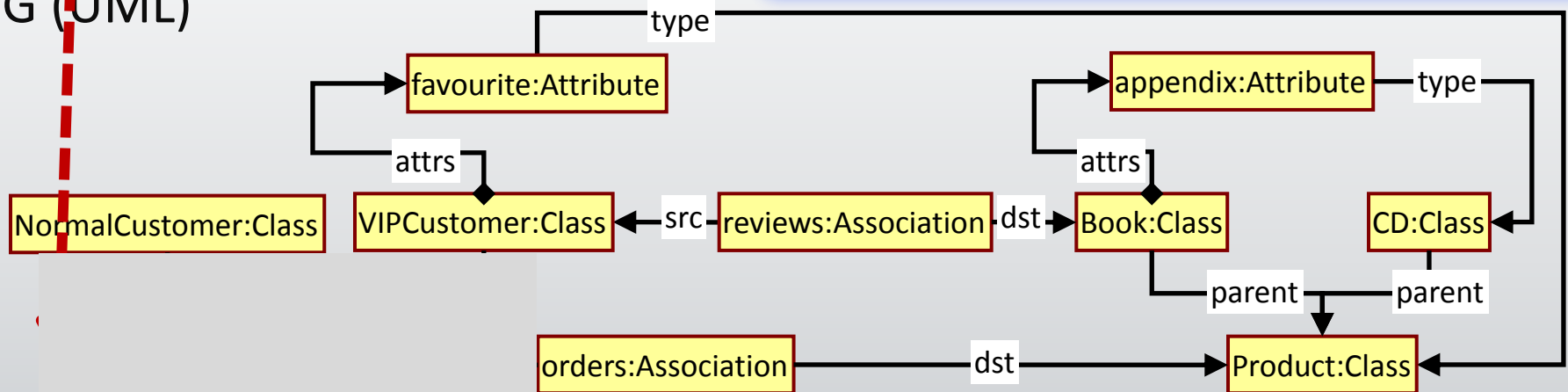
Application of GT rules

4. Deletion

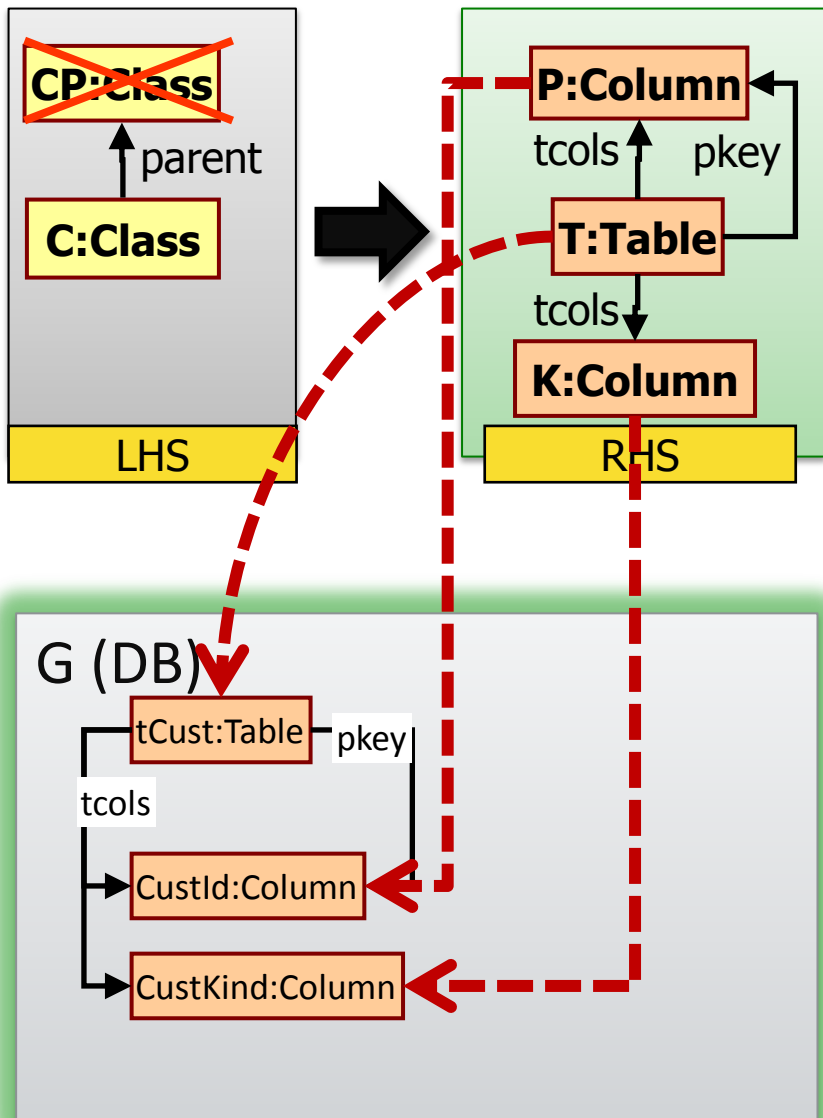
- Deletion of LHS \ RHS from G
- In LHS yes, in RHS no



G (UML)



Application of GT rules



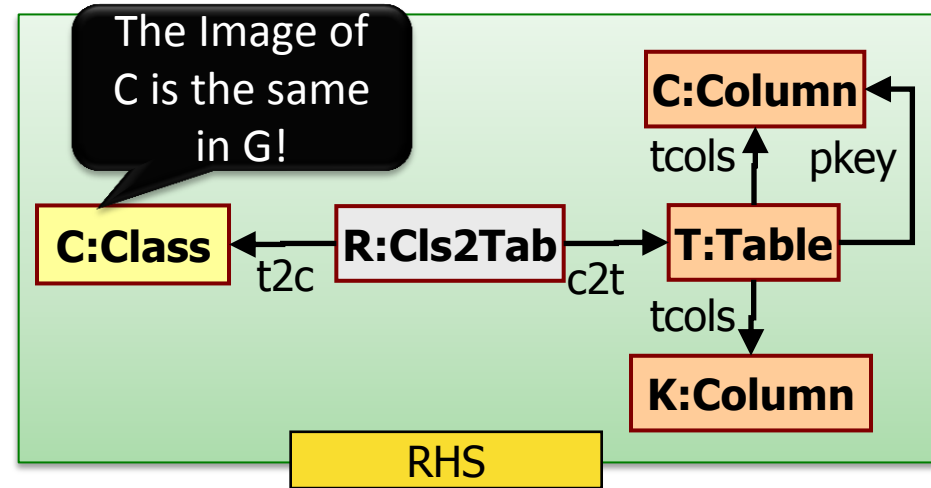
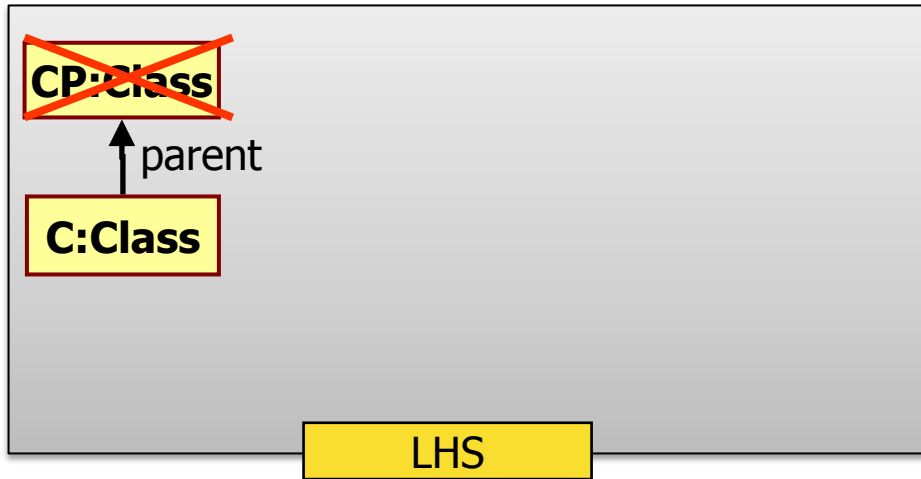
5. Creation (and binding)

- Creation of RHS \ LHS in G with their corresponding relations
- Output: a „match” of RHS in G

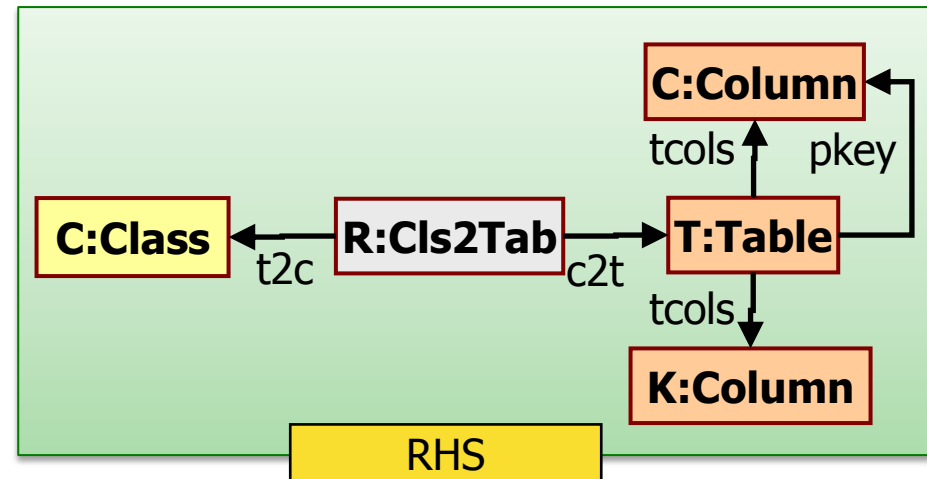
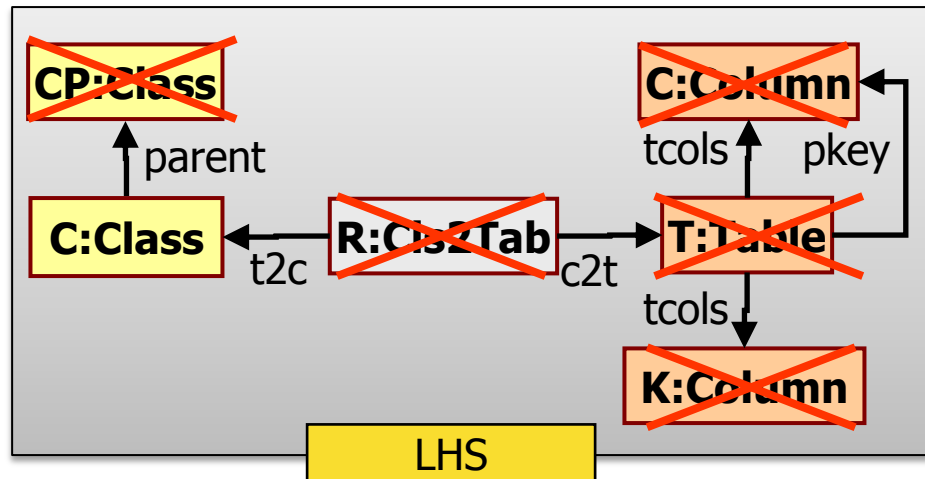
Customer	
PK	<u>id</u>
	kind

Typical problems...

1) Saving the source model, traceability

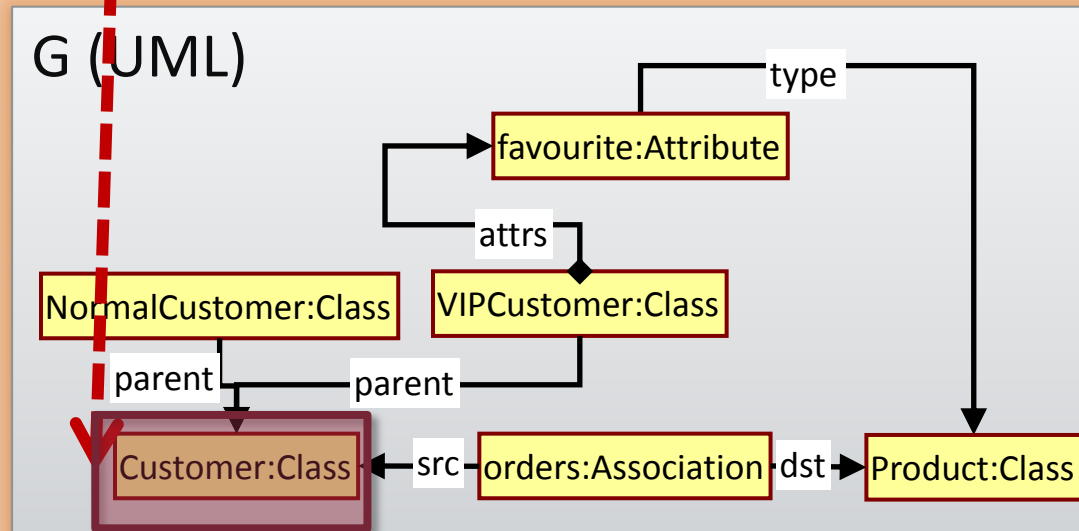
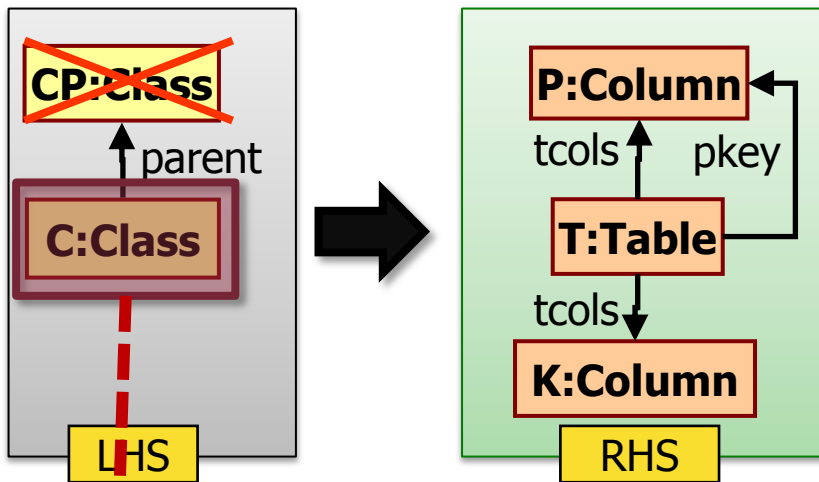


2) Application of the same rule along the same match



5. Different Semantics

Semantics : Handling of Dangling edges



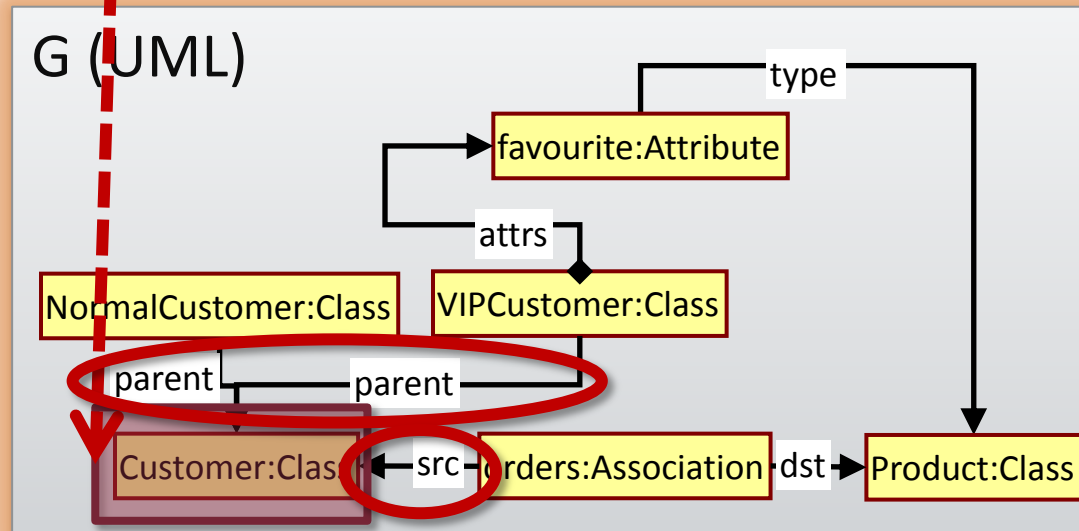
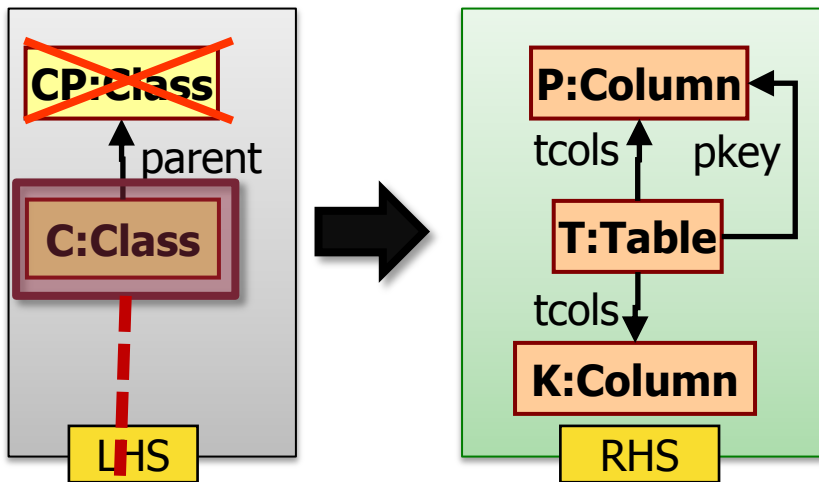
■ Dangling edges:

- Delete a node
 - What to do with the dangling edges?

■ Greedy approach

- Delete all dangling edges
- **Pro:**
 - Intuitive for engineers
 - Easy to implement
- **Con:**
 - Verification is hard (side effect of rules)

Semantics : Handling of Dangling edges



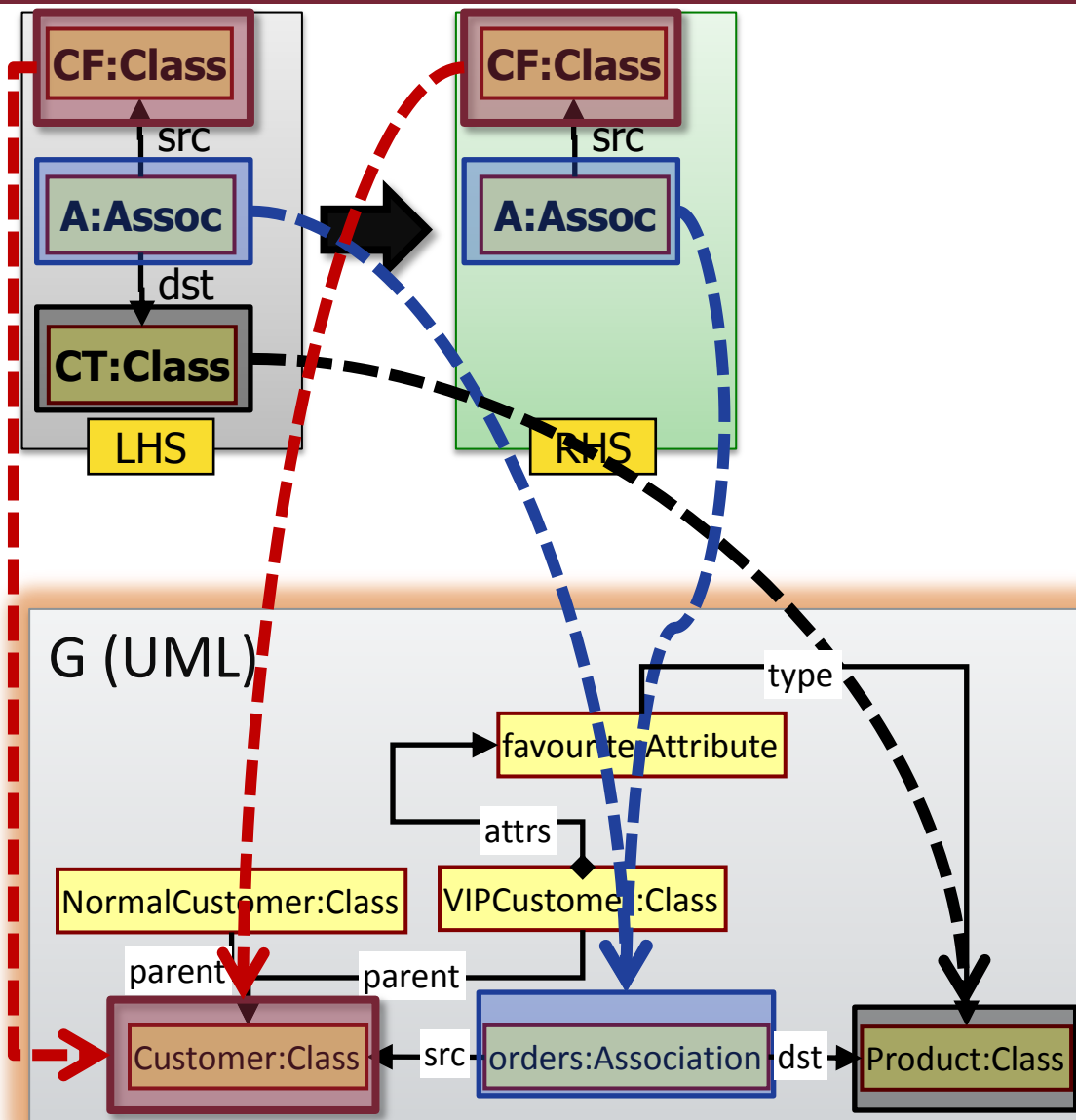
■ Dangling edges:

- Delete a node
 - What to do with the dangling edges?

■ Conservative approach

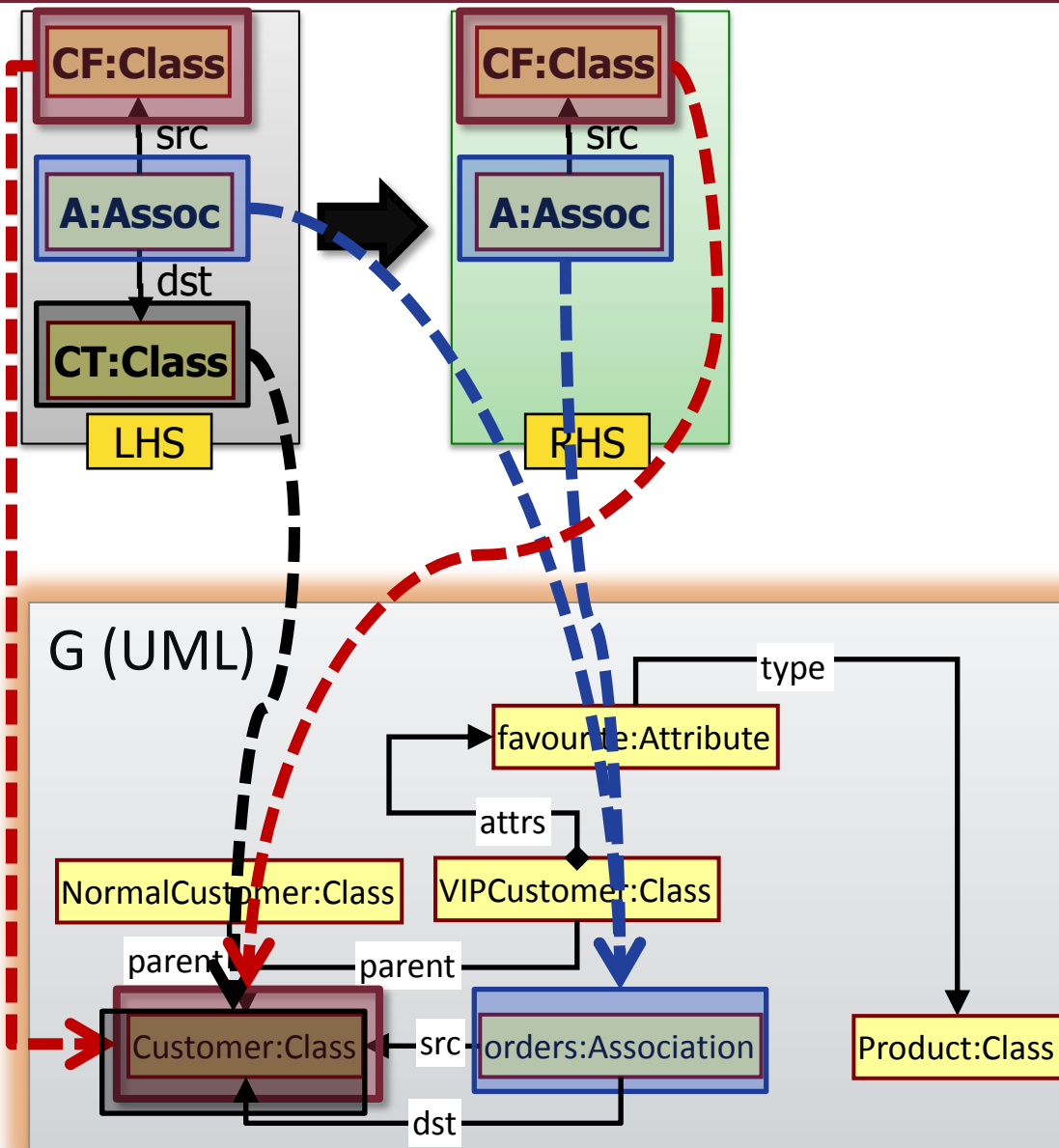
- The rule cannot be applied if it would produce a dangling edge
- **Pro:**
 - Side effect free rules
 - Helps verification
- **Con:**
 - Harder to implement
 - What is its meaning for engineers (not mathematicans)

Semantics: Injective matching



- **Injective matching („kisajátító”)**
 - For all nodes in the LHS → separate nodes are matched in G
- **Pro:**
 - Intuitive for engineers
- **Con:**
 - Verbose specification of rules (many alternate subrules)

Semantics: Non-injective matching



■ Non-Injective matching („közösködő”)

- For multiple nodes in the LHS → the same node can be matched in G

■ Con:

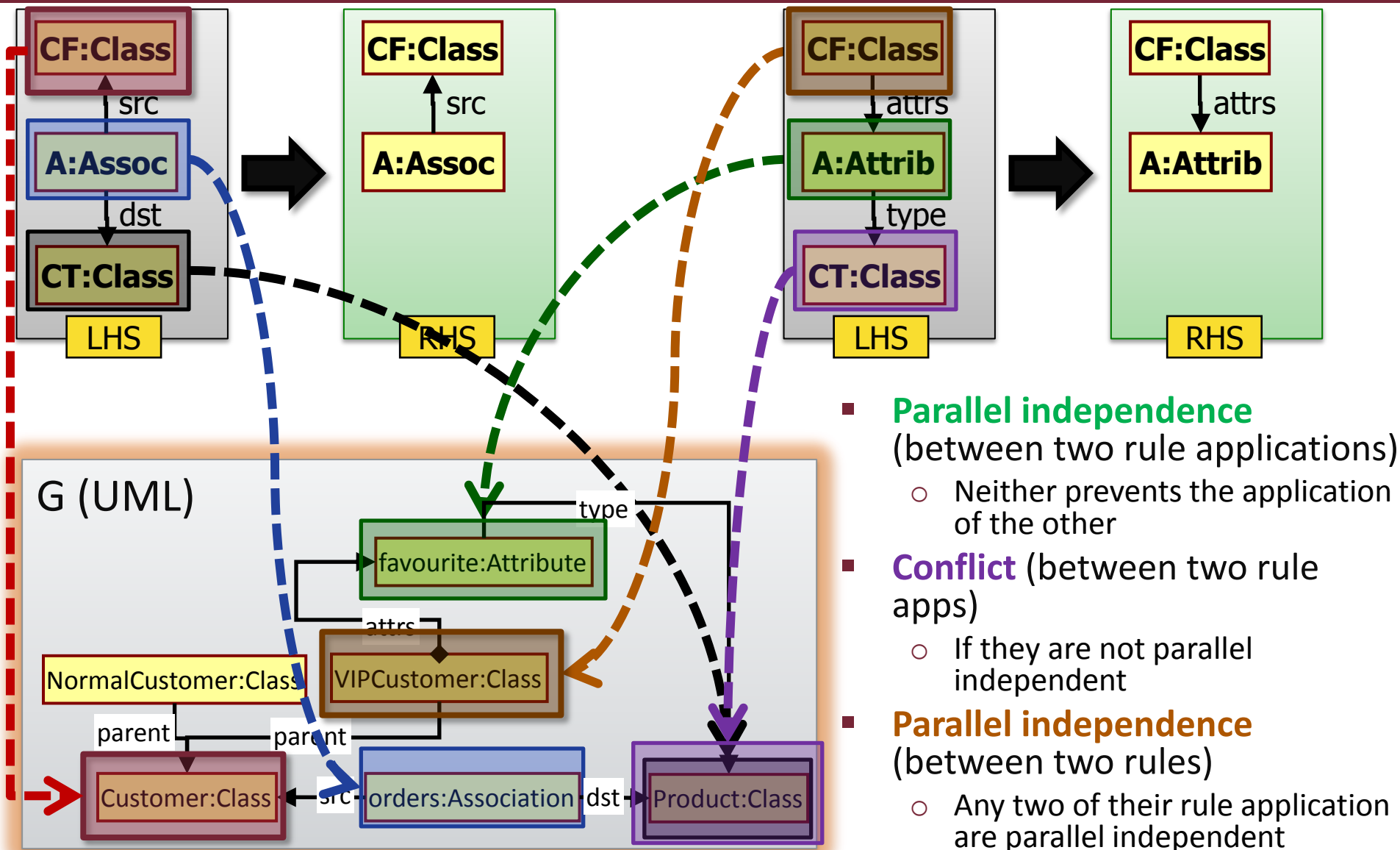
- Contradictory specification for a node
 - For **CF** : keep it
 - For **CT** : delete

■ Solution:

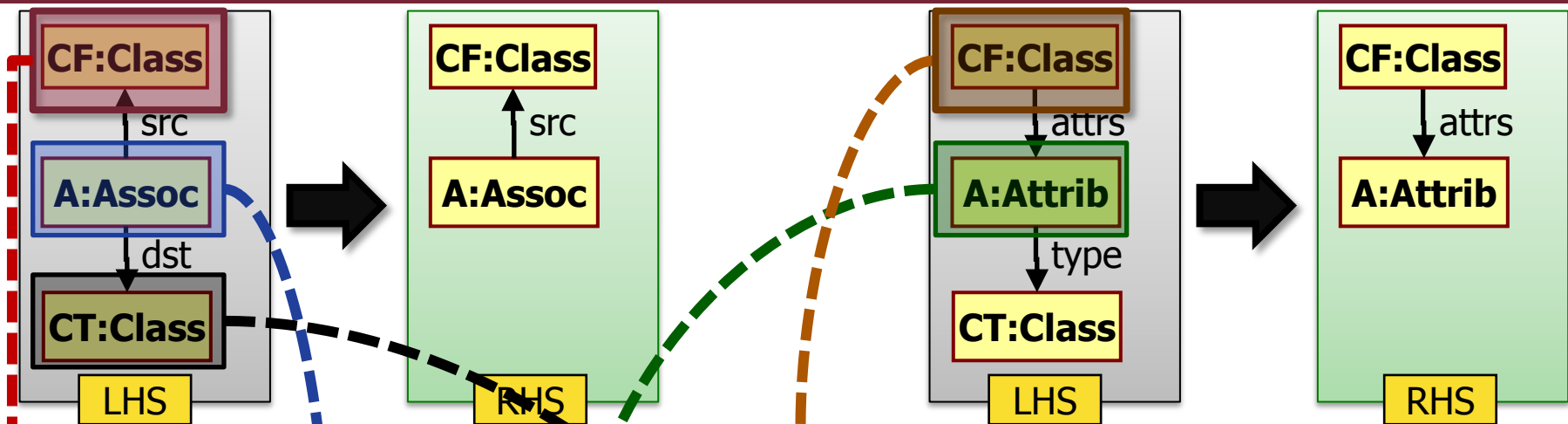
- Nodes to be deleted in LHS are matched with injective semantics

6. Effects of Multiple GT Rules

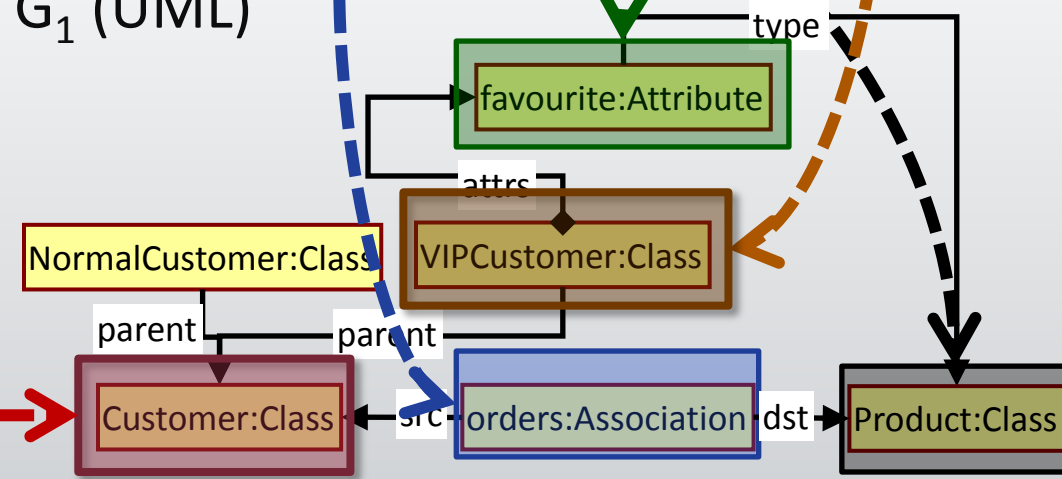
Conflict / Parallel independence



Sequential independence



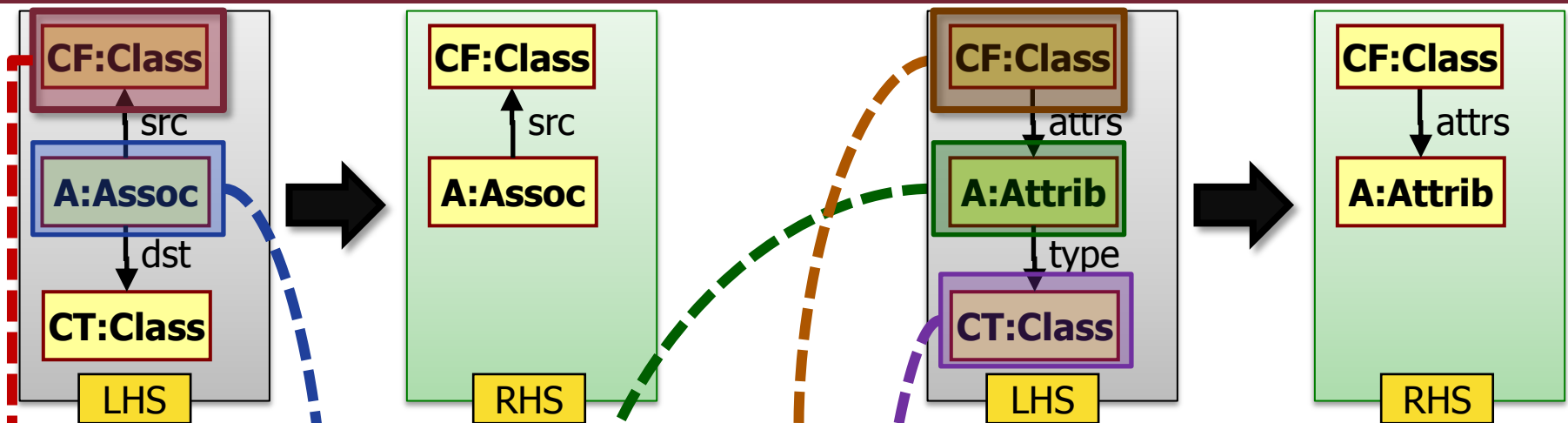
G_1 (UML)



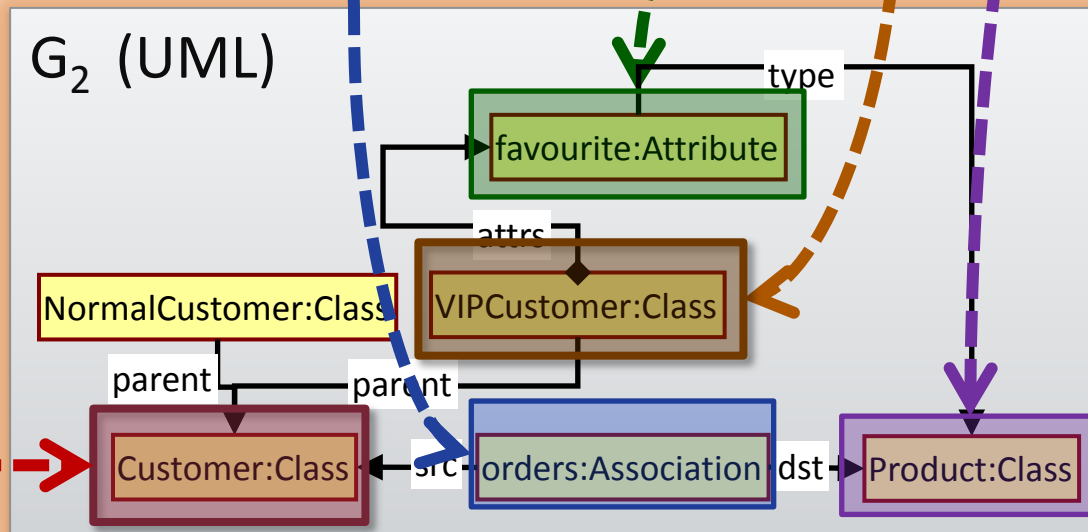
Sequential independence
(two following rule applications)

- Their order can be swapped without any effect on their final result

Sequential independence



G₂ (UML)

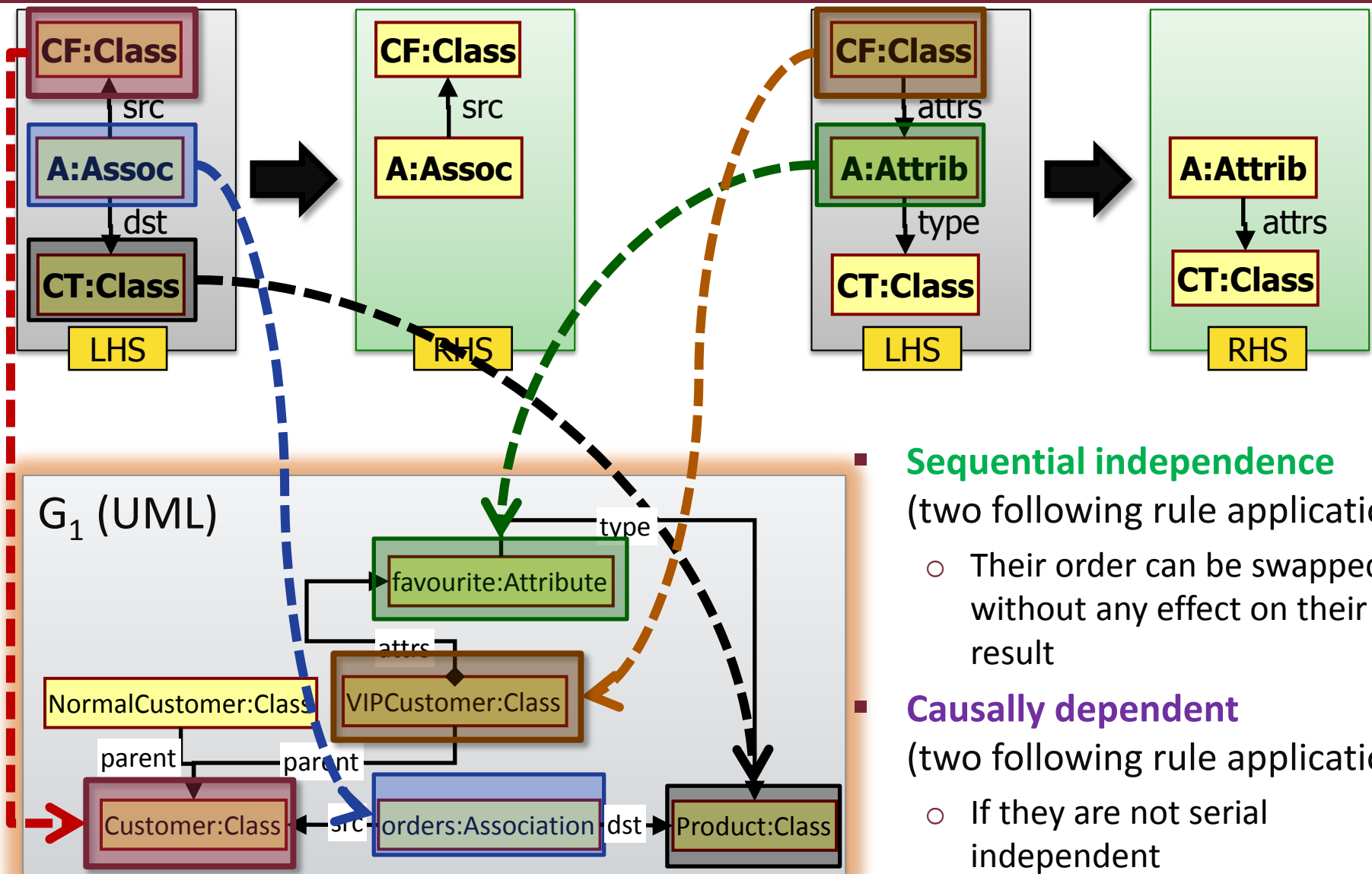


Sequential independence
(two following rule applications)

- Their order can be swapped without any effect on their final result

Example

Causal dependence I.



Sequential independence

(two following rule applications)

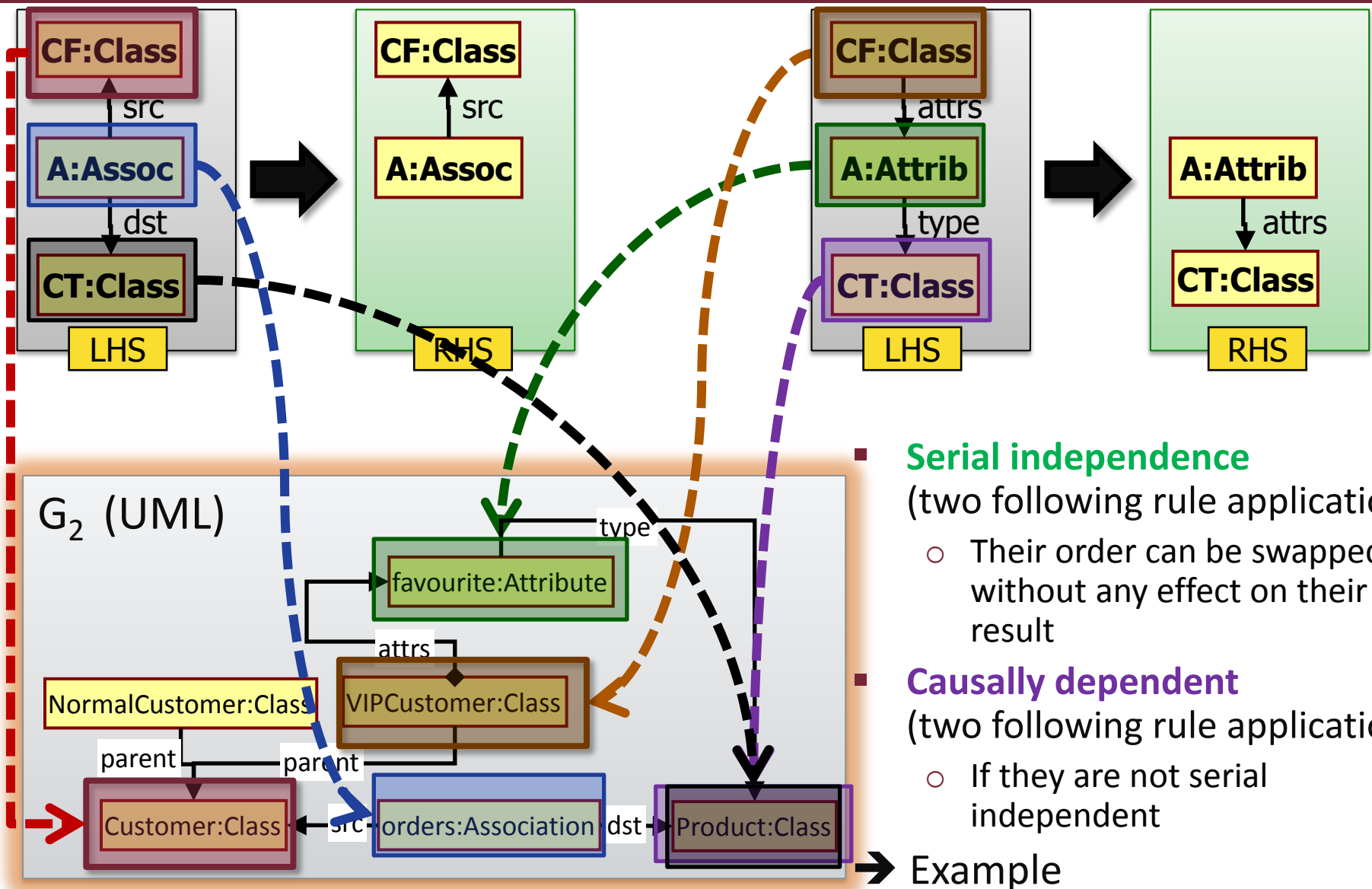
- Their order can be swapped without any effect on their final result

Causally dependent

(two following rule applications)

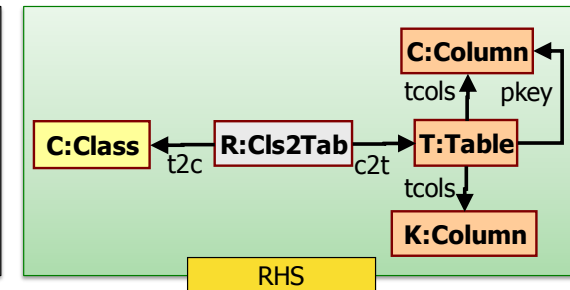
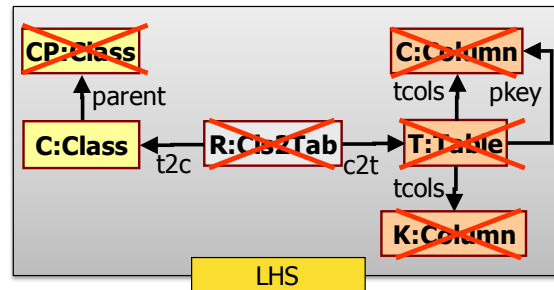
- If they are not serial independent

Causally dependence II.



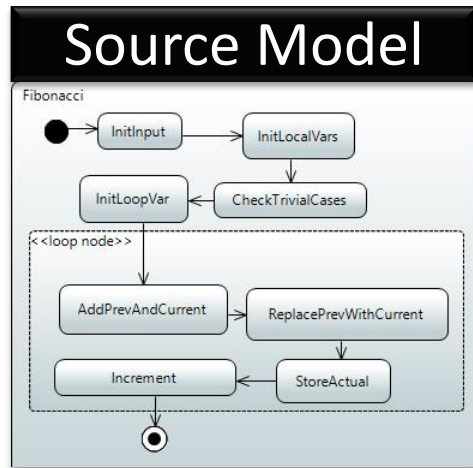
Summary

- **Graphtransformation,**
as a modeltransformation paradigm
 - Rule and pattern based formal specification
 - Querying and manipulating graph based models
 - Intuitive graph based specification
- **Structure**
 - LHS graph pattern: precondition
 - RHS graph pattern: postcondition
 - NAC: negative condition
- **Rule application**
 - Graph pattern matching
 - Deletion + Creation
 - Dangling edges and injectivity
 - Affect of multiple rule application (conflicts and causality)

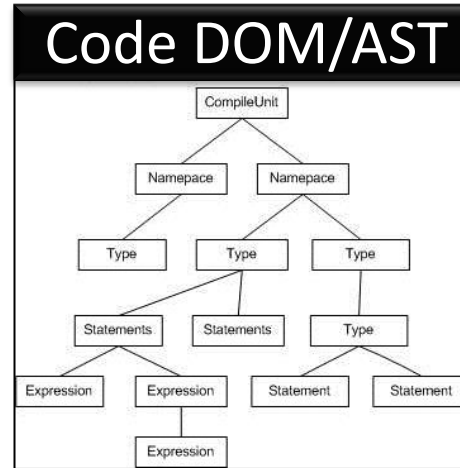


Chaining and Traceability of Model Transformations

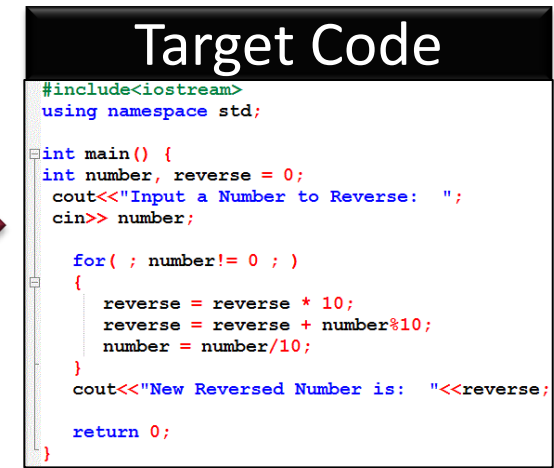
Code Generation by Model Transformations



M2M



M2T



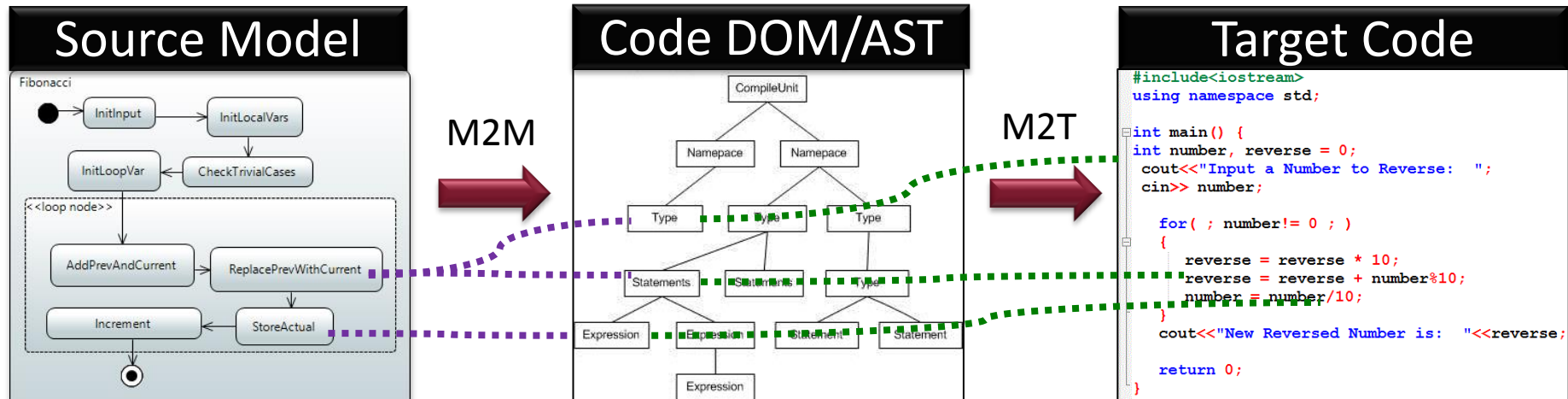
Model-to-Model (M2M) Transformation

- SRC: In-memory model (objects)
- TRG: In-memory model (objects)

Model-to-Text (M2T) Transformation

- SRC: In-memory model (objects)
- TRG: Textual code (string)

Traceability in Model Transformations



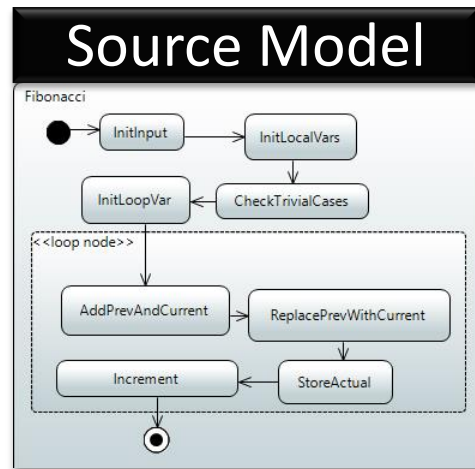
Traceability links:

- Additional links (edges)
- Connect SRC and TRG models

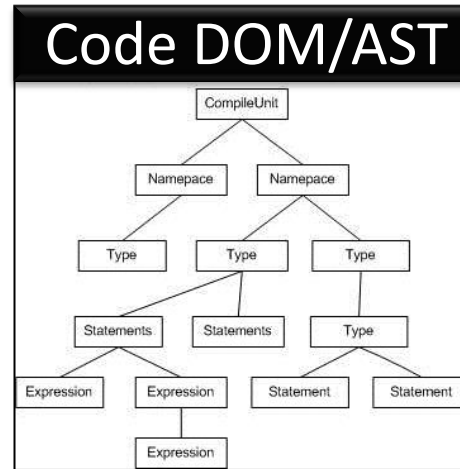
Objective:

- Support end-to-end traceability
- REQ \Leftrightarrow Model \Leftrightarrow Code

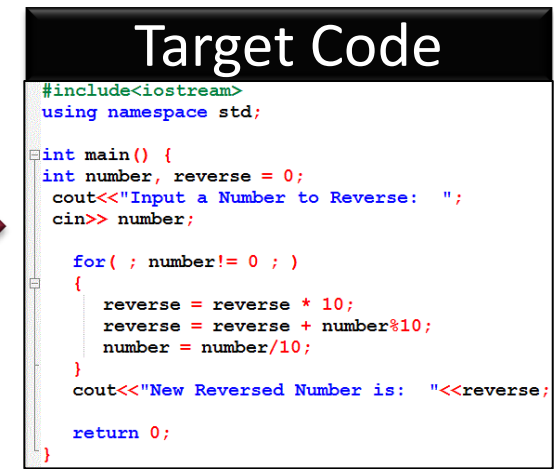
Chaining of Model Transformations



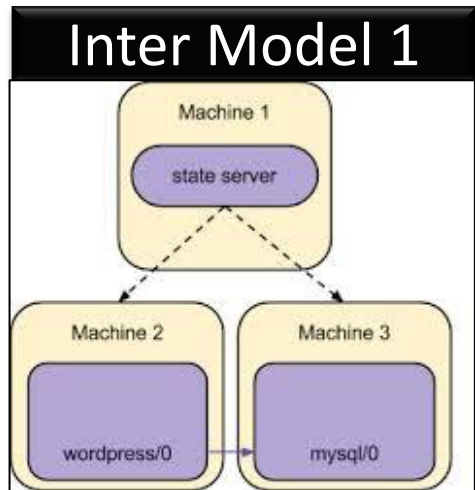
M2M



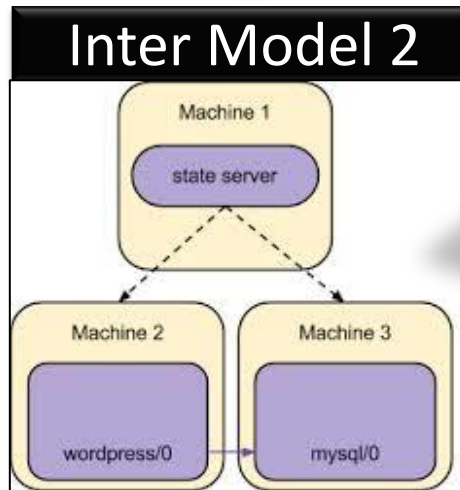
M2T



M2M



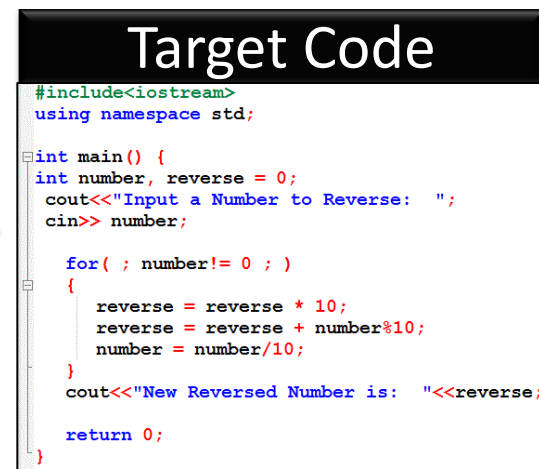
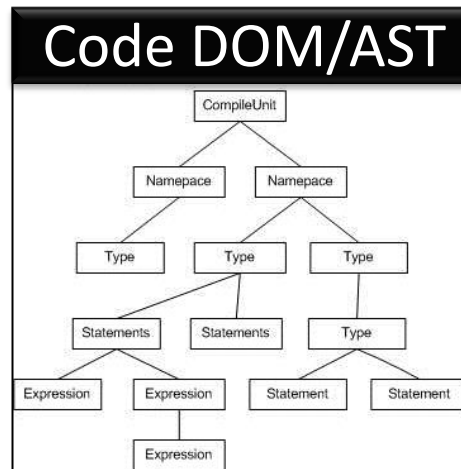
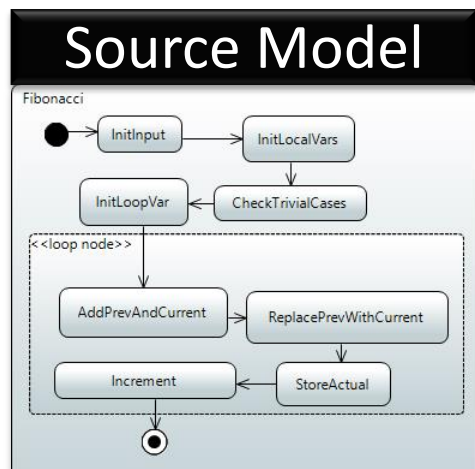
M2M



Goal:

- Reduce abstraction gap by „divide and conquer“
- Intermediate models
- Chain of model transformations

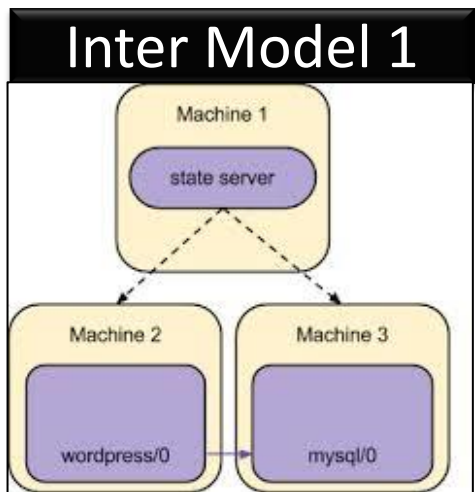
Model Transformation Flows / Chains



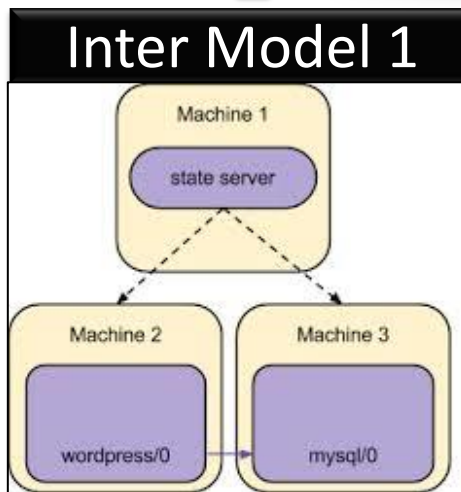
M2T



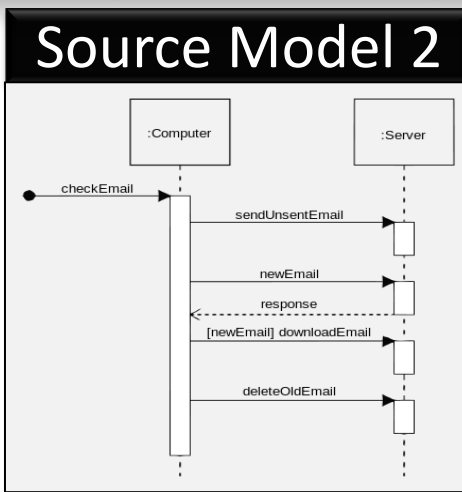
M2M



M2M



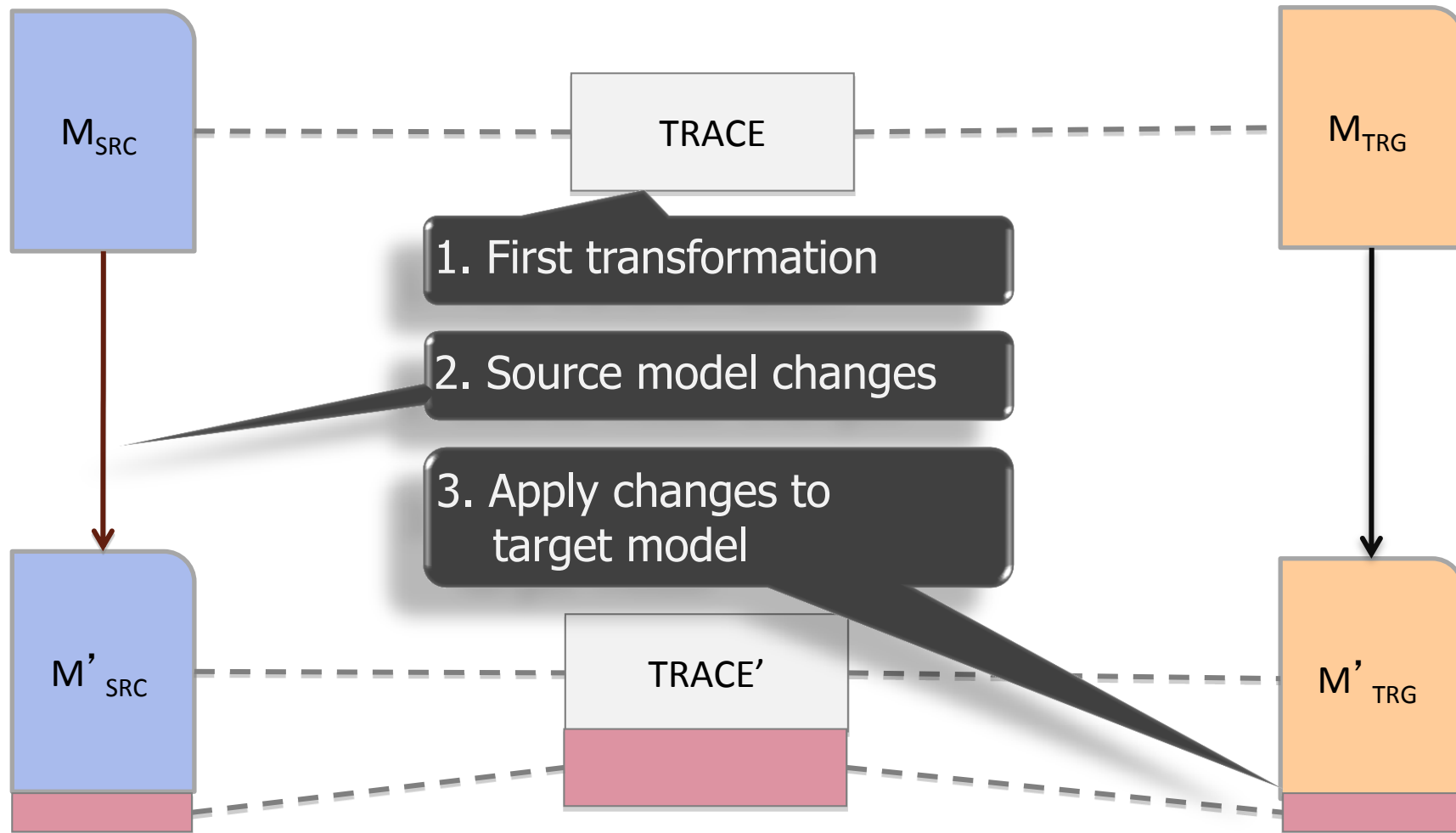
M2M



Joint optimization steps

Incrementality in model transformations

Incremental Forward Transformation



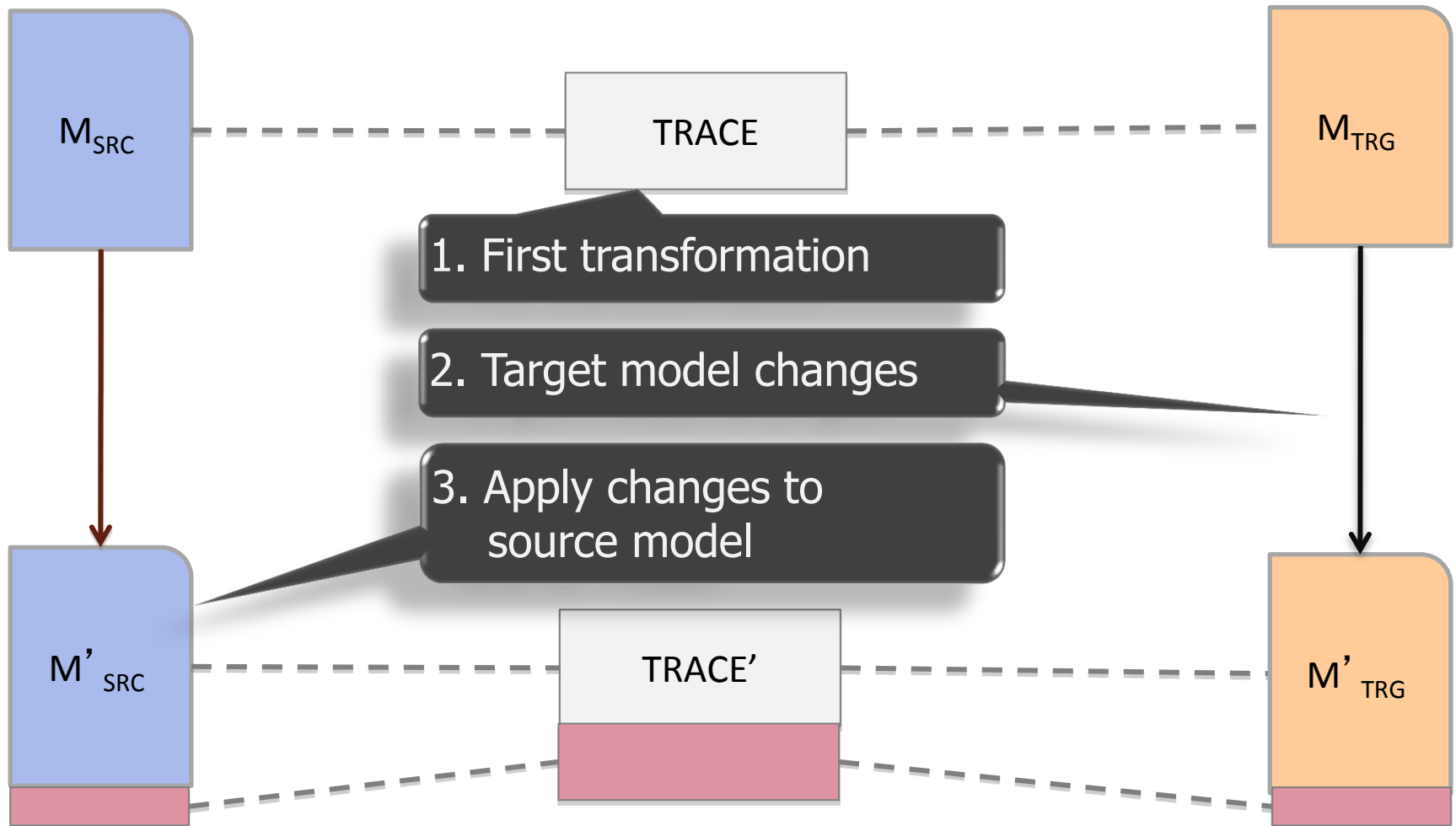
Practical application scenarios:

- Incremental model synchronization
- Tool integration

Solutions:

- Bidirectional transformations
- Change-driven transformations

Incremental Backward Transformation



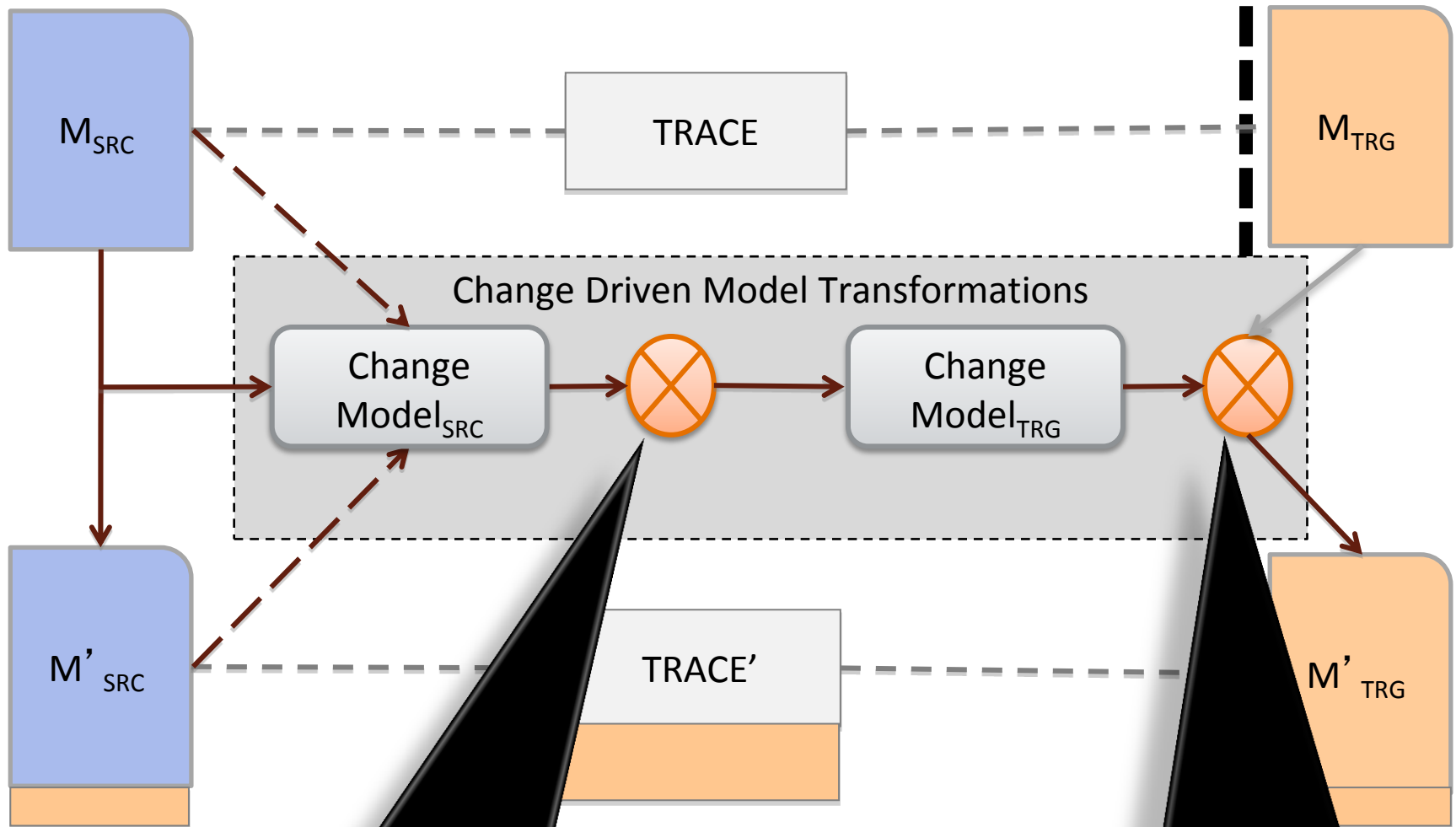
Challenge:

SRC \rightarrow TRG specified
TRG \rightarrow SRC inferred

Recent Approaches:

A. Schürr, P. Stevens, N. Foster, T. Hettel,
Cicchetti&Pierantonio, Czarnecki&Diskin

Change Driven Model Transformations



Change Driven Transformation

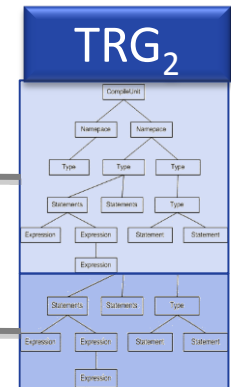
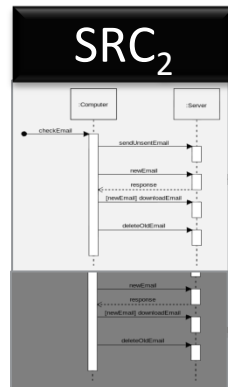
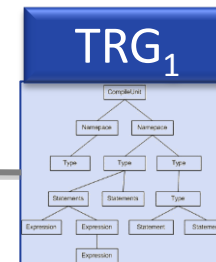
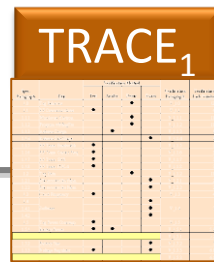
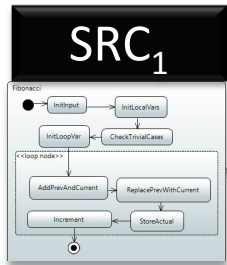
- Input: consumes change model
- Output: produces change model

Apply Target Change Model

- via an API with little trace info
- target model is not materialized!

Levels of Incrementality in Model Transformations

No Incrementality: Batch Transformations

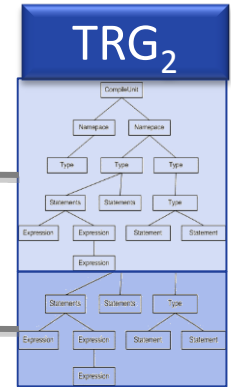
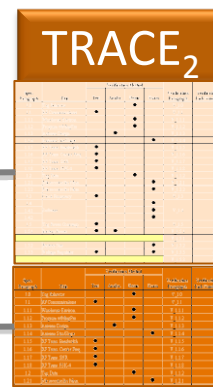
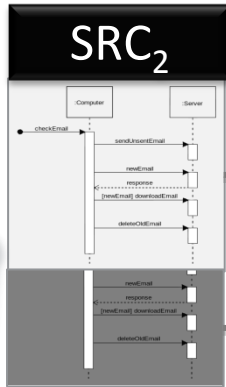
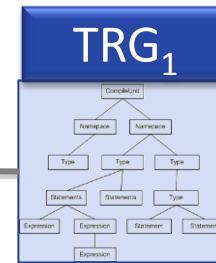
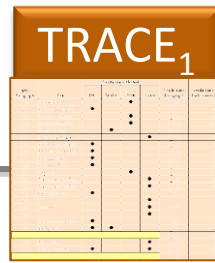
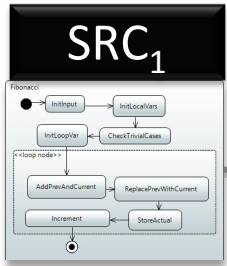


1. First transformation

2. Source model changes

3. Re-execute from scratch
for all source models

Dirty Incrementality



Pros:

- Large-step incrementality
- Avoids continuous exec.

Cons:

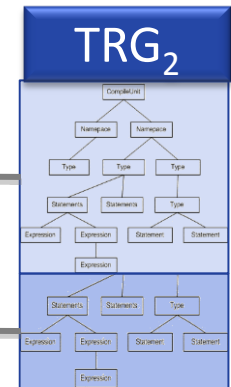
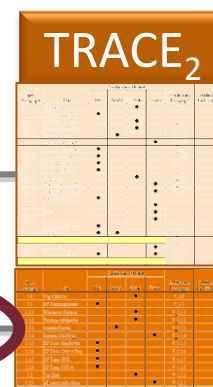
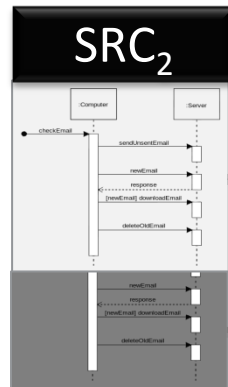
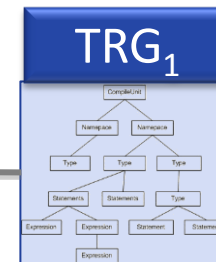
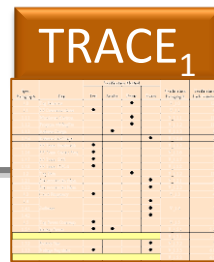
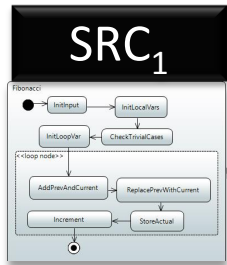
- Complex MT can be slow
- Cleanup (after an error)?
- Chaining?

1. First transformation

2. Source model changes

3. Re-execute from scratch only for changed models

Incrementality by Traceability



Pros:

- Small-step incrementality
- Better performance

Cons:

- Highly depends on traceability links
- Smart matcher needed

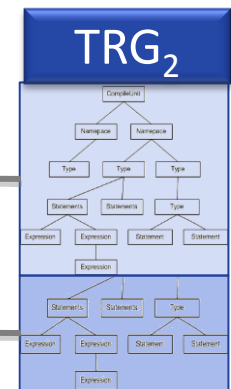
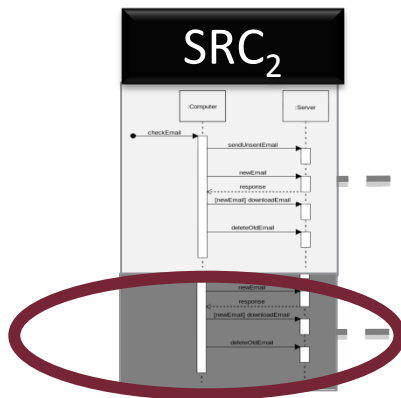
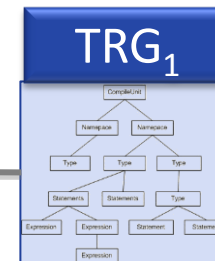
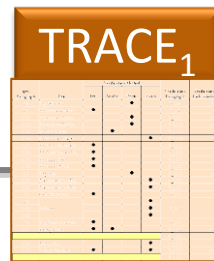
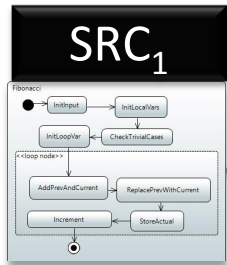
1. First transformation

2. Source model changes

3. Detect missing trace links

4. Re-execute MT only for untraceable elements

Event Driven Transformations



Pros:

- Refined context: driven by changes of query result set
- Chaining
- Avoids continuous comp.

Cons:

- Language-level restrictions

1. First transformation

2. Source model changes

3. Detect new activations

4. Fire rule activations
(in relevant context)



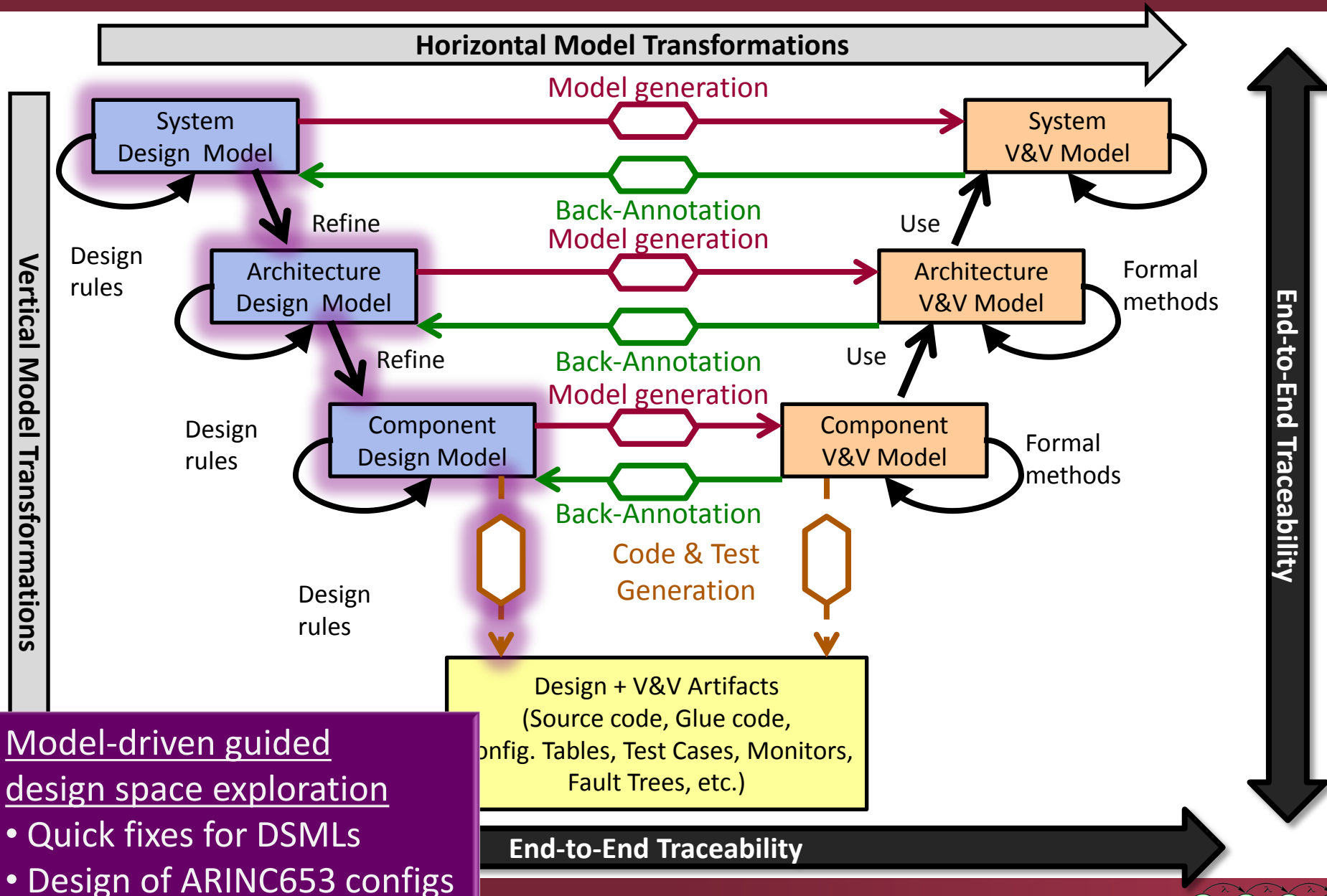
Design Space Exploration

Á. Hegedüs, Á. Horváth, D. Varró:

A model-driven framework for guided design space exploration.
Automated Software Engineering (August 2014)

DOI: 10.1007/s10515-014-0163-1

Model-Driven Guided Design Space Exploration



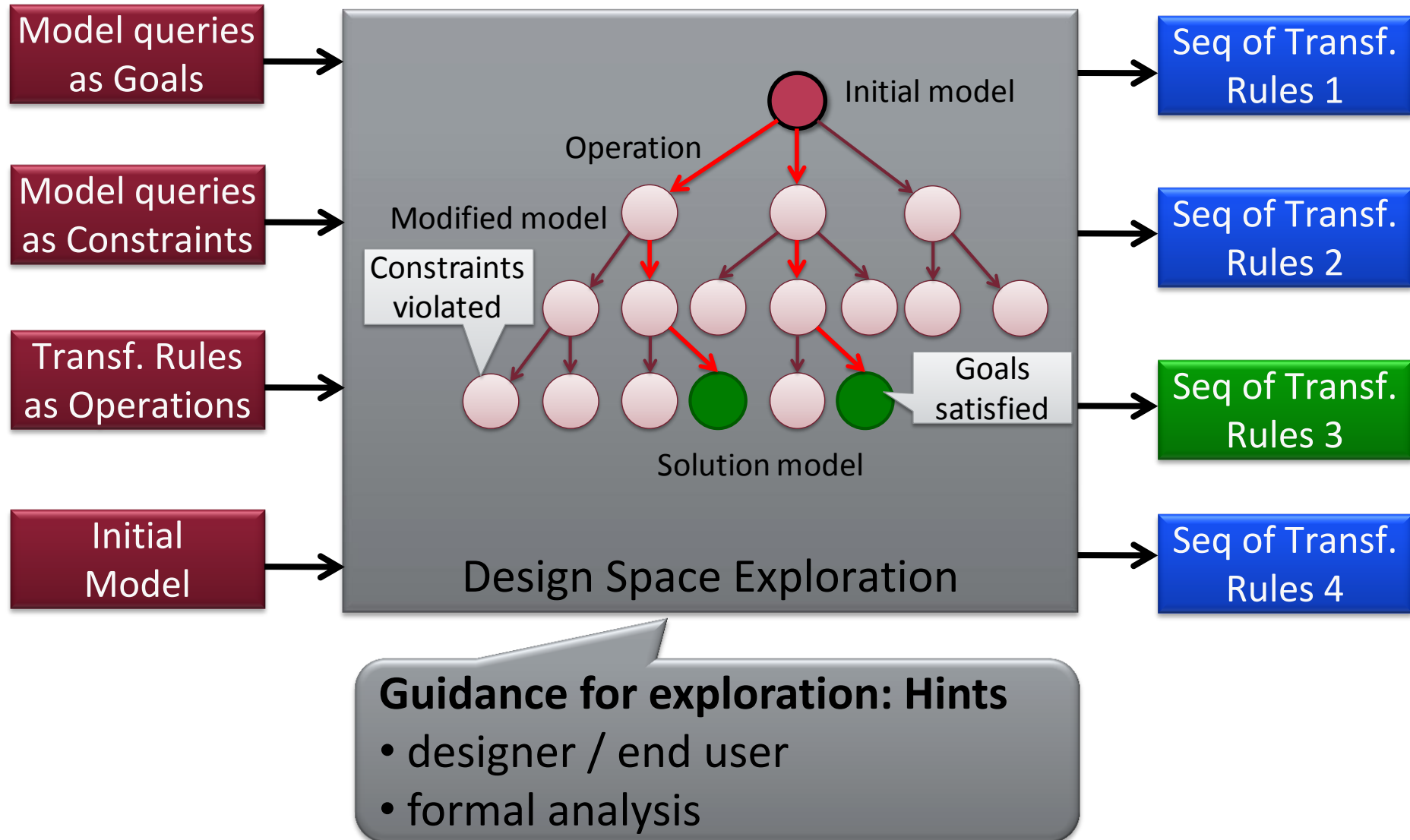
Design Space Exploration



Special state space exploration

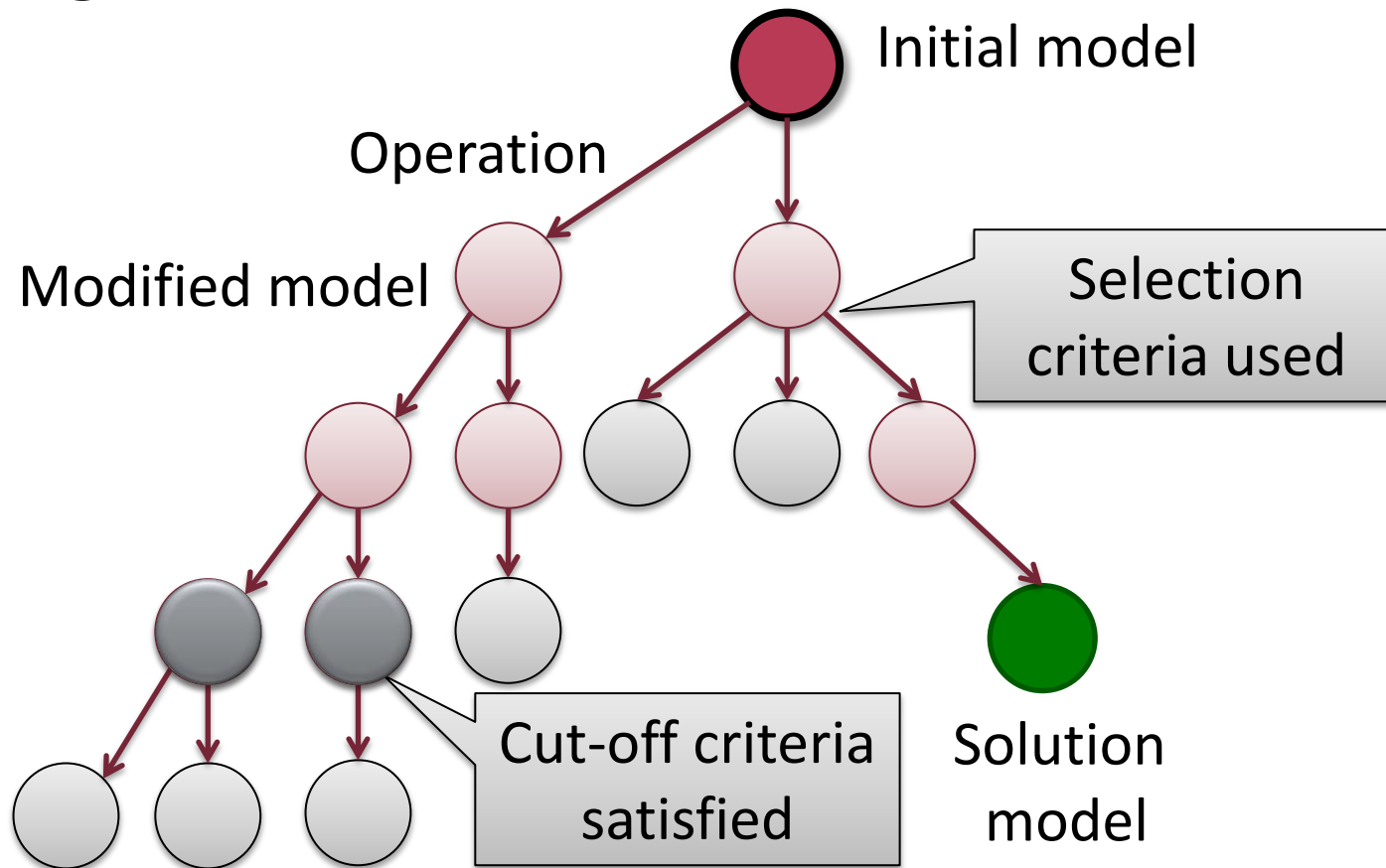
- potentially infinite state space
- „dense” solution space

Model Driven Guided Design Space Exploration



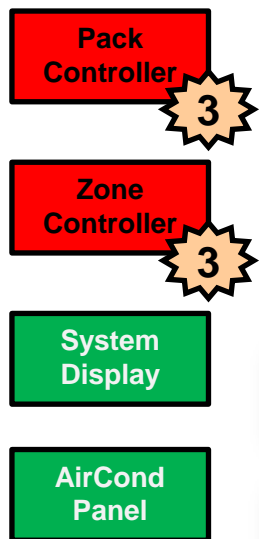
Guided Design Space Exploration

- High-level overview

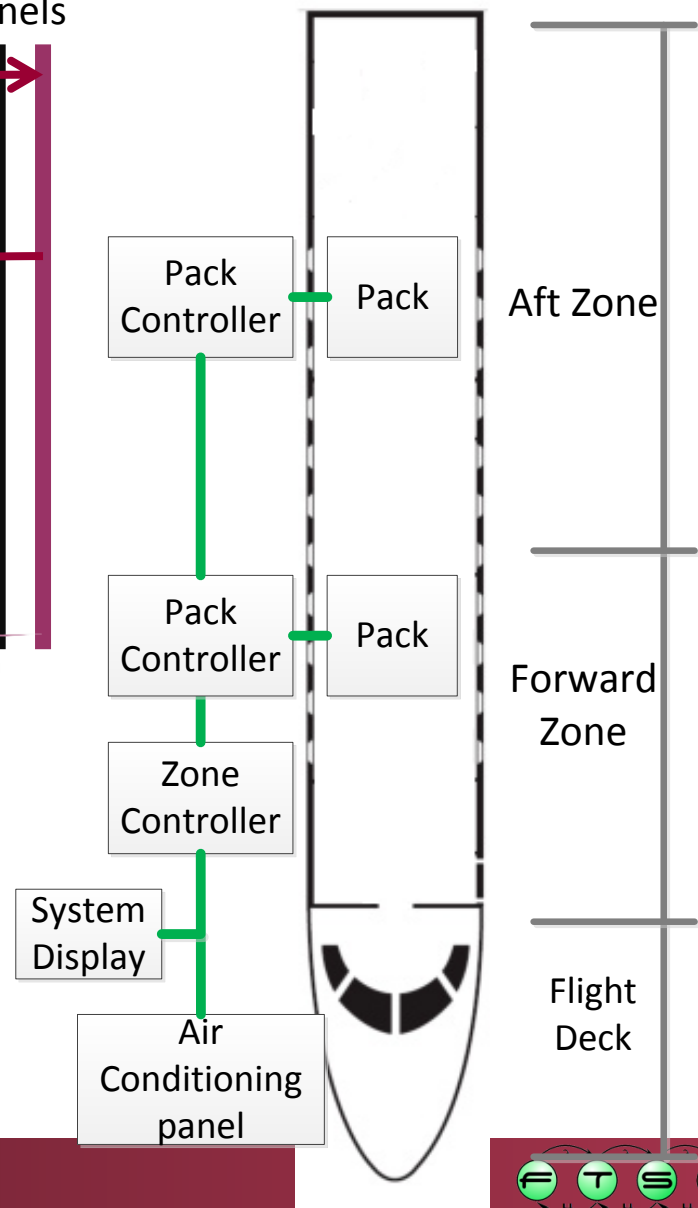
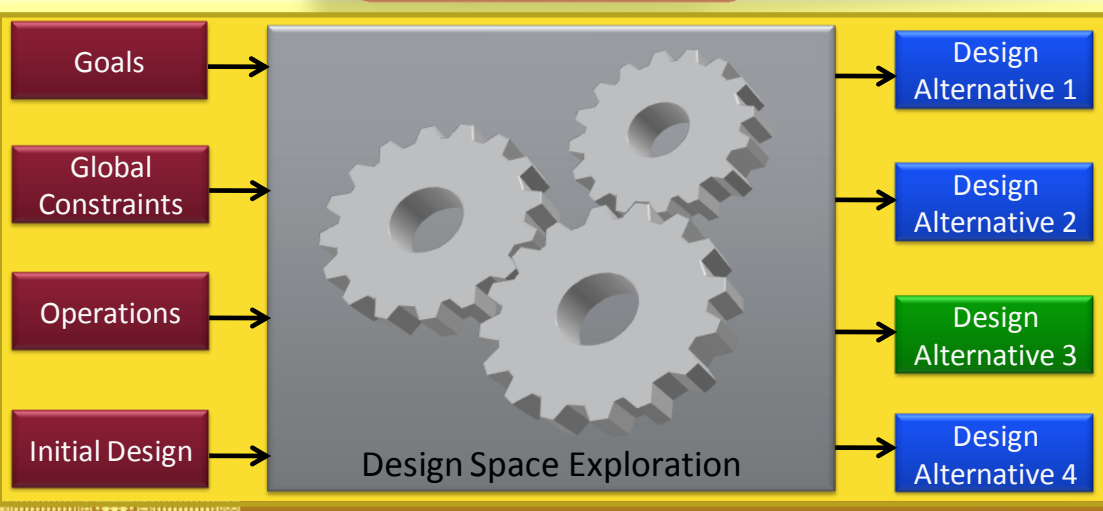
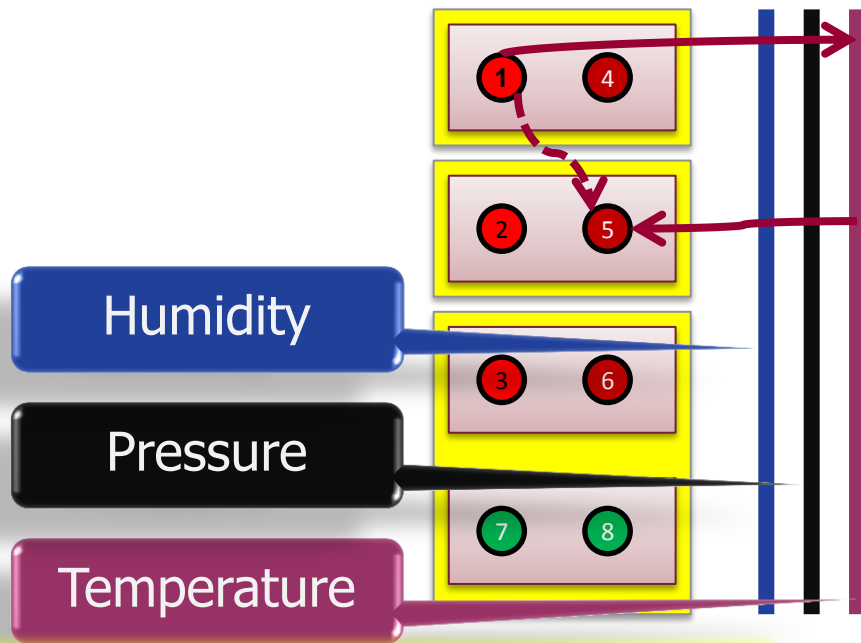


Design Space Exploration for IMA Config. Design

SW functionality

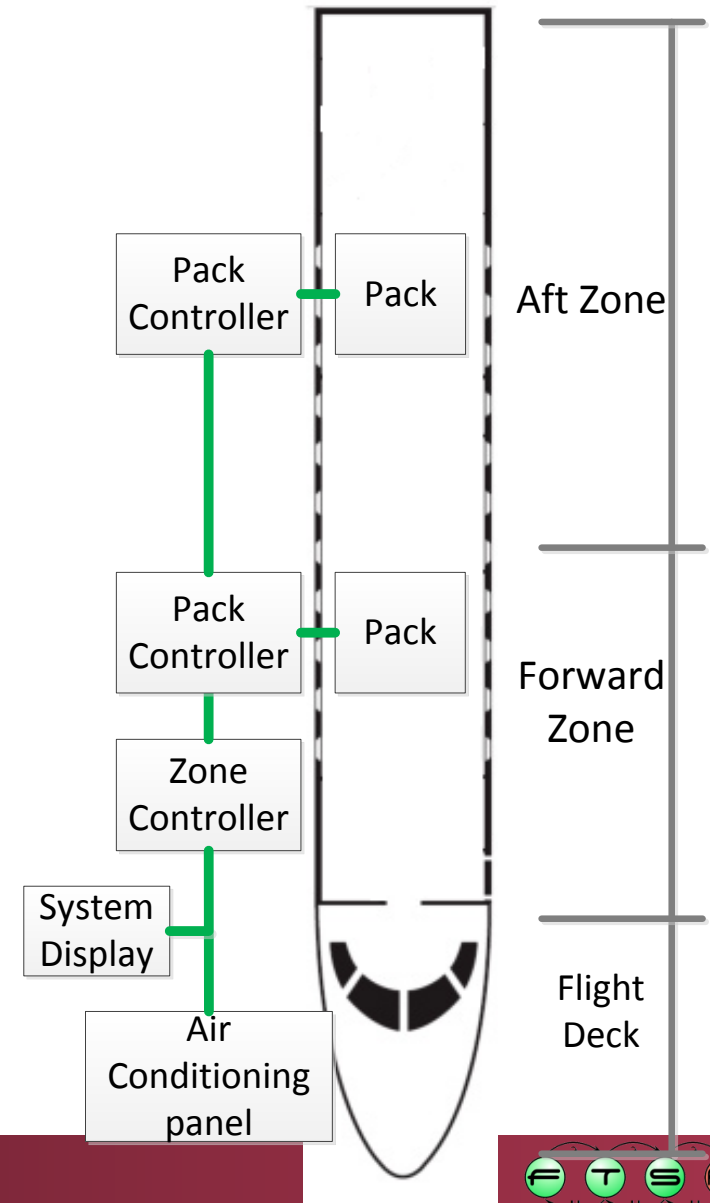
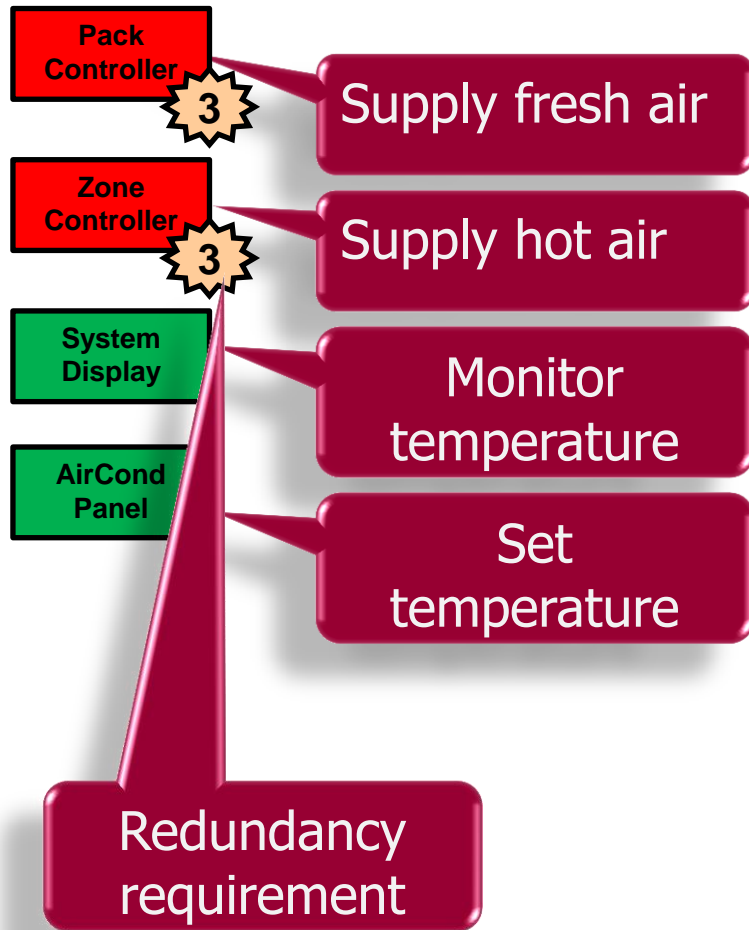


Communication channels



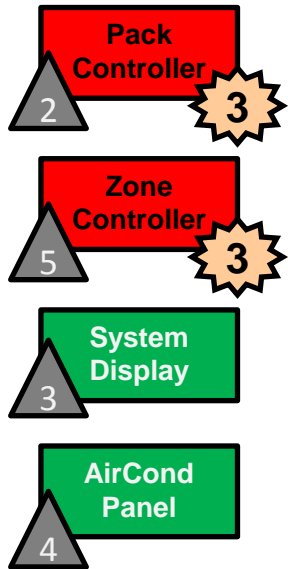
Designing ARINC653 configurations

SW functionality
(critical + non-critical)

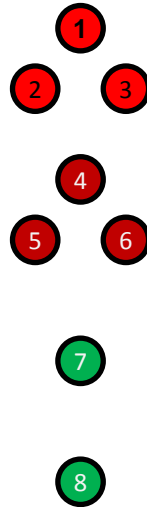


Job instances, Partitions, Modules

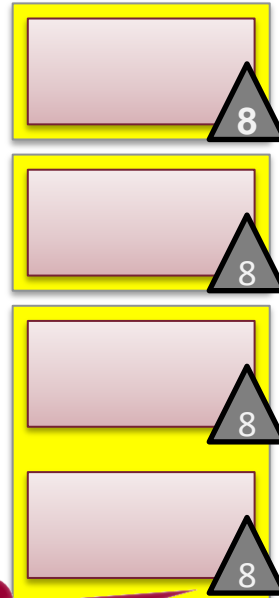
SW functionality
(critical + non-critical)



Job instances



Partitions

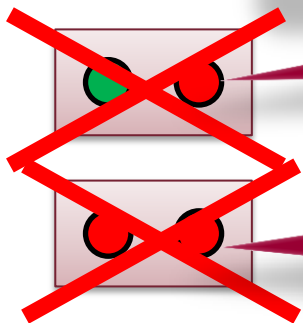


Modules

Additional constraints

- WCET,
- scheduling, etc.
- interfaces
- datatypes

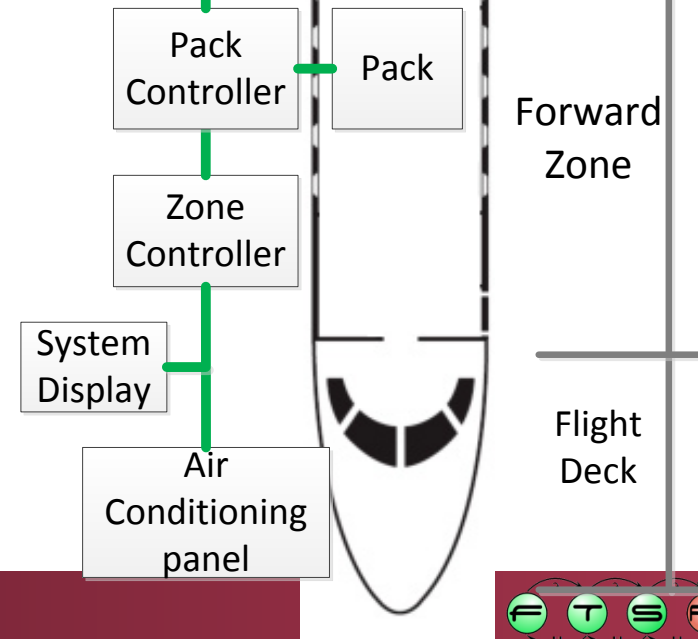
Constraints



Memory needs
+ constraints

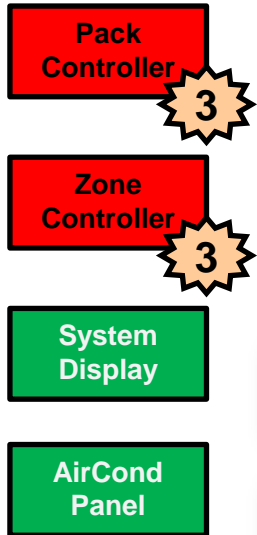
Do not mix critical
and non-crit. jobs

Do not mix
instances of the
same critical job

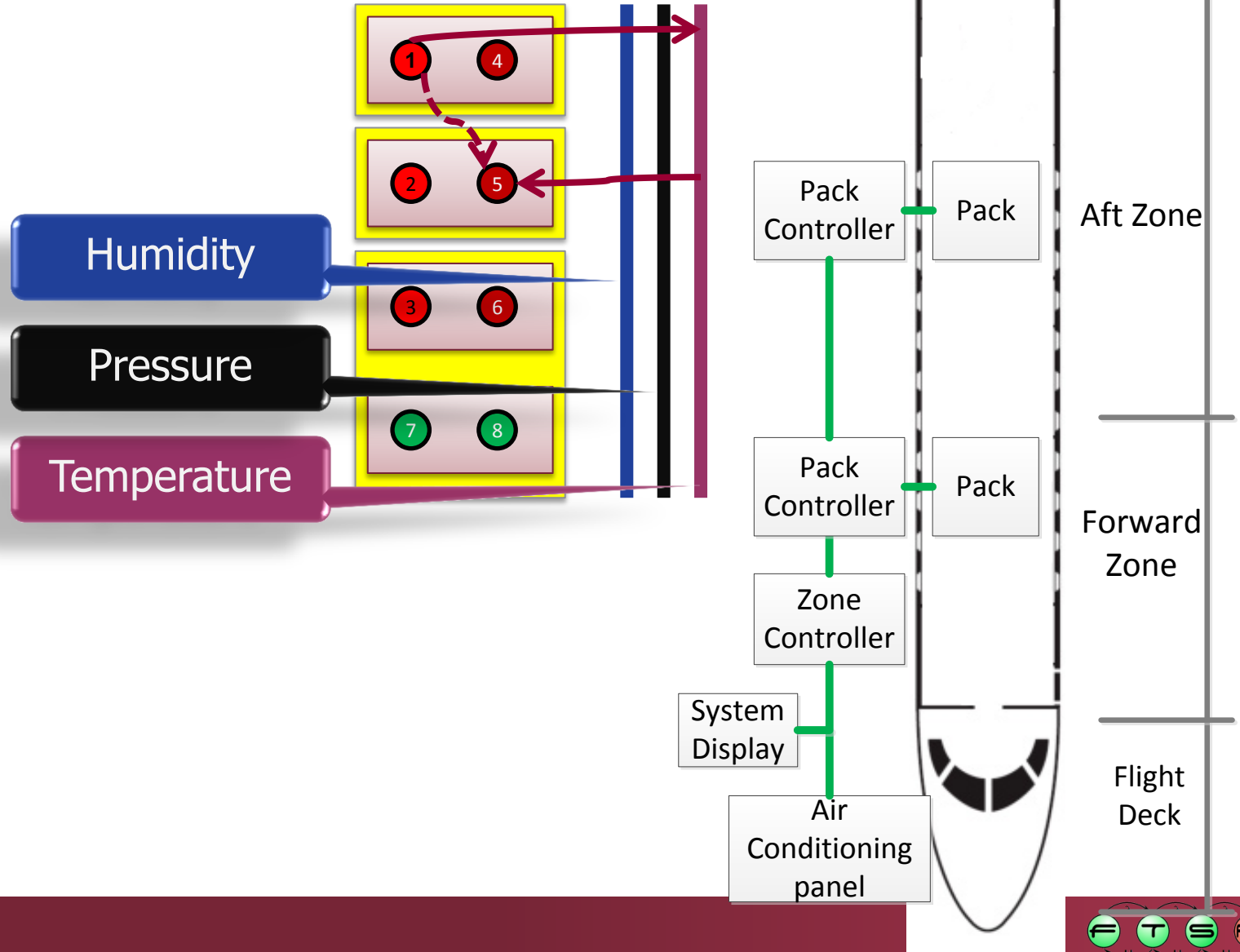


Allocating communication channels

SW functionality



Communication channels



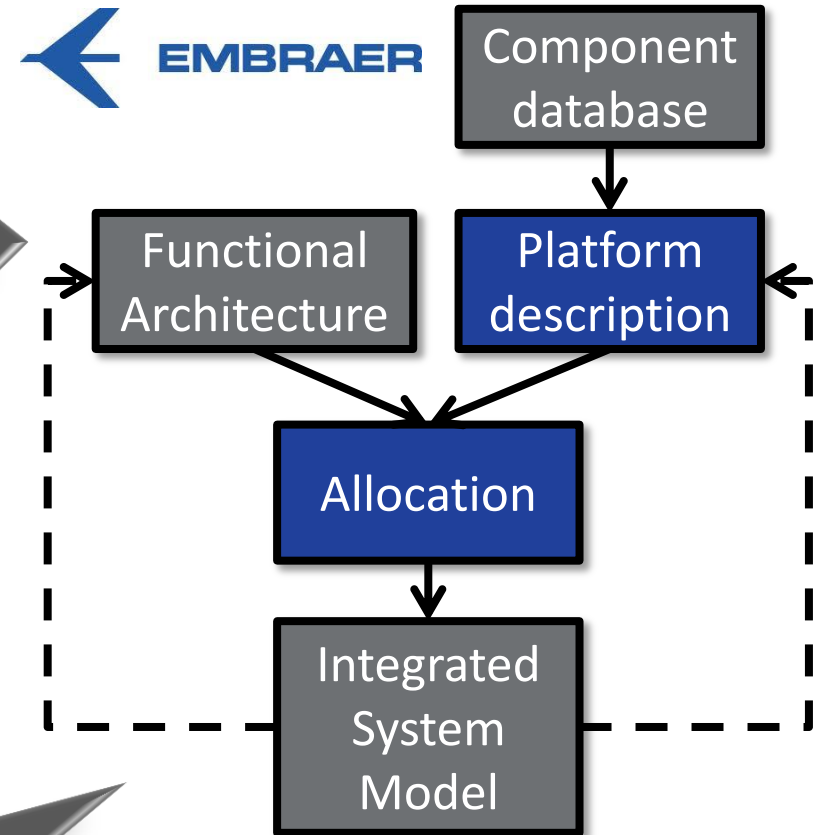
Model Driven Development of IMA Configs

Inputs:

- Platform Independent Model (PIM) (functional + nonfunc. reqs; Simulink)
- Platform Description Model (PDM) for ARINC 653 (DSML)

Output:

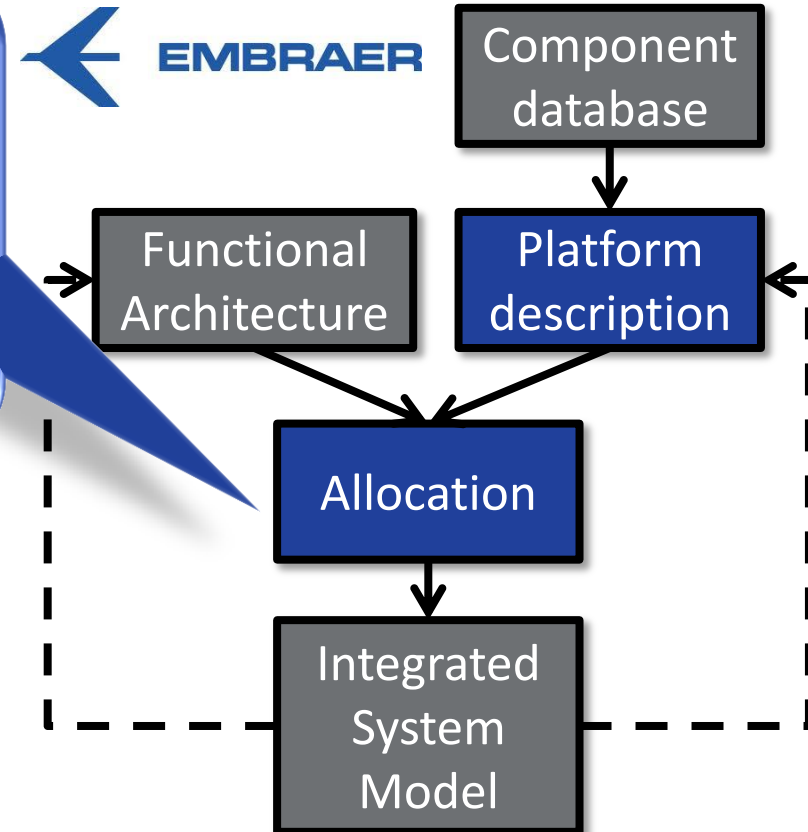
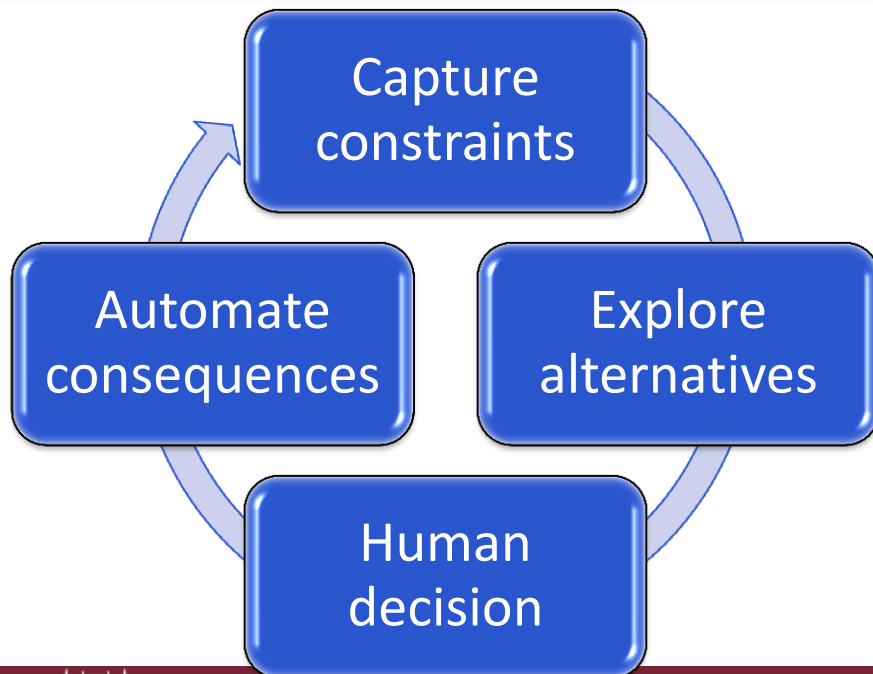
- Integrated system model
- Ready for simulation
- End-to-end traceability



Model Driven Development of IMA Configs

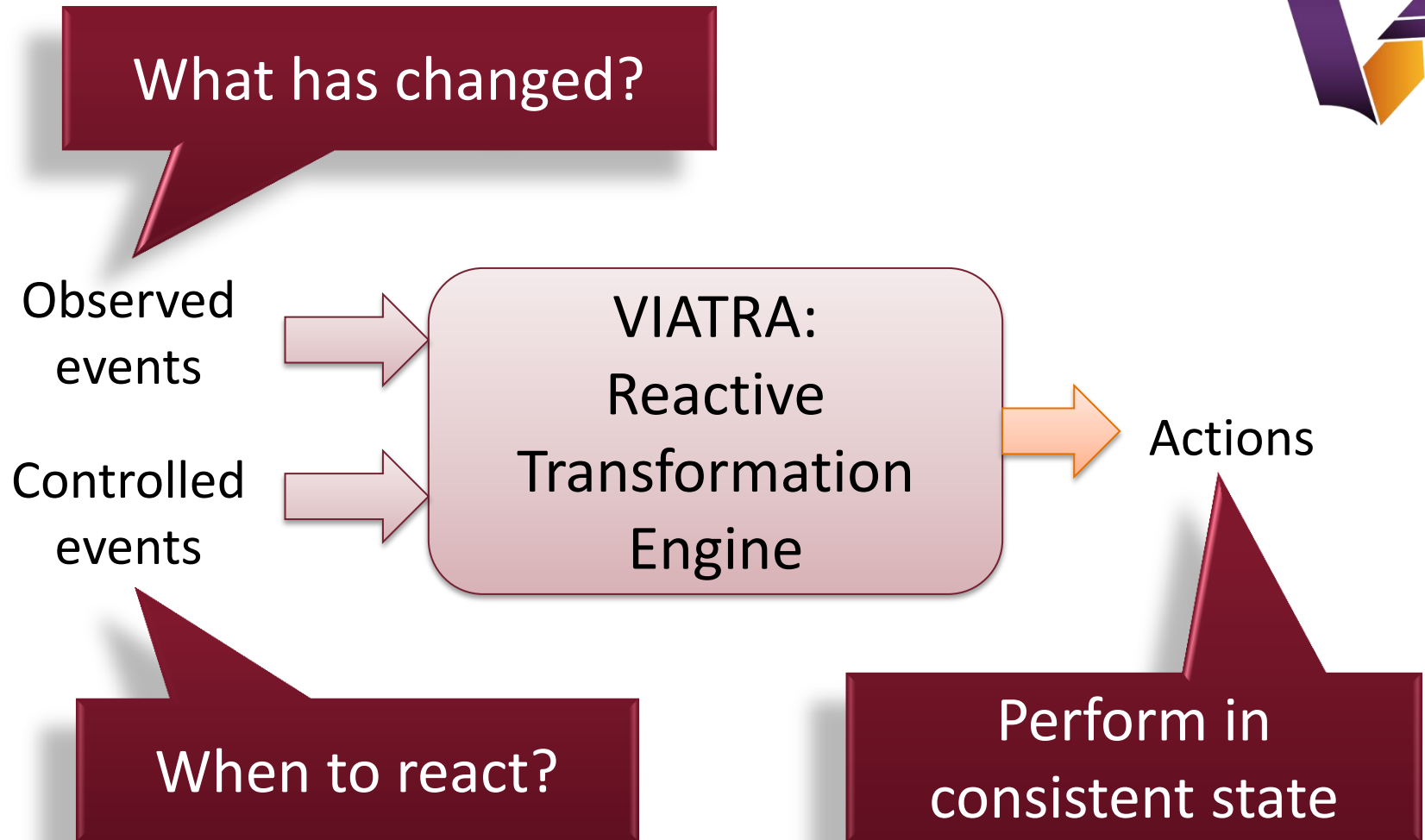
Model transformation chains:

- Designer-guided manual steps
- Automated steps
 - design space exploration
 - optimization
 - code generators
- Continuous validation of design rules

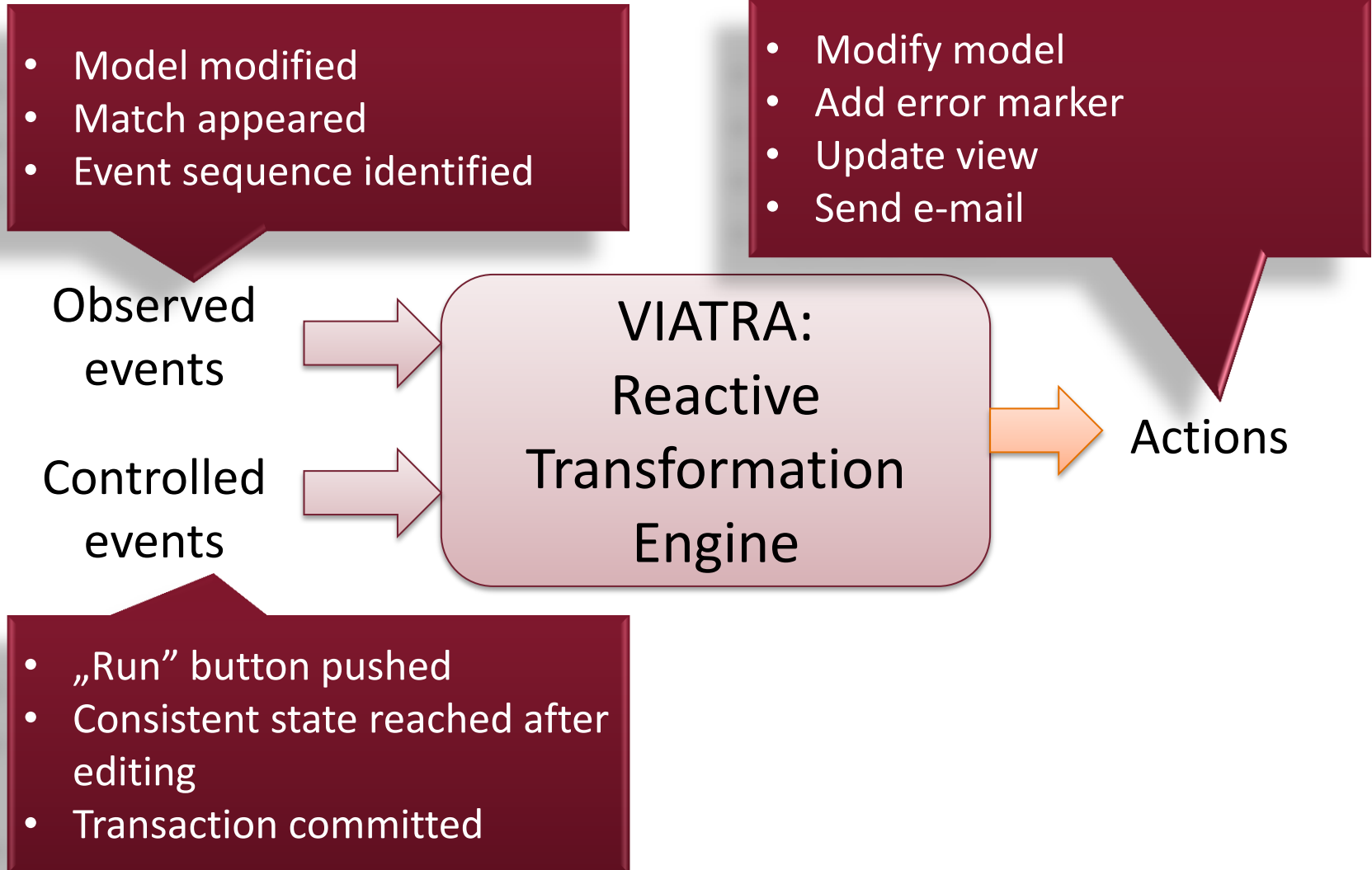


VIATRA: A Reactive Incremental Transformation Platform

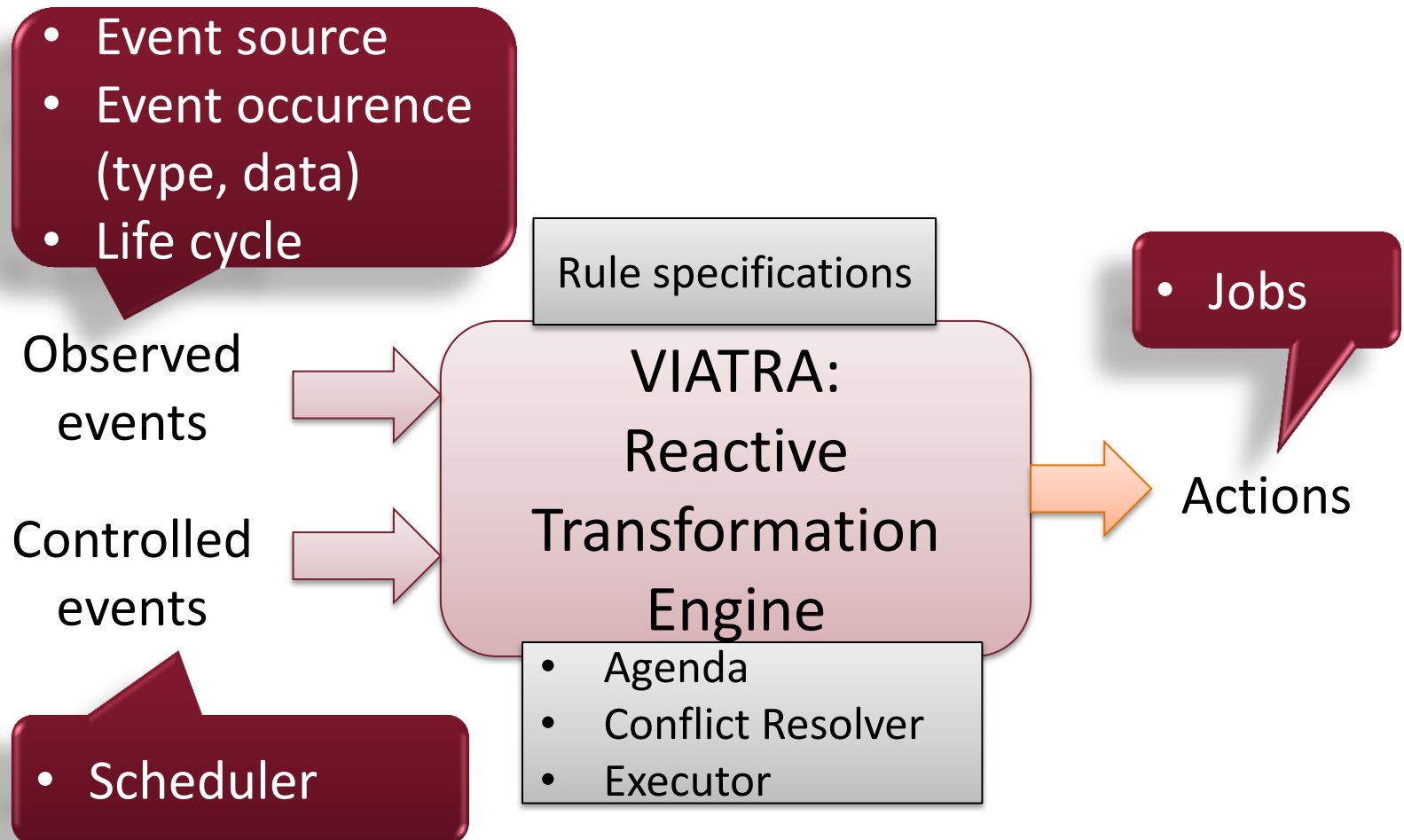
Reactive Event Driven Transformations



Reactive Event Driven Transformations



Reactive Event Driven Transformations



Language Example

pattern someCondition(param1, param2) {...} Query language

Xtend (Java)

val rule = createRule().precondition(**match** | // perform action).

action[**match** | // perform action].build

val incrRule = createRule().precondition(someCondition).

lifecycle(ActivationLifecycles.incremental).

action(::Appeared)[
 match | // perform action].

action(::Disappeared)[
 match | // perform action].

build

Event data

- Event source
- Event occurrence (type, data)
- Life cycle

Observed events

Controlled events

• Scheduler

Rule specifications

Reactive Transformation Framework

- Agenda
- Executor

• Conflict resolver

• Jobs

Actions

Language Example

pattern `someCondition(param1, param`

Rule specification

Xtend (Java)

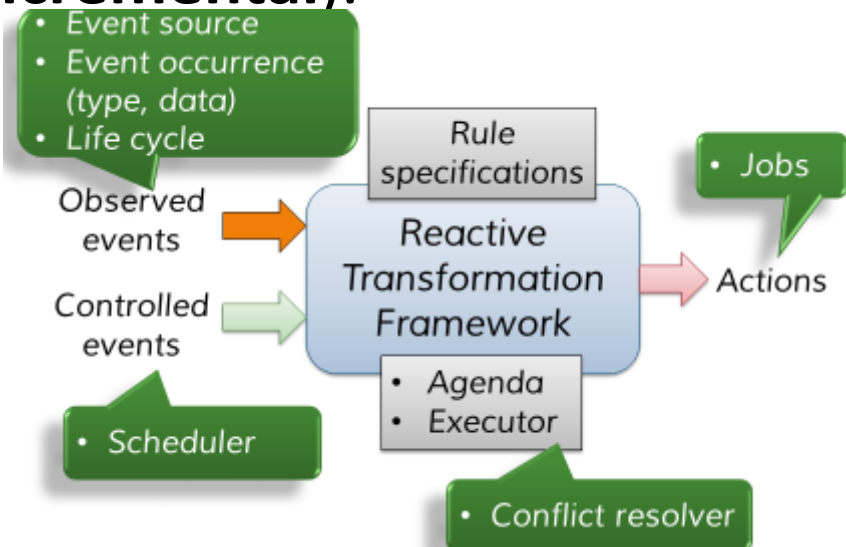
```
val rule = createRule().precondition(someCondition).  
  action[ match | // perform action ].build
```

```
val incrRule = createRule().precondition(someCondition).  
  lifecycle(ActivationLifecycles.incremental).
```

```
  action(::Appeared)[  
    match | // perform action].
```

```
  action(::Disappeared)[  
    match | // perform action].
```

```
  build
```



Language Example

pattern **someCondition**(**param1**, **param2**) {...} Query language

Xtend (Java)

```
val rule = createRule().precondition(someCondition).
```

```
  action[ match | // perform action ].build
```

```
val incrRule = createRule().precondition
```

Observed events

```
  lifecycle(ActivationLifecycles.incremental).
```

```
  action(::Appeared)[
```

```
    match | // perform action].
```

```
  action(::Disappeared)[
```

```
    match | // perform action].
```

```
  build
```

- Event source
- Event occurrence (type, data)
- Life cycle

Observed events

Controlled events

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Reactive Transformation Framework

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Actions

Language Example

pattern someCondition(param1, param2) {...} Query language

Xtend (Java)

val rule = createRule().precondition(someCondition).

action[match | // perform action].build

val incrRule = createRule().precondition(someCondition).

lifecycle(IncrementalLifecycles.incremental).

action Job specification

match | // perform action].

action(::Disappeared)[

match | // perform action].

build

- Event source
- Event occurrence (type, data)
- Life cycle

Observed events

Controlled events

• Scheduler

Rule specifications

Reactive Transformation Framework

- Agenda
- Executor

• Conflict resolver

• Jobs

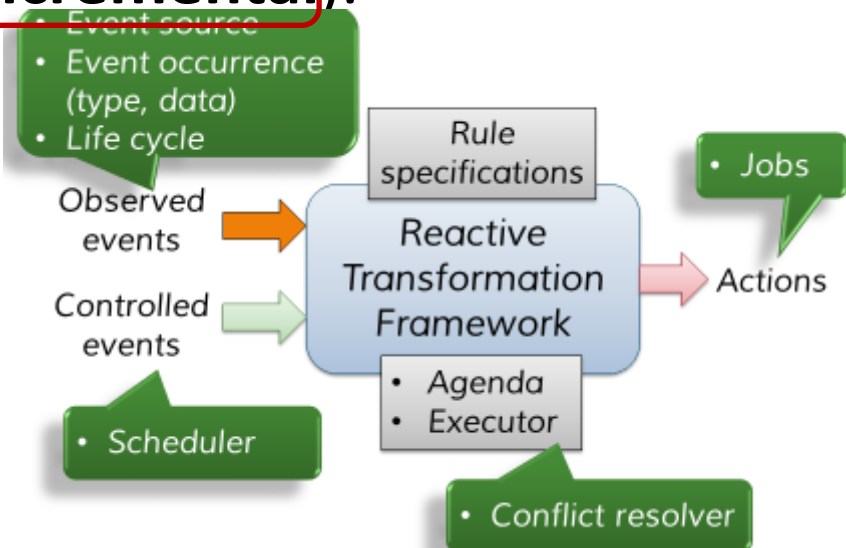
Actions

Language Example

```
pattern someCondition( param1, param2 ) { ... }
```

```
val rule = createRule().precondition(someCondition).  
    action[ match | // perform action ].build  
val incrRule = createRule().precondition(someCondition).  
    lifecycle(ActivationLifecycles.incremental).  
    action(::Appeared)[  
        match | // perform action].  
    action(::Disappeared)[  
        match | // perform action].  
    build
```

Activation
state-event
transitions



Language Example

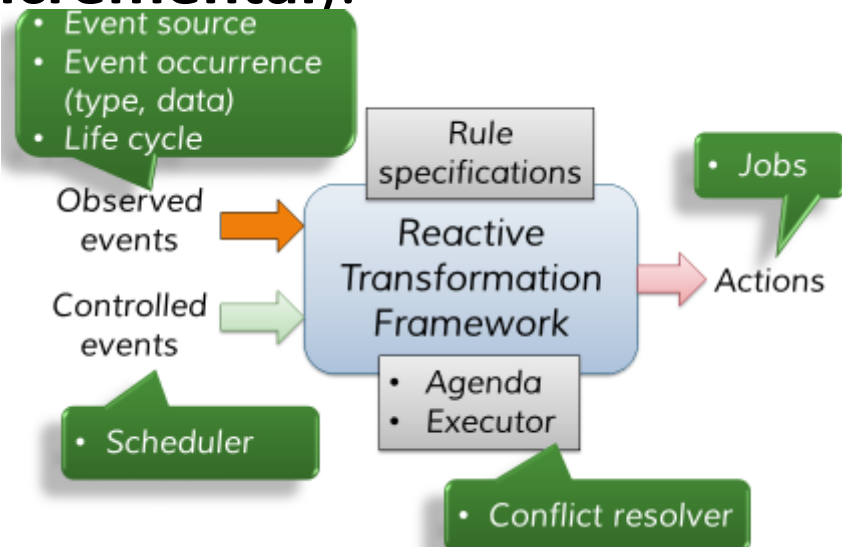
pattern **someCondition**(**param1**, **param2**) {...} Query language

Xtend (Java)

Jobs associated with event types
`Rule().precondition(someCondition).
// perform action].build`

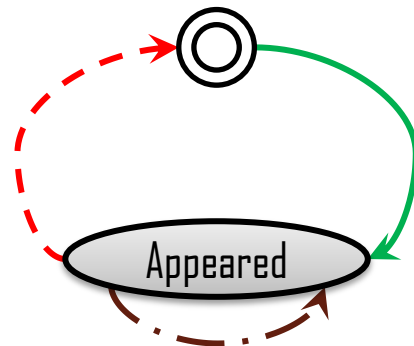
`var Rule = createRule().precondition(someCondition).
lifecycle(ActivationLifecycles.incremental).`

`action::Appeared[
 match | // perform action].
action::Disappeared[
 match | // perform action].
build`

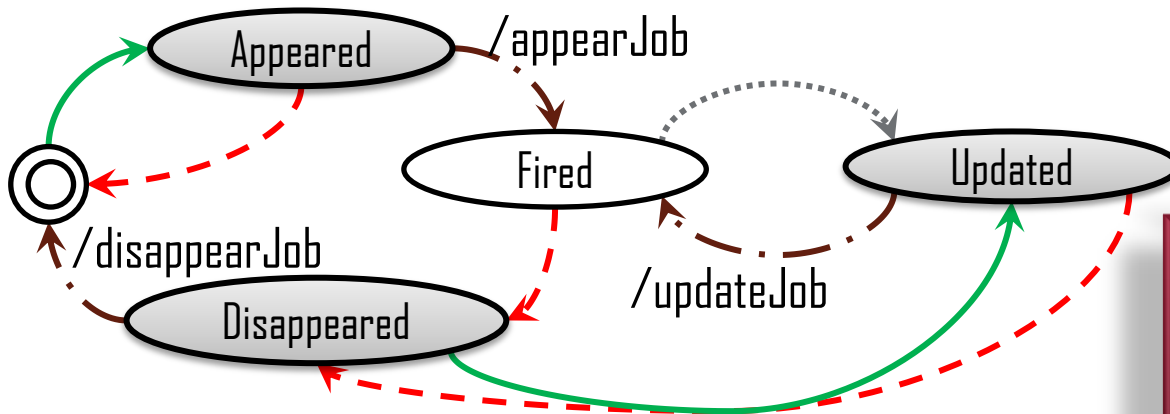


Activation Lifecycles

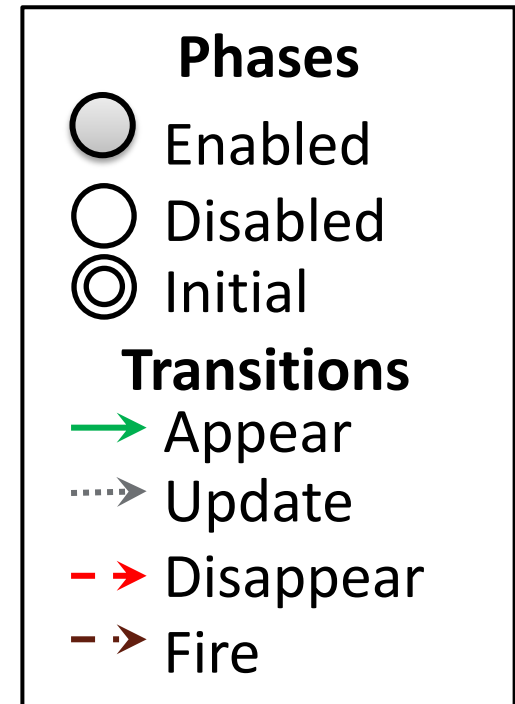
- Batch transformation



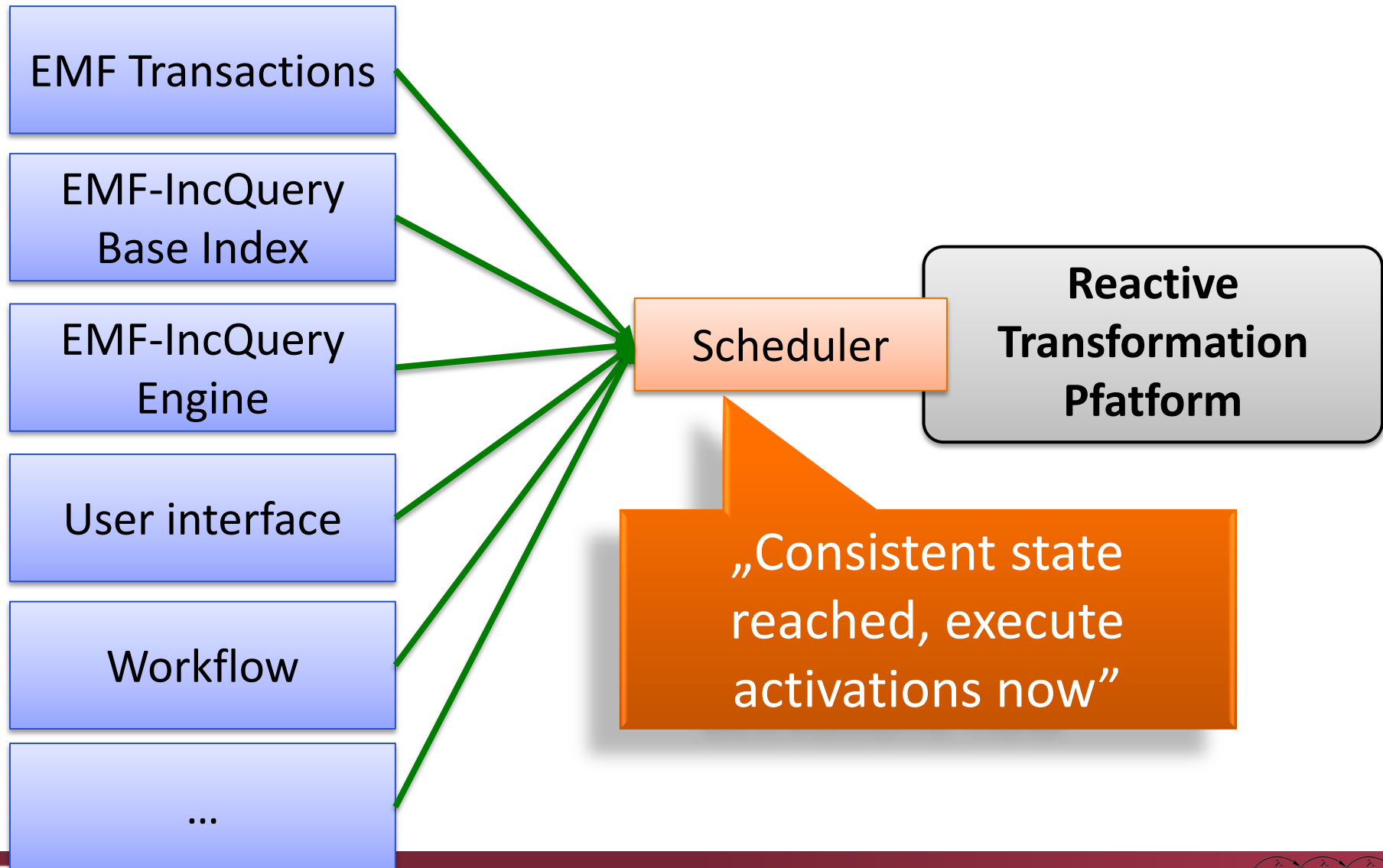
- Event-driven transformation



Only feature of
event data object
has changed



Scheduling



Conflict Resolution

- Multiple actions available
 - Different activations in the same rule
 - Activations of different rules
- Which activation to execute next?
- Conflict resolver can be selected
 - Global conflict set: deals with all rules
 - Scoped conflict set: selected rules

VIATRA: Overview of Features

■ Reactive MT Platform

○ MT Language:

- Internal DSL over Xtend
- Transformation API

○ MT Engine:

- Event-driven virtual machine
- Batch + Incremental MTs
- Control flow library
- Compiles to Java
- Debugger
- High performance

○ Integrations:

- EMF, IncQuery, Xtend, EMF-UML, ...

Design Space Exploration

- Explore design model candidates
- Satisfying multiple criteria
- Rule based exploration
- Optimization

Complex Event Processing

- Detect complex event sequences
- Rule based reaction
- Xtext based language

Model Obfuscator

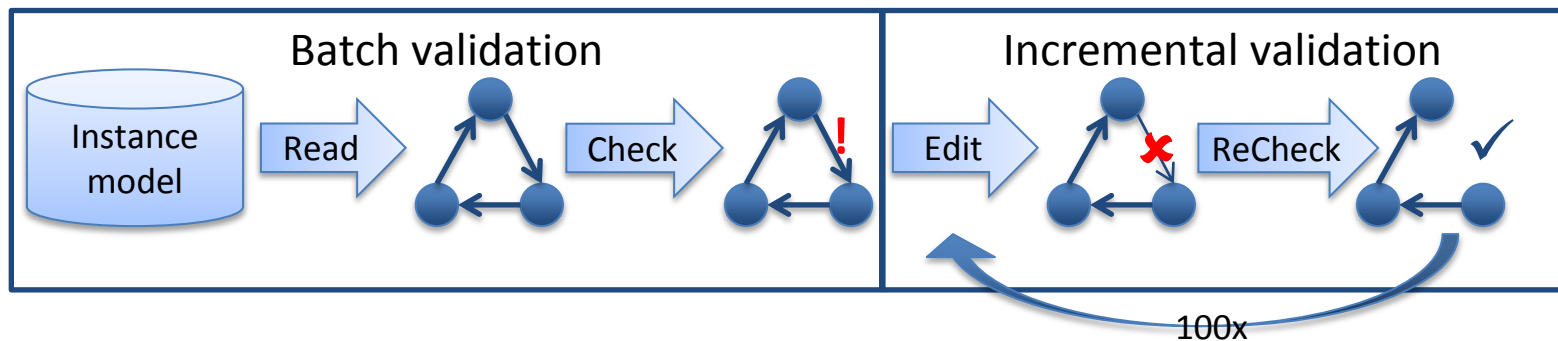
- Remove sensitive information from confidential models
- Original model → Obfuscated model

Cross-technology benchmark for model validation

The Train Benchmark Case for Incremental Model Validation
(Transformation Tool Contest 2015)

The Train Benchmark

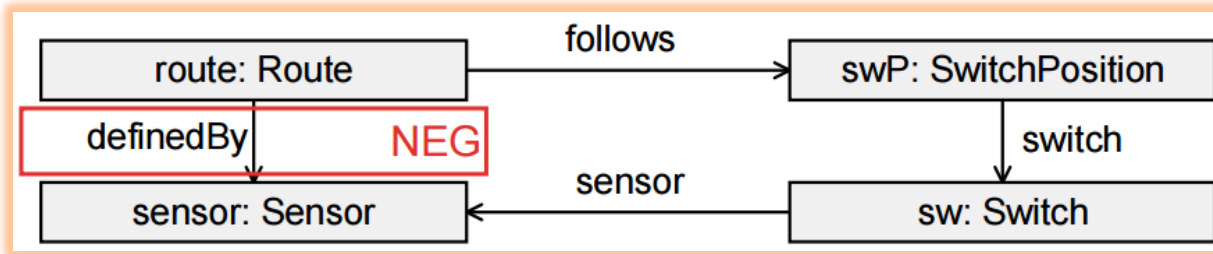
- Model validation workload:
 - User edits the model
 - Instant validation of well-formedness constraints
 - Model is repaired accordingly
- Scenario:
 - Load + Check
 - Edit + Re-Check
- Models:
 - Randomly generated
 - Close to real world instances
 - Following different metrics
 - Customized distributions
 - Low number of violations
- Queries:
 - Two simple queries (<2 objects, attributes)
 - Two medium queries (4-7 joins, negation)
 - Two complex queries (7+ joins, transitive closure)
 - Validated match sets
- Transformations:
 - Modifies the model
 - Repair: remove existing violations
 - Inject: create new violations (matches)



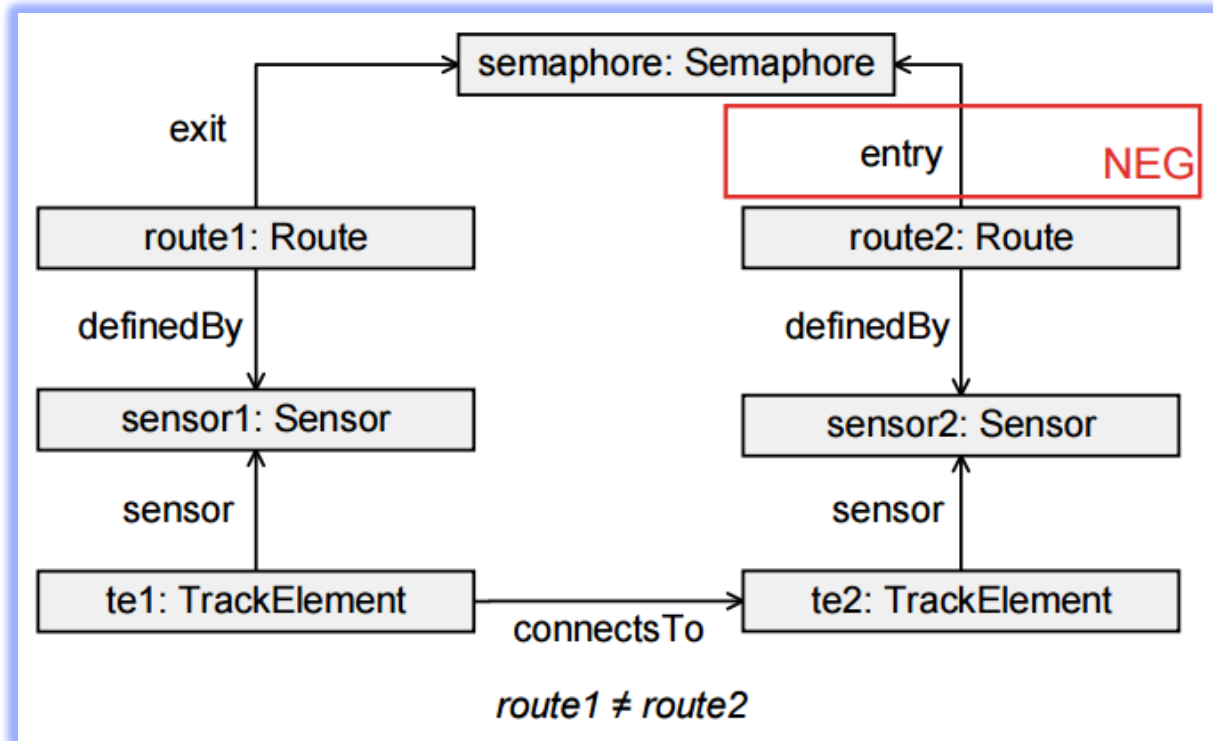
What Tools are Compared?



Selected Queries

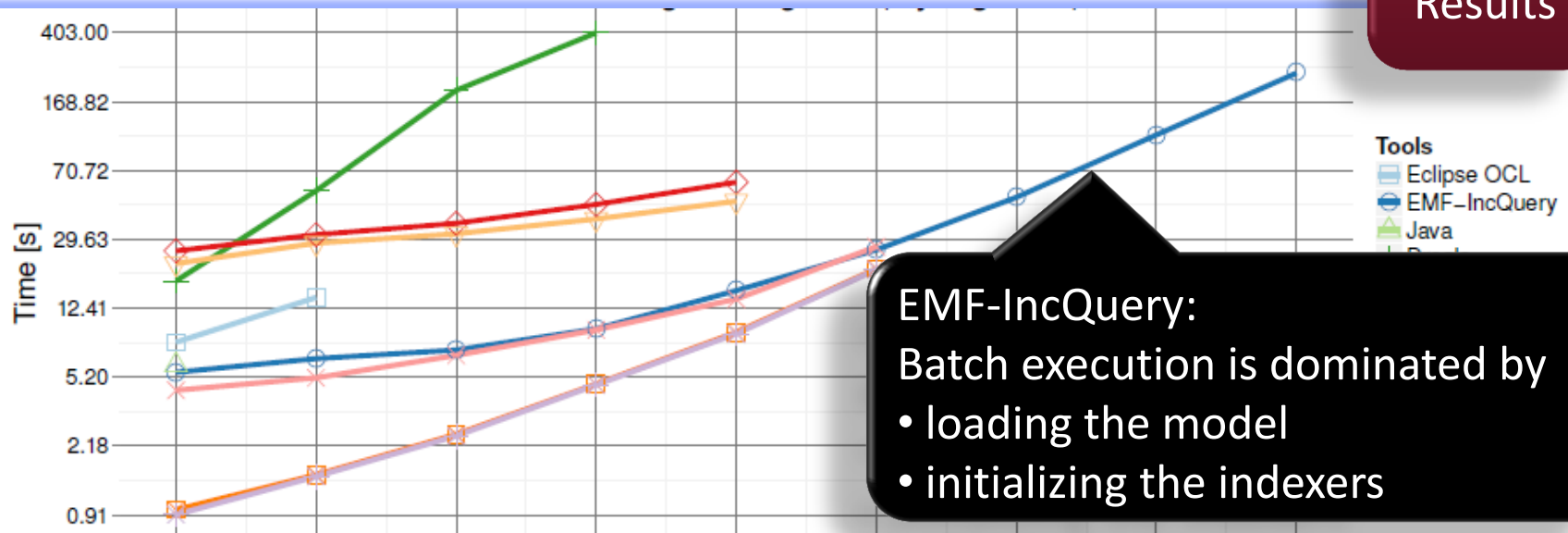
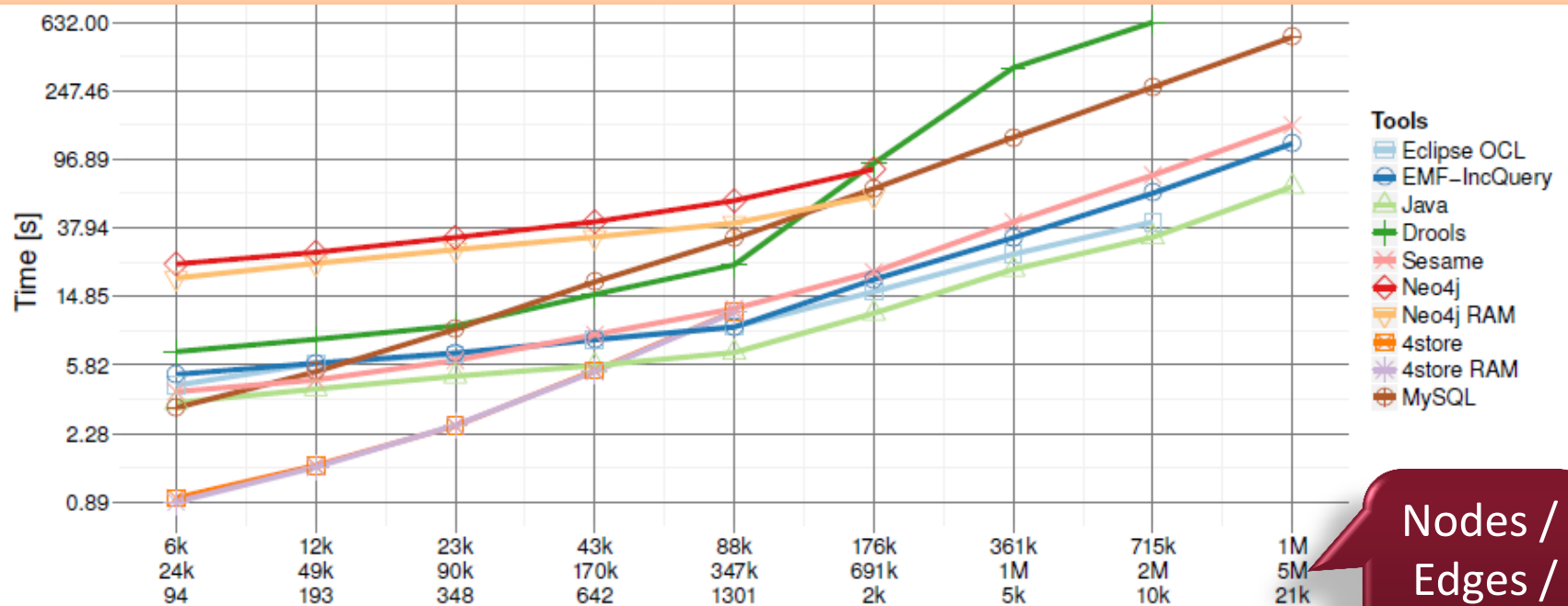


RouteSensor



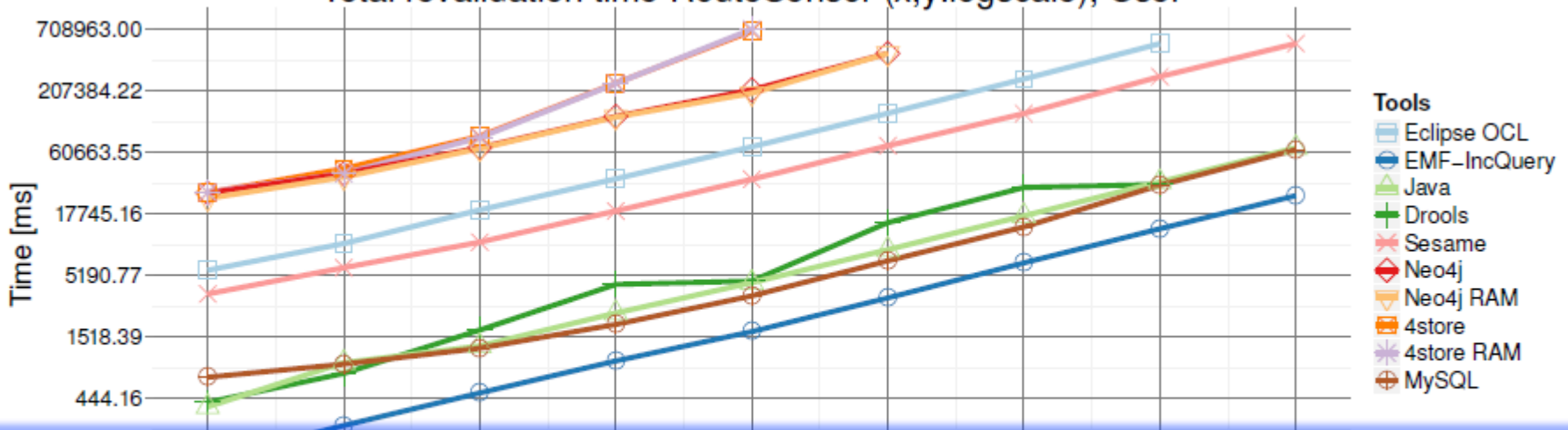
SemaphoreNeighbor

Batch validation runtime

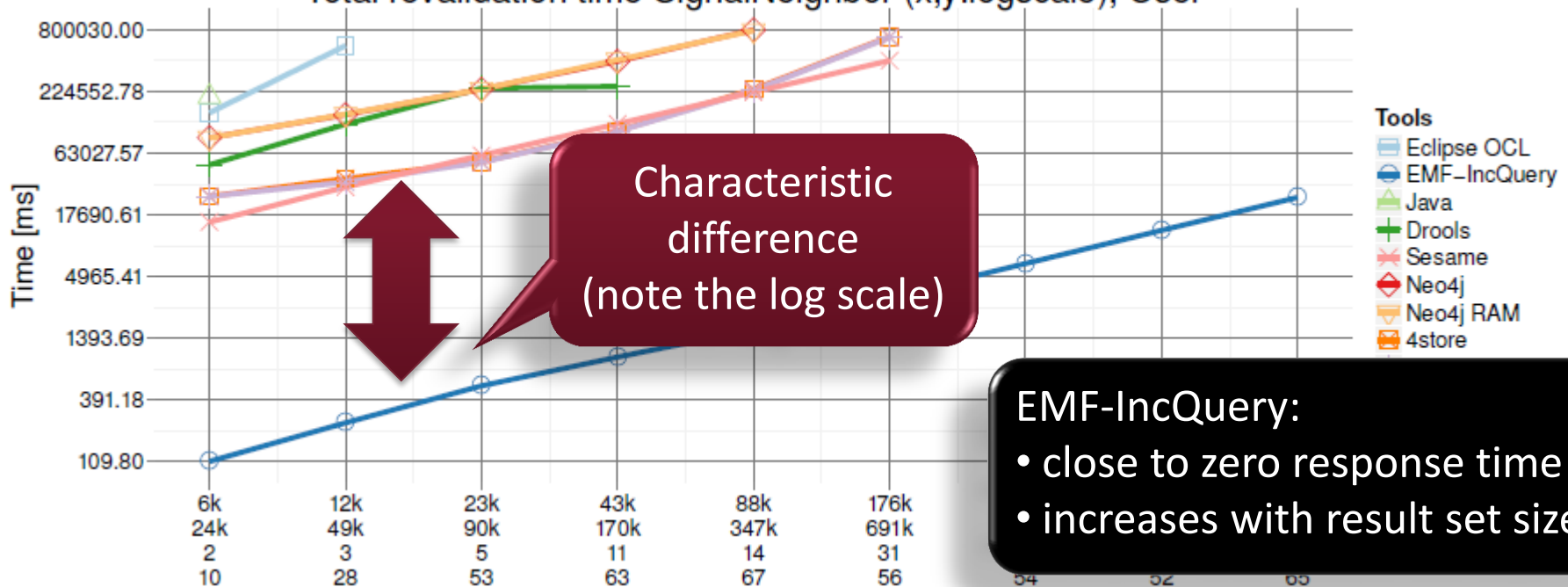


Re-validation time (complex queries)

Total revalidation time RouteSensor (x,y:logscale), User



Total revalidation time SignalNeighbor (x,y:logscale), User

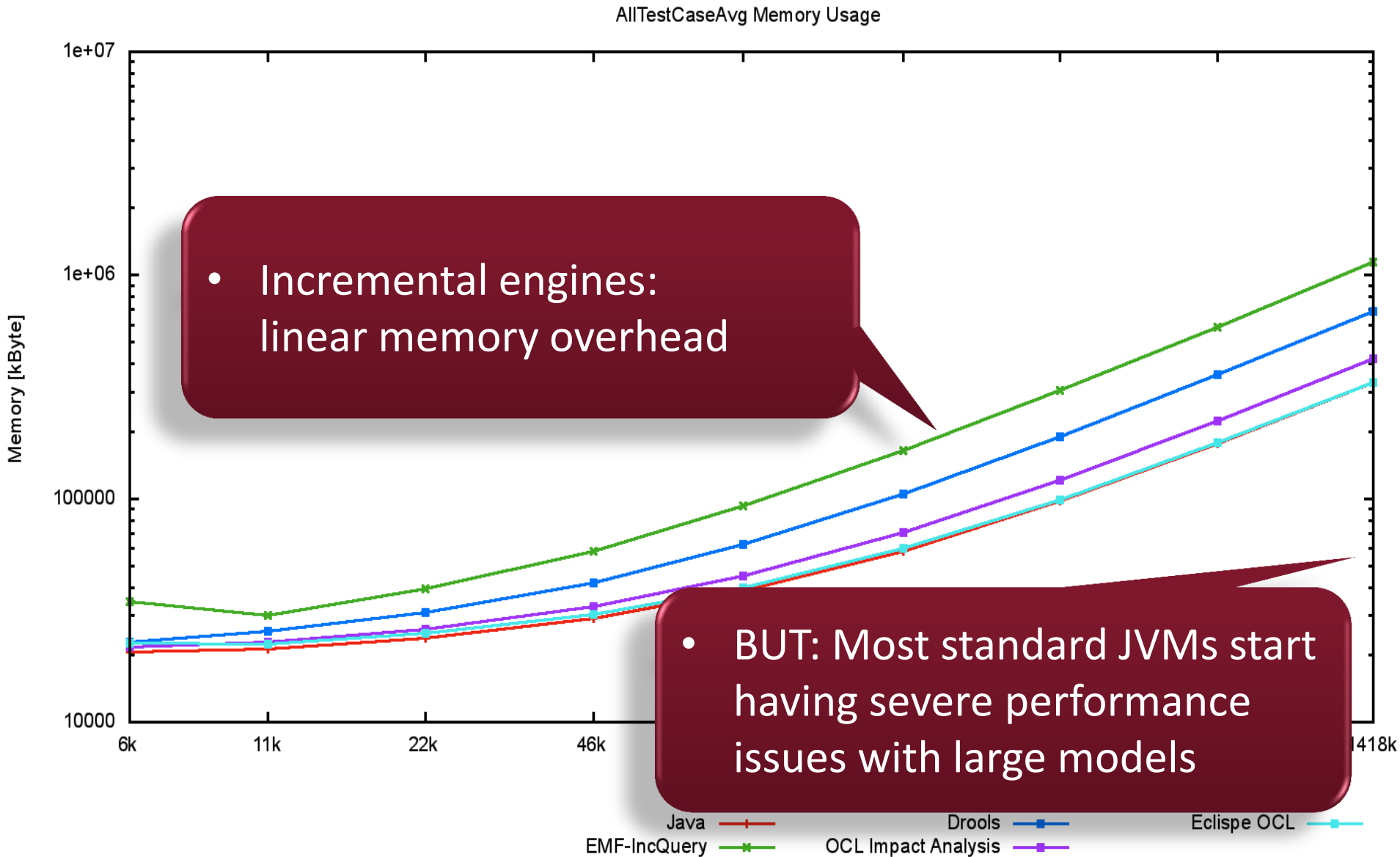


Characteristic difference
(note the log scale)

EMF-IncQuery:

- close to zero response time
- increases with result set size

Memory usage



See also a MT benchmark: <https://github.com/viatra/incquery-examples-cps/>

Performance benchmarks

<https://github.com/viatra/incquery-examples-cps>

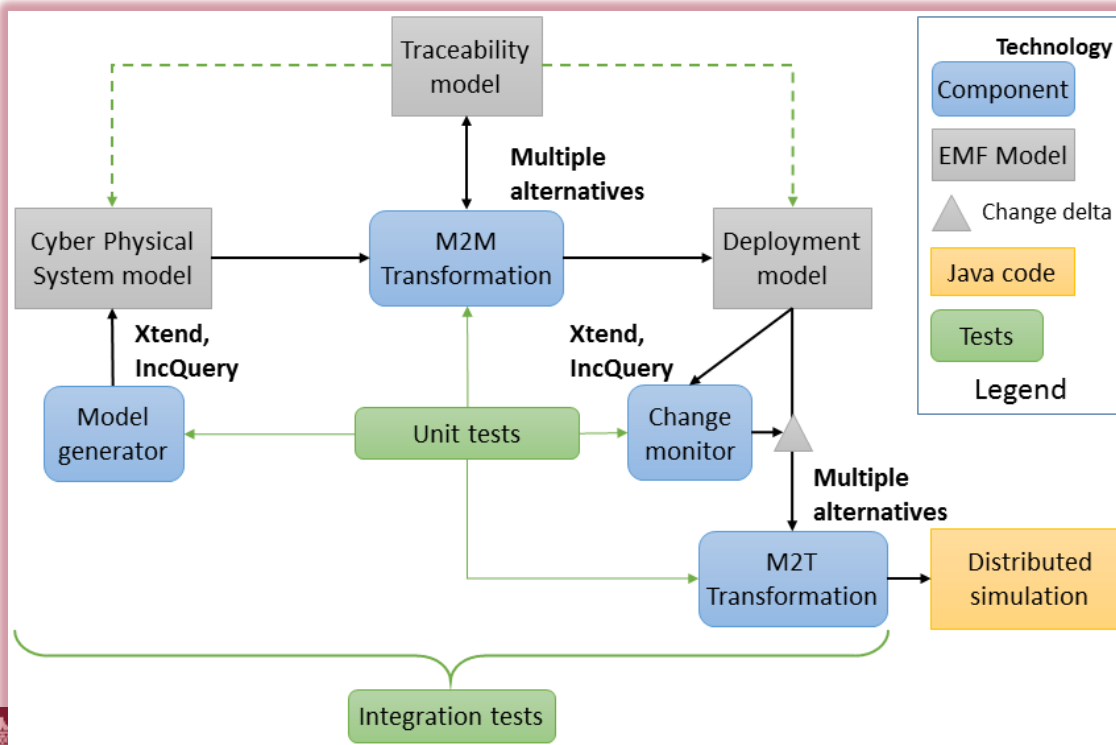
CPS Reallocation Benchmark

■ Benchmark setup

- Rule-based redeployment for cloud-based CPS
 - Model generator + Unit tests
 - M2M + M2T transformations

■ Different target architecture / platform

- Industrial (Low-Synch)
- Client-Server
- Publish-Subscribe



Test Scenario

- Different transformation variants
 - Batch
 - Xtend (2 versions)
 - IncQuery+Xtend
 - Incremental
 - Dirty (2 approaches)
 - Explicit traceability
 - Query-driven
 - Change-driven (VIATRA-EVM)
- Executions
 - First transformation execution
 - Small modification + (re)execution
- Environment
 - New machine with 16 GB RAM
- Parameters
 - 10 GB Heap
 - Maximum 10 minutes execution times for complete chain

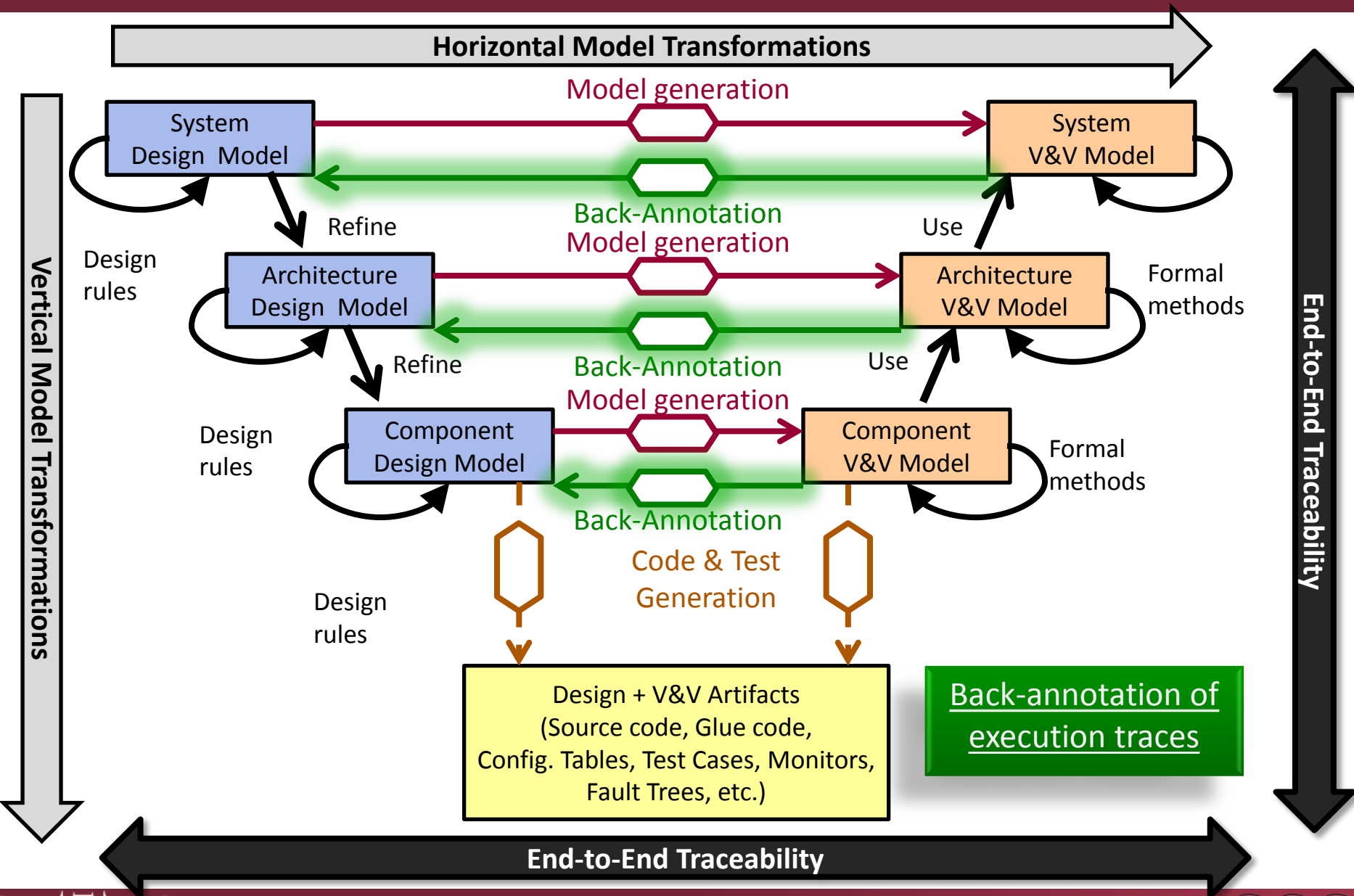
Scale	SRC Objects	SRC References	TRG Objects	TRG References	Trace Objects	Trace References	SUM Objects	SUM References
1	395	772	366	736	354	720	1 115	2 228
2	780	1544	732	1472	708	1440	2 384	4 891
4	1560	3088	1464	2944	1416	2880	4 750	10 725
8	3120	6176	2928	5888	2832	5760	10 124	29 739
16	6240	12352	5856	11776	5664	11520	22 056	115 824
32	12480	24704	11712	23552	11328	23040	50 319	651 623
64	24960	49408	23424	47104	22656	46080	125 703	4 556 465

Trace model's size
similar to target model

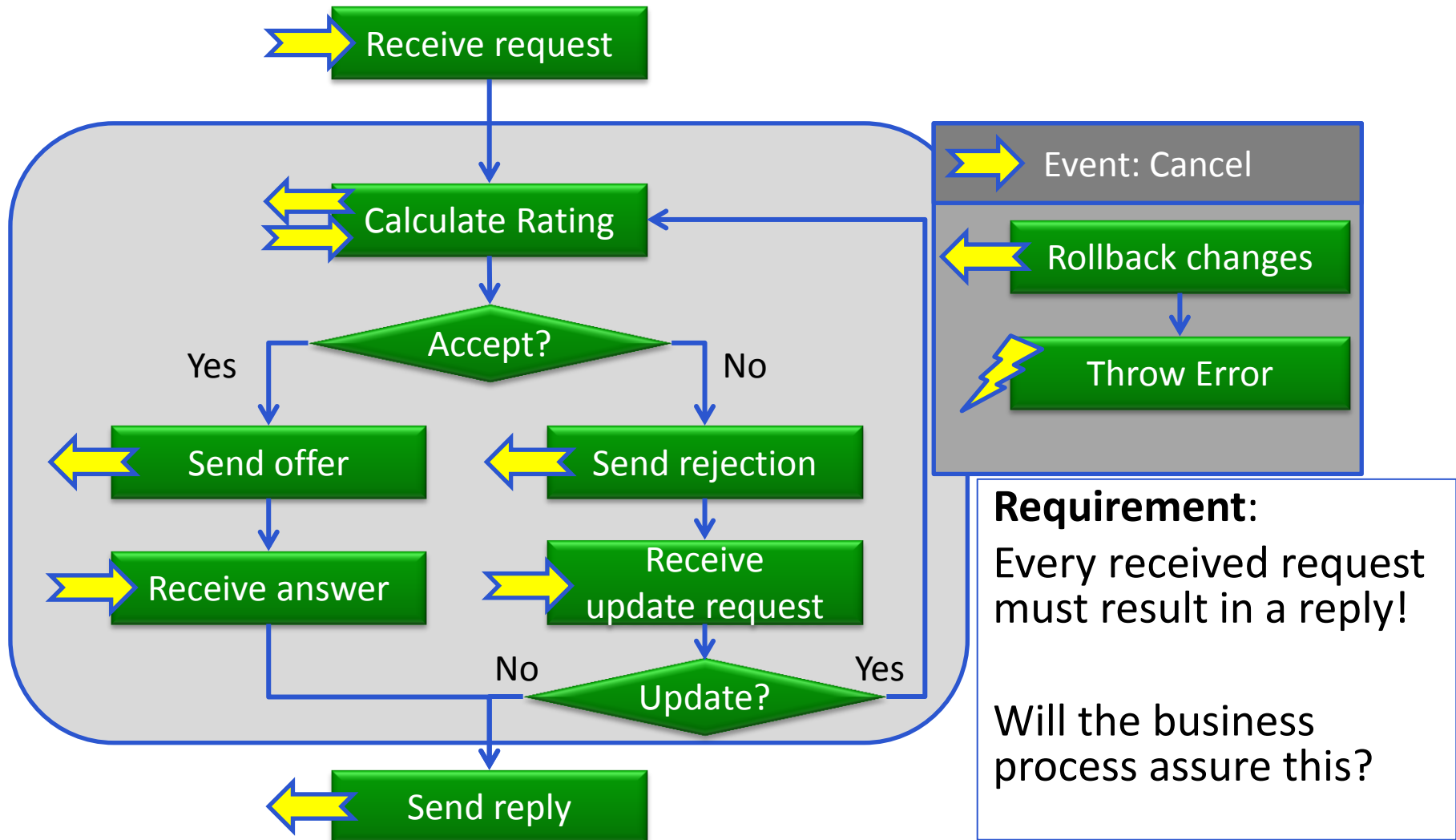
Back-annotation of Verification Results

Based on Ábel Hegedüs' PhD thesis

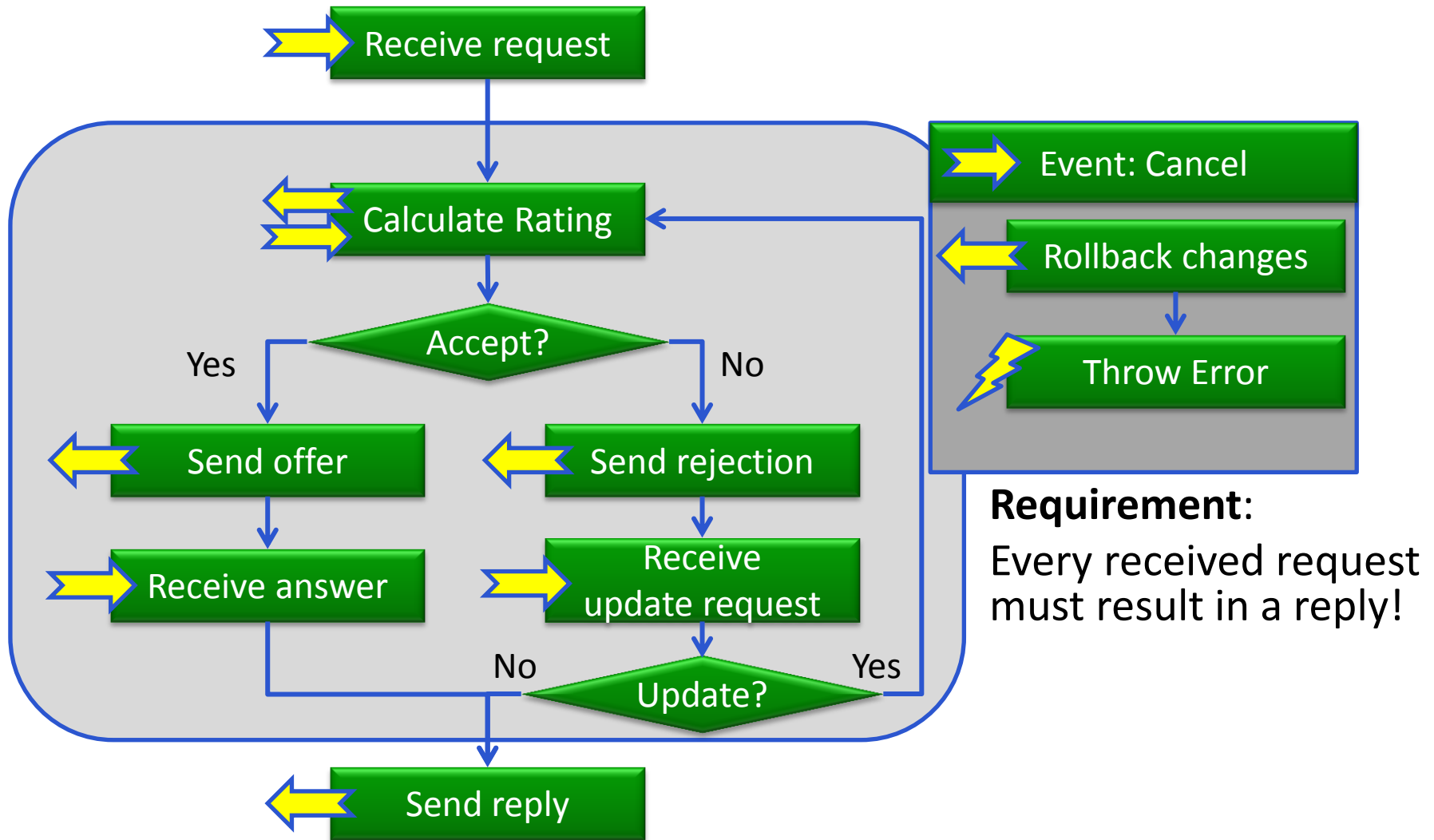
Overview: Back-Annotation of Execution Traces



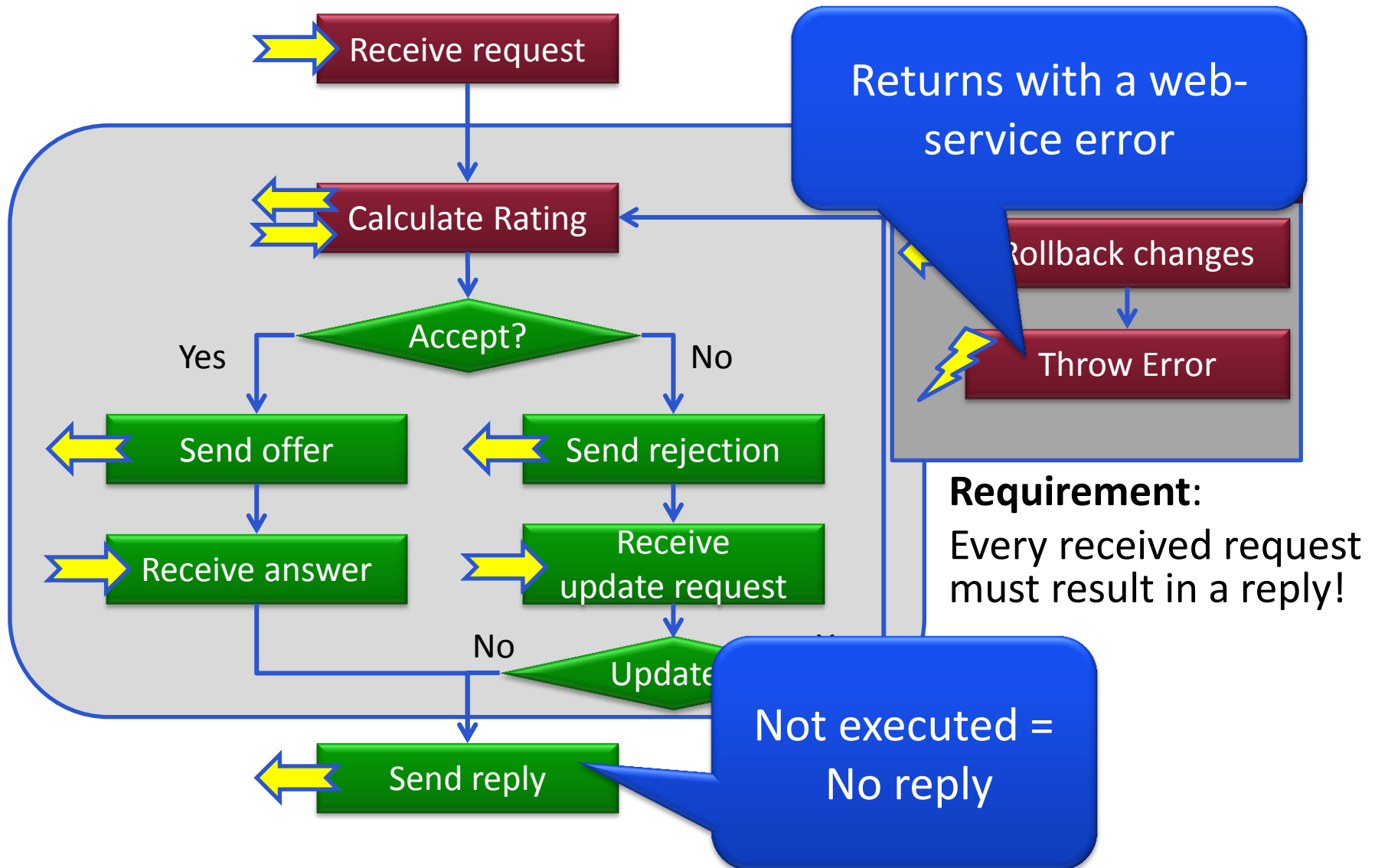
Model Analysis: Motivation for BPEL



Motivating scenario (cont.)



Motivating scenario (cont.)

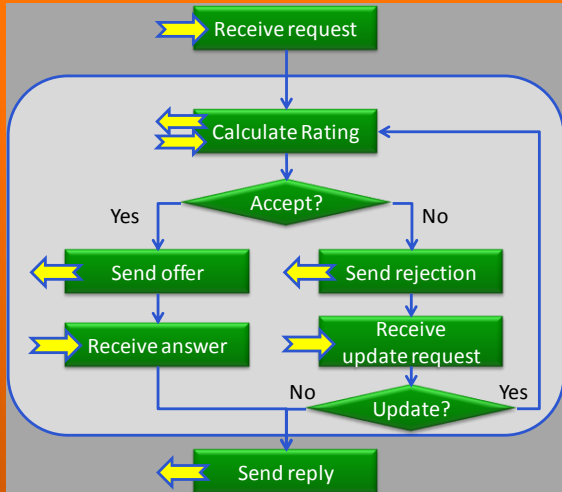


Model Based Analysis

System design

Mathematical analysis

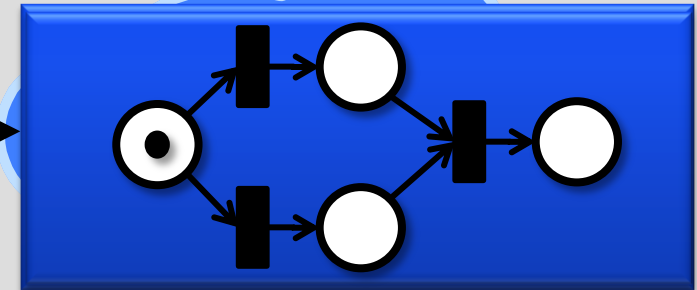
High-level System Model



Model generation

Fix problem

List of inconsistencies



Step 9:

--- System Variables (assignments) ---

login_receive16 = receiveFinished

var_creditProcess_loginData[creditProcess] = variableWritten

Counter-example / Execution traces

Counterexample:

=====

Path

=====

Step 0:

--- System Variables (assignments) ---

ba-pc!1 = 1

Accept_sequence36 = sequenceNotYetStartable

calculateOverAllRating_invoke33 = invokeNotYetStartable

Cancel_sequence22 = sequenceNotYetStartable

createNewCreditRequest_receive32 = receiveNotYetStartable

creditProcess_Process = scopeNotYetStartable

creditProcess_ProcessFaultCont = faultName_invalidVariables

creditProcess_ProcessFaultVar =

varType_tns_creditProcessRequestMessage

creditProcess_ProcessFinished = scopeNotFinished

cycleCore_sequence31 = sequenceNotYetStartable

Decline_sequence41 = sequenceNotYetStartable

else40 = elseNotYetStartable

getCustomerData_invoke17 = invokeNotYetStartable

login_receive16 = receiveNotYetStartable

Login_sequence15 = sequenceNotYetStartable

logout_invoke46 = invokeNotYetStartable

main_scope18 = scopeNotYetStartable

main_scope18FaultCont = faultName_invalidVariables

main_scope18FaultVar =

varType_tns_creditProcessResponseMessage

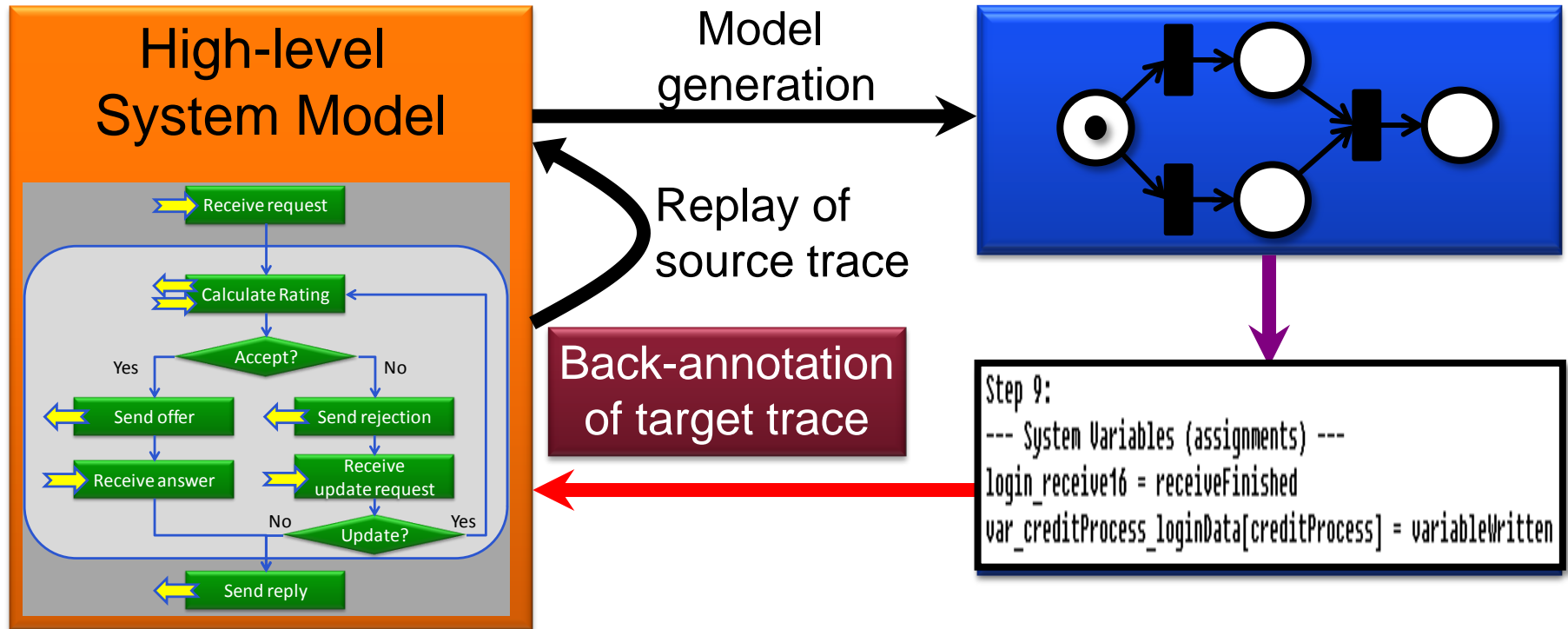
main_scope18Finished = scopeNotFinished

main_sequence14 = sequenceNotYetStartable

offer_reply38 = replyNotYetStartable



Back-Annotation of Counter Example Traces



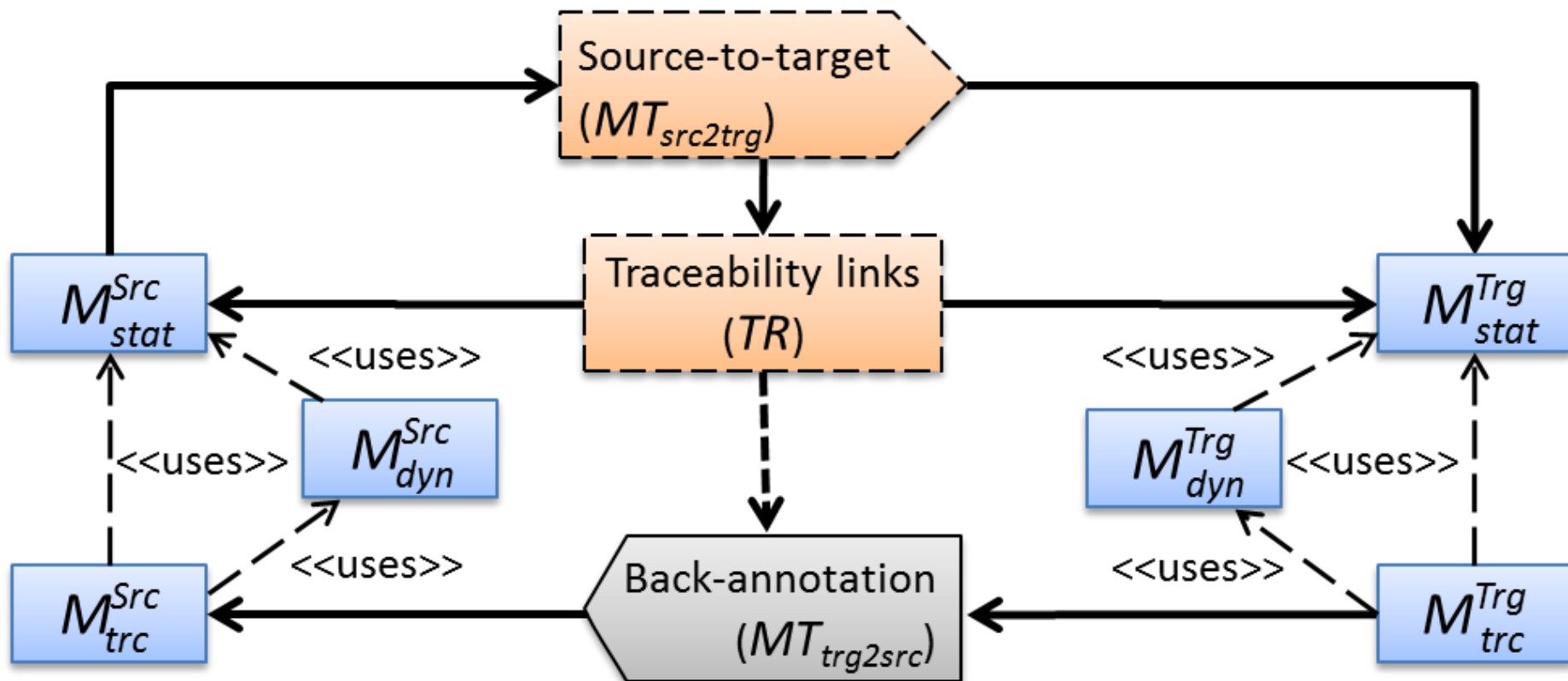
Overview of Back-annotation

Legend:

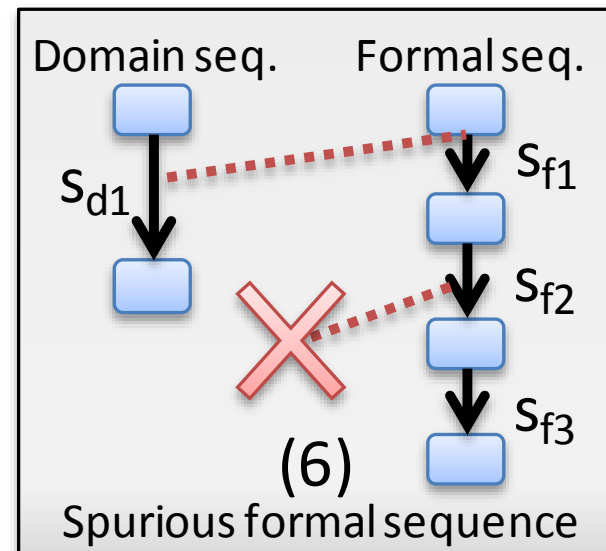
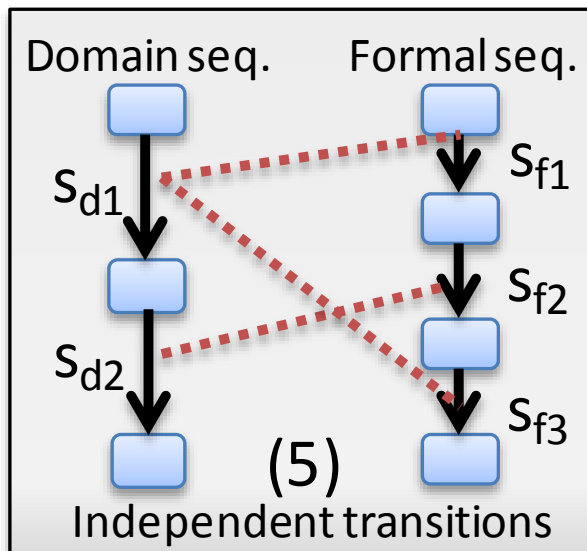
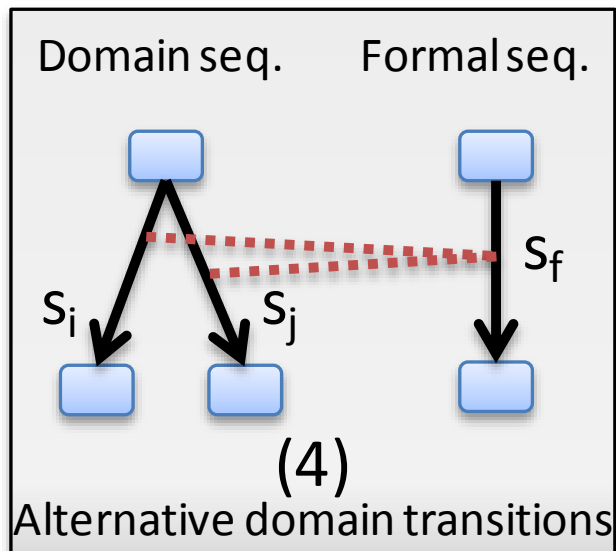
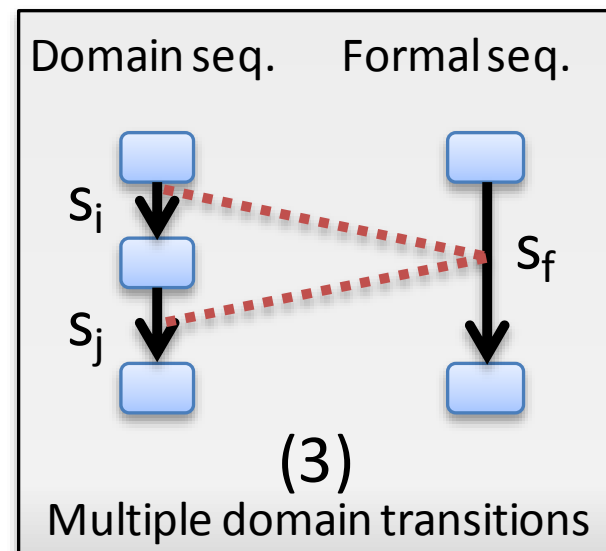
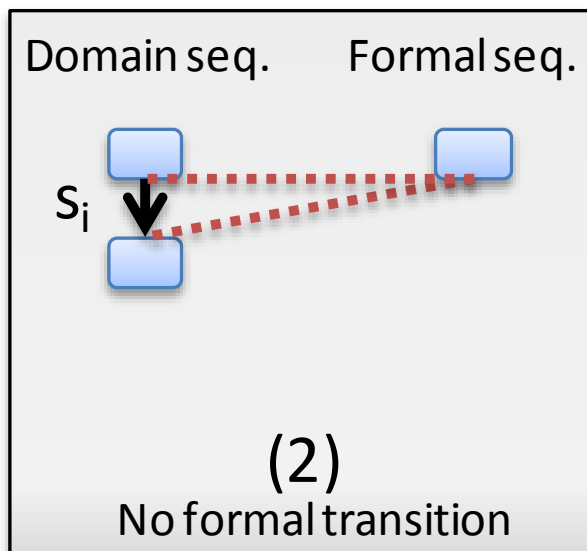
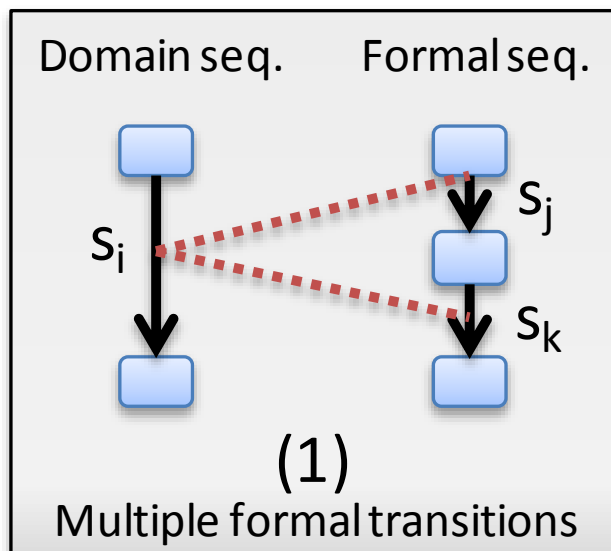
Model

Forward
Transformation

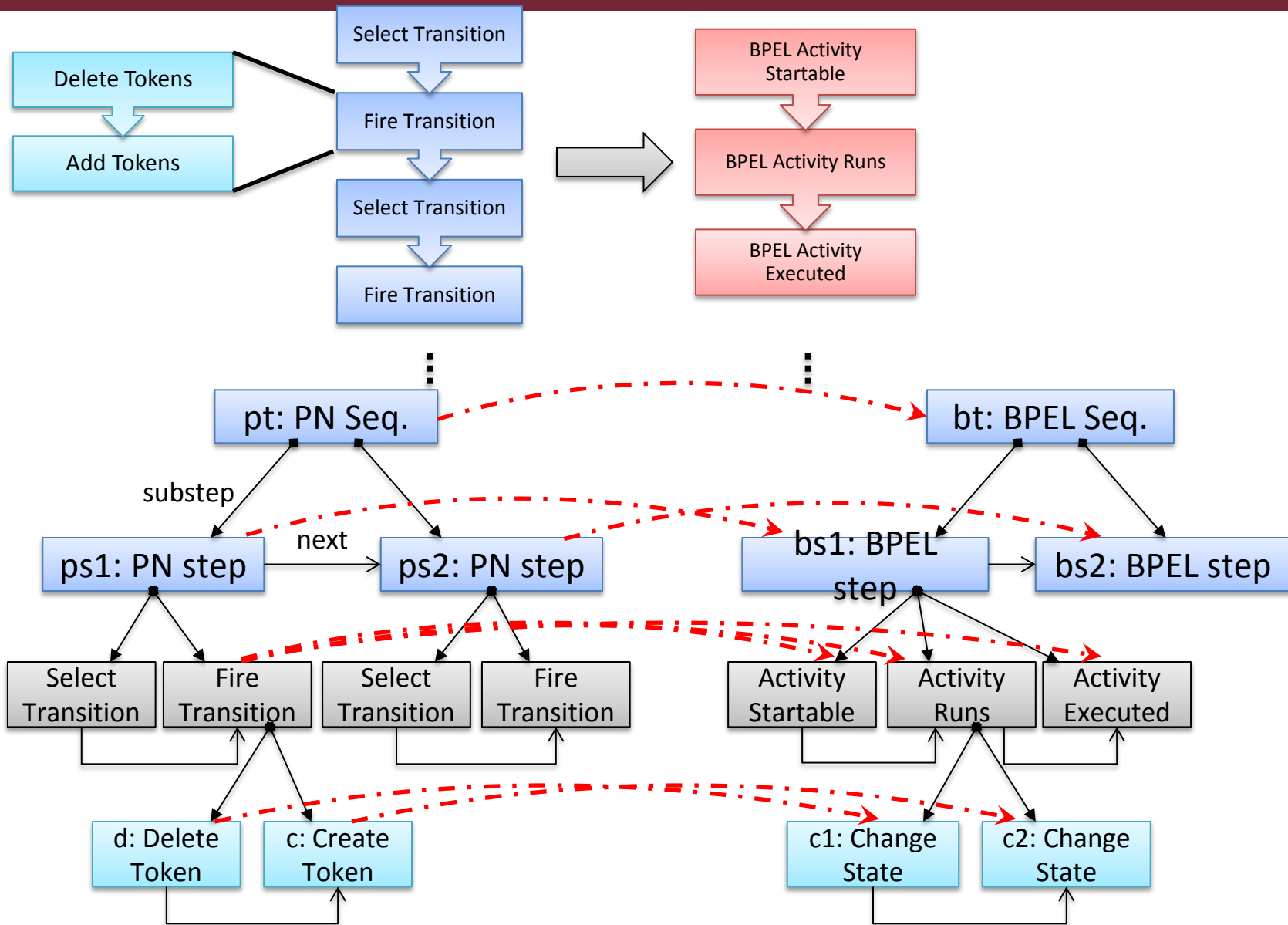
Back-annotation
Transformation



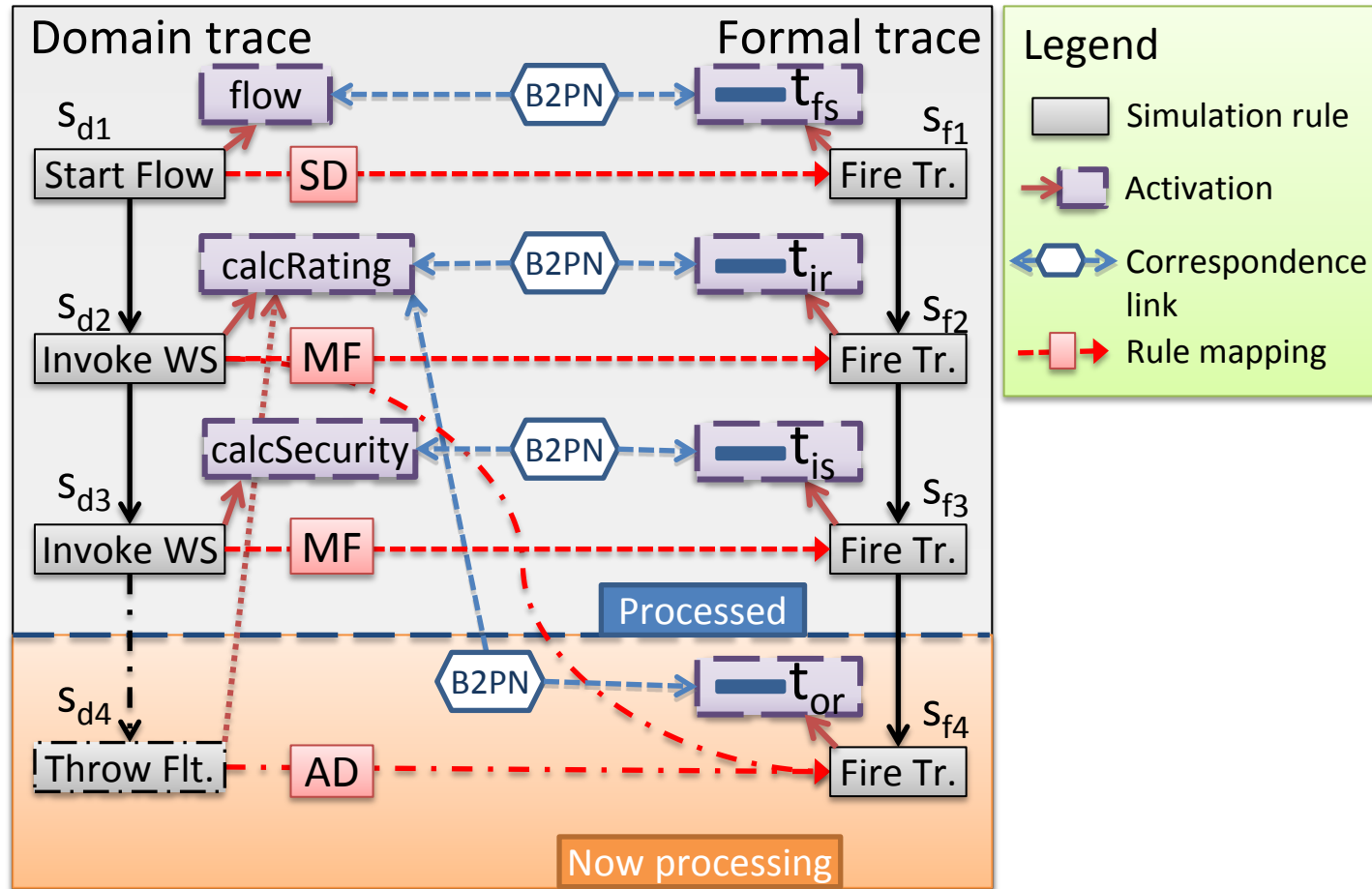
Challenges of Back-annotation



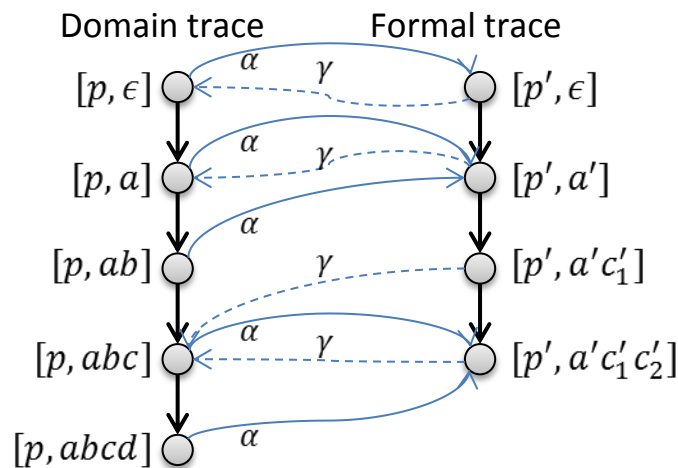
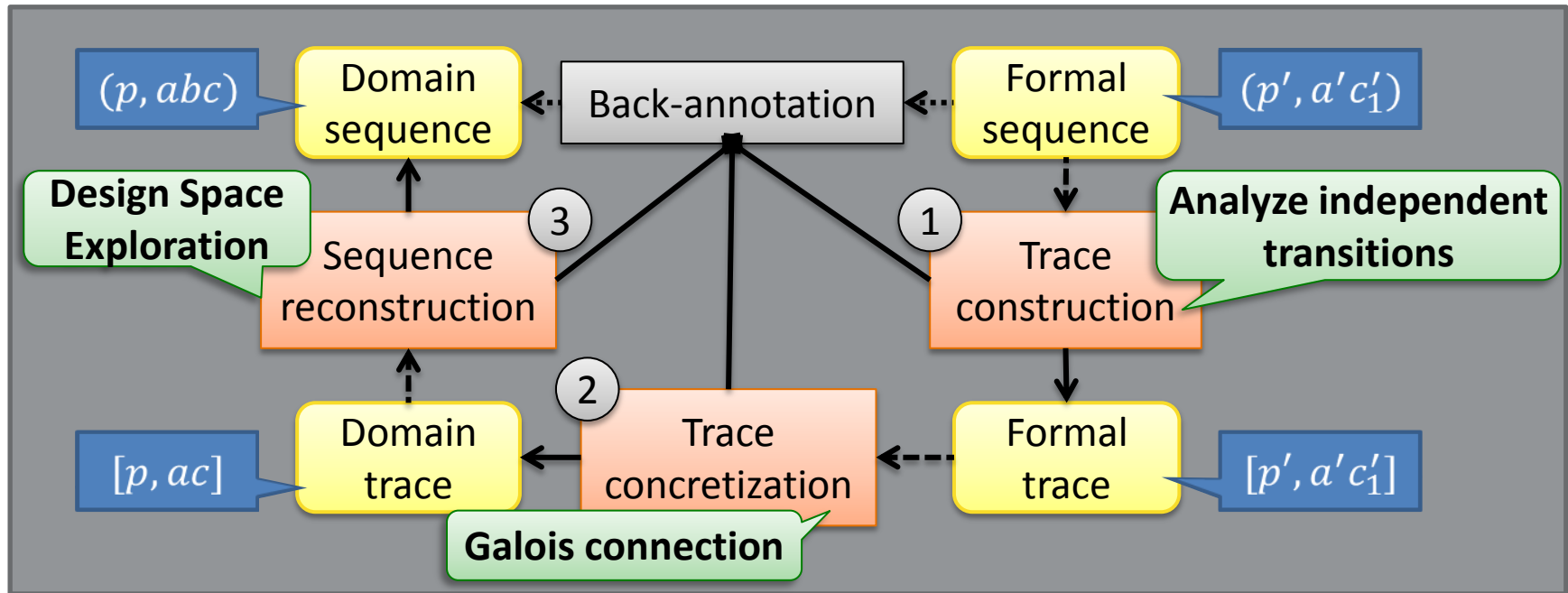
Example: Back-Annotation



Example: Back-annotation



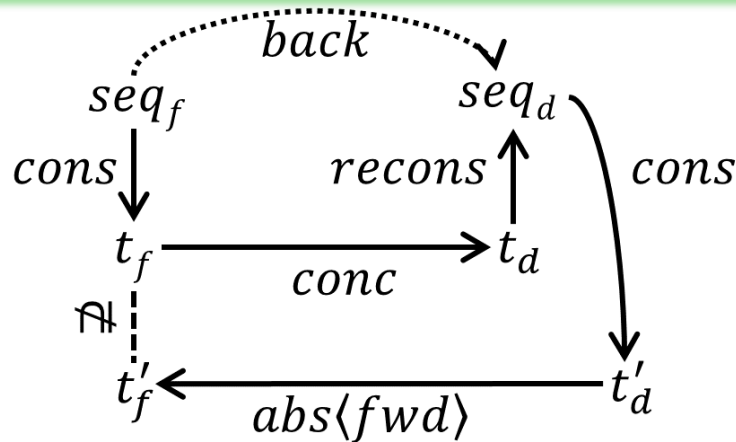
Our Back-Annotation Approach



Semantic Properties of Back-Annotation

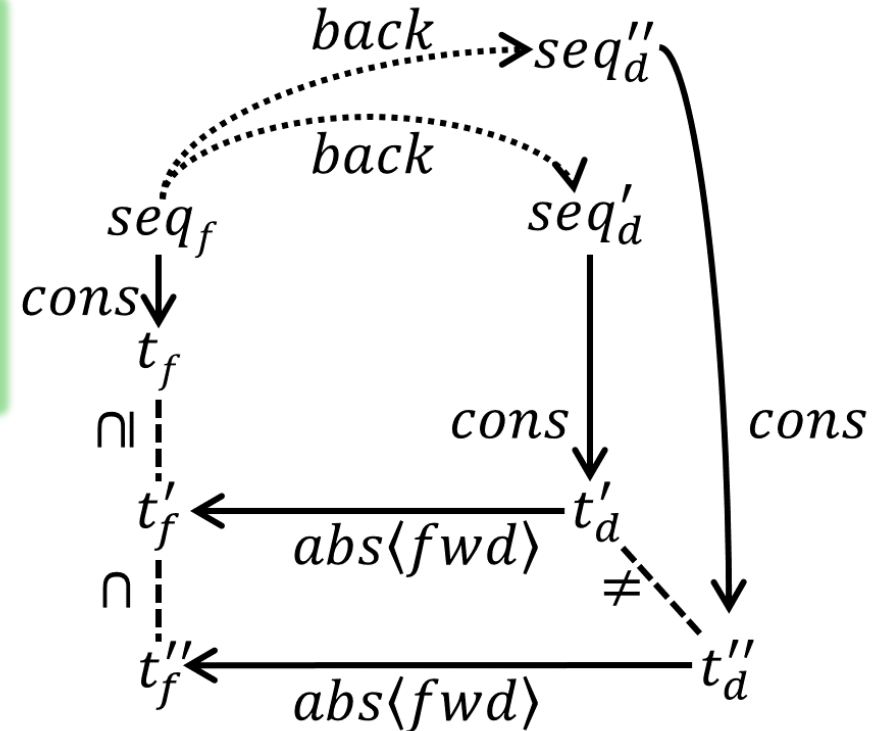
Correctness

Definition 16. The *correctness* of a forward transformation is defined as a simulation property between domain and formal traces implied by the abstraction function $abs\langle fwd \rangle$. For each (stuttering equivalent) domain trace $t_d = [G_d, w_d]$, there is a uniquely defined formal trace $t_f = [G_f, w_f]$ with $t_f = abs\langle fwd \rangle(t_d)$.



Definition 17. A back-annotation $back : seq_f \rightarrow 2^{seq_d}$ is *correct* wrt. a correct abstraction function $abs\langle fwd \rangle$ if for each $seq_f = (G_f, w_f)$ and $seq_d = (G_d, w_d)$ with $seq_d \in back(seq_f)$, then $[G_f, w_f] \subseteq abs\langle fwd \rangle([G_d, w_d])$.

Minimality



Definition 18. A back-annotation $back : seq_f \rightarrow 2^{seq_d}$ is *minimal* wrt. a correct abstraction function $abs\langle fwd \rangle$ if for each $seq_f = (G_f, w_f)$ and $seq_d = (G_d, w_d)$ with $seq_d \in back(seq_f)$, there does not exist another domain sequence $seq'_d = (G_d, w'_d)$ with $seq'_d \in back(seq_f)$ such that $[G_f, w_f] \subseteq abs\langle fwd \rangle([G_d, w'_d]) \subset abs\langle fwd \rangle([G_d, w_d])$.