

$$\boxed{12} \quad \sum_{n=1}^{\infty} \frac{3^n}{n} (3-2x)^n = \sum_{n=1}^{\infty} \underbrace{\frac{(-2)^n \cdot 3^n}{n}}_{a_n} \cdot \left(x - \frac{3}{2}\right)^n \quad x_0 = \frac{3}{2} \quad \textcircled{3}$$

$$\sqrt[n]{|a_n|} = \sqrt[n]{\frac{6}{n}} \rightarrow 6 \Rightarrow R = \frac{1}{6} \quad \textcircled{3}$$

Viegpunkt 1:  $x_1 = x_0 + R = \frac{3}{2} + \frac{1}{6} = \frac{10}{6} = \frac{5}{3}$  esetén

$$\sum_{n=1}^{\infty} \frac{(-6)^n}{n} \cdot \underbrace{\left(\frac{10}{6} - \frac{3}{2}\right)^n}_{-\frac{1}{6}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \text{konvergens, nélküliz.} \quad \textcircled{2}$$

$$x_2 = x_0 - R = \frac{3}{2} - \frac{1}{6} = \frac{8}{6} = \frac{4}{3} \quad \text{esetén}$$

$$\sum_{n=1}^{\infty} \frac{(-6)^n}{n} \cdot \underbrace{\left(\frac{4}{3} - \frac{3}{2}\right)^n}_{-\frac{1}{6}} = \sum_{n=1}^{\infty} \frac{1}{n} = \infty \quad \text{diverg.} \quad \textcircled{2}$$

Tehát K.T. =  $\left[\frac{4}{3}, \frac{5}{3}\right]$   $\quad \textcircled{2}$

$$\boxed{8} \quad 2, a, \quad f_1(x) = \ln(1+x) ; \quad x_0 = 0 \quad f_1'(x) = \frac{1}{1+x}, \quad \text{tehát}$$

$$f_1(x) = \underbrace{f_1(0)}_{0} + \int_{t=0}^{x} \frac{1}{1+t} dt = \int_{t=0}^{x} \left( \sum_{n=0}^{\infty} (-t)^n \right) dt = \sum_{n=0}^{\infty} \int_{t=0}^{x} (-t)^n dt =$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} = \underbrace{\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}}_{\text{, ha } |x| < 1}. \quad \textcircled{5}$$

Viegpunktban a Taylor-sor konvergenciáját vizsgáljuk:

$$x_1 = -1 - \ln : \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{-1}{n} = -\infty \quad \text{div.}$$

$$x_2 = +1 - \ln : \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = \text{konv. (Leibniz)}$$

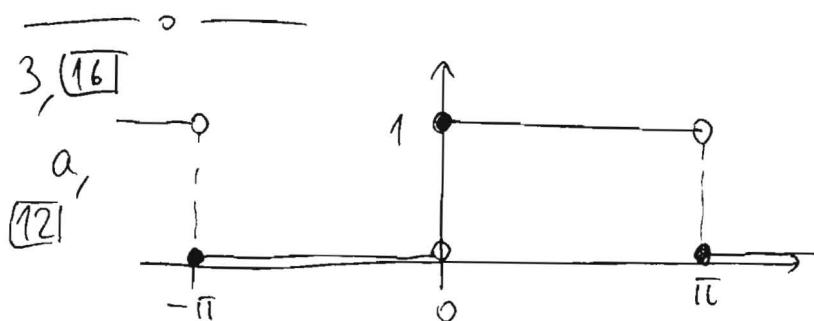
Tehát K.T. =  $(-1, +1)$   $\quad \textcircled{3}$

(- 2 -)

2, b, 7  $f_2(x) = \frac{1}{3+5x} = \frac{1}{3+5(x-2)+10} = \frac{1}{13} \cdot \frac{1}{1-\left(-\frac{5}{13}(x-2)\right)} =$   
 $x_0 = 2$   $= \frac{1}{13} \sum_{n=0}^{\infty} \left(\frac{-5}{13}\right)^n (x-2)^n$  ④, ha  $|q| < 1$ , aran ha ③  
 $|x-2| < \frac{13}{5}$ , K.T. =  $\left(2-\frac{13}{5}, 2+\frac{13}{5}\right)$   
 $= \left(-\frac{3}{5}, \frac{23}{5}\right)$

2, c, 7  $f_3(x) = \frac{1}{3+5x^2} = \frac{1}{3} \cdot \frac{1}{1-\left(-\frac{5}{3}x^2\right)}$   $= \frac{1}{3} \sum_{n=0}^{\infty} \left(-\frac{5}{3}\right)^n x^{2n}$ , ha  $|q| < 1$   
 $x_0 = 0$  aran ha  $|x^2| < \frac{3}{5}$ ; K.T. =  $\left(-\sqrt{\frac{3}{5}}, +\sqrt{\frac{3}{5}}\right)$  ③

2, d, 7  $f_4(x) = e^{2x} \quad ch(2x) = e^{2x} \frac{e^{2x} + e^{-2x}}{2} = \frac{1}{2} e^{4x} + \frac{1}{2} =$   
 $x_0 = 1$   $= \frac{1}{2} + \frac{1}{2} e^4 \cdot e^{4(x-1)} = \frac{1}{2} + \frac{1}{2} \cdot e^4 \sum_{n=0}^{\infty} \frac{4^n}{n!} (x-1)^n$  ⑤  
K.T. =  $\mathbb{R}$ . ②



12  $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = 1$  ②

$f(x) - \frac{1}{2}$  paralel, tebah  $a_m = 0$ , ha  $m \geq 1$ . ③

(Uppr  $a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(mx) dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos(mx) dx = \frac{1}{\pi} \left[ \frac{\sin(mx)}{m} \right]_{-\pi}^{\pi} = 0$ )

$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(mx) dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin(mx) dx = \frac{1}{\pi} \left[ \frac{-\cos(mx)}{m} \right]_{-\pi}^{\pi} = \frac{1 - (-1)^m}{m \cdot \pi} = \begin{cases} \frac{2}{m\pi}, & \text{ha } m \text{ pris.} \\ 0, & \text{ha } m \text{ gen.} \end{cases}$  ③

$$a_0 = 1, \quad a_m = 0, \quad b_m = \begin{cases} \frac{2}{m\pi}, & \text{for } m \text{ odd} \\ 0, & \text{for } m \text{ even} \end{cases} \quad (m \geq 1)$$

$$\left| \begin{aligned} \phi(x) &= \frac{1}{2} + \sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} \sin((2k+1)x) = \\ &= \frac{1}{2} + \frac{2}{\pi} \sin x + \frac{2}{3\pi} \sin(3x) + \dots \end{aligned} \right\} \quad (4)$$

b, Dirichlet - Test etablichen

$$(4) \quad \phi(x) = \begin{cases} f(x), & \text{for } x \neq k\pi \\ \frac{1}{2}, & \text{for } x = k\pi \end{cases} \quad (4)$$

$$(4) \quad (22) \quad f(x, y) = \begin{cases} \frac{x \sin(y)}{\sqrt{x^2+y^2}}, & \text{for } (x, y) \neq (0, 0) \\ 0, & \text{for } (x, y) = (0, 0) \end{cases}$$

$$(5) \quad \lim_{(x, y) \rightarrow (0, 0)} f(x, y) = \lim_{r \rightarrow 0^+} \frac{x \cos \varphi \cdot \sin(r \sin \varphi)}{x} = \underbrace{\lim_{r \rightarrow 0^+} \cos \varphi}_{\text{const.}} \cdot \underbrace{\sin(r \sin \varphi)}_{\substack{\uparrow \\ 0}} = 0$$

Teilt f folgbar an originale!

$$(6) \quad f'_x(x, y) = \frac{y \cdot \sqrt{x^2+y^2} - x \sin(y) \cdot \frac{xx}{\sqrt{x^2+y^2}}}{x^2+y^2} \quad (3) \quad = \frac{y^2 \sin(y)}{(x^2+y^2)^{3/2}}$$

$$f'_y(x, y) = \frac{x \cos(y) \sqrt{x^2+y^2} - x \sin(y) \cdot \frac{xy}{\sqrt{x^2+y^2}}}{x^2+y^2} \quad (3) \quad (\text{for } (x, y) \neq (0, 0))$$

$$(5) \quad f'_x(0, 0) = \lim_{x \rightarrow 0} \frac{f(x, 0) - f(0, 0)}{x} = \lim_{x \rightarrow 0} \frac{0 - 0}{x} = 0 \quad (3)$$

$$f'_y(0, 0) = \lim_{y \rightarrow 0} \frac{f(0, y) - f(0, 0)}{y} = \lim_{y \rightarrow 0} \frac{0 - 0}{y} = 0 \quad (2)$$

$$\boxed{6} \quad \frac{df(0,0)}{dx} = \lim_{t \rightarrow 0^+} \frac{f(t,0) - f(0,0)}{t} \stackrel{(2)}{=} \lim_{t \rightarrow 0^+} \frac{1}{t} \cdot \frac{\sqrt{2}x \sin(\sqrt{2}t)}{t} =$$

$$= \lim_{t \rightarrow 0^+} 2 \cdot \frac{\sin(\sqrt{2}t)}{\sqrt{2}t} = \underline{\underline{2}} \quad (2)$$

5, (14)  $f(x,y) = \frac{x(2y+1)}{e^y}; D_f = \mathbb{R}^2$

a, (7)  $f'_x(x,y) = \frac{2y+1}{e^y} \quad (2)$

$$f'_x(x,y) = \frac{2xe^y - x(2y+1)e^y}{e^{2y}} = \frac{x - 2xy}{e^y} \quad (2)$$

A parciális deriváltai mindenütt láthatók és polinomok, tehát  $\mathbb{R}^2$ -n  $\exists \text{ grad } f(x,y) = \begin{bmatrix} f'_x \\ f'_y \end{bmatrix}$ . (3)

6  $f(x,y) = g(2xy + y^2)$

7  $f'_x(x,y) = 2y g'(2xy + y^2); f'_y = (2x+2y)g'(2xy + y^2) \quad (2)$

$$f''_{yy}(x,y) = 2g'(2xy + y^2) + 4(x+y)^2g''(2xy + y^2) \quad (3)$$

5/b, (7)  $\|v\| = \sqrt{3^2 + (-4)^2} = 5; \underline{e} = \frac{v}{\|v\|} = \begin{bmatrix} 3/5 \\ -4/5 \end{bmatrix} \quad (2)$

$$\text{grad } f(P) = \begin{bmatrix} f'_x(2,0) \\ f'_y(2,0) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (2)$$

$$\left. \frac{df}{dx} \right|_P = \underline{e} \cdot \text{grad } f(P) = \begin{bmatrix} 3/5 \\ -4/5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{3}{5} - \frac{8}{5} = \underline{\underline{-1}} \quad (1)$$

Reihenentwicklungen:

(-3-1)

7, 10  $f(x, y) = \ln(2x + 3y^2)$   $P(-1, 1)$

Approximation:  $z - f(P) = f'_x(P)(x - x_0) + f'_y(P)(y - y_0)$  ②

Merk:  $f(P) = f(-1, 1) = \ln(-2 + 3) = \ln 1 = 0$  ②

$$f'_x(x, y) = \frac{2}{2x + 3y^2} ; \quad f'_x(-1, 1) = 2 \quad ②$$

$$f'_y(x, y) = \frac{6y}{2x + 3y^2} ; \quad f'_y(-1, 1) = 6 \quad ②$$

Einfach mit:  $z = 2(x+1) + 6(y-1) = 2x + 6y - 4$  ②

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8, 10 Binomialreihe verfeinert herstellen:

$$f(x) = \sqrt{1+x} = (1+x)^{1/2} = \sum_{n=0}^{\infty} \binom{1/2}{n} x^n \quad ⑤$$

$$\begin{aligned} T_3(x) &= \binom{1/2}{0} + \binom{1/2}{1} x + \binom{1/2}{2} x^2 = 1 + \frac{1}{2} x + \frac{\frac{1}{2} \cdot (-\frac{1}{2})}{2} x^2 = \\ &= 1 + \frac{x}{2} - \frac{x^2}{8} \quad ⑤ \end{aligned}$$